

# **ELECTROMAGNETISM**

## **in MATTER**

**M Apostol**

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Moon's Problem 2005

Hadronization of the Quark-Gluon Plasma 2006

Curved Space, Covariance, Motion and Quantization 2007

Coherence in Matter Interacting with Radiation 2008

Electromagnetism in Matter 2009

NanoScience and NanoTechnology 2004 (Pitesti)

## **PART I**

### **Electromagnetism**

Fresnel, Coulomb, Ampere, Gauss, Weber, Kirchhoff, Hertz, Poincare,  
Lorentz    **Integral Representations**

Faraday, Maxwell    **Differential Equations**

Two lines of thought

## Motivation

**Refraction** Fresnel-Huygens theory; **extinction theorem**

"Effective Medium Permittivity" Theory: **the problem of  $\varepsilon$**

**Phenomenological** response functions:  $\varepsilon, \mu, \sigma$

The role of matter, its dynamics: **plasmons, polaritons; diffraction**

The **enhancement** in structures with finite geometries

**Metamaterials**

## Maxwell Equations in Vacuum

$$\mathit{div}\mathbf{E} = 4\pi\rho \quad (\text{Gauss})$$

$$\mathit{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t} \quad (\text{Faraday})$$

$$\mathit{div}\mathbf{H} = 0 \quad (\text{Gauss})$$

$$\mathit{curl}\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j} \quad (\text{Ampere})$$

Continuity:  $\frac{\partial\rho}{\partial t} + \mathit{div}\mathbf{j} = 0$ ; convection current ( $\mathbf{j} = \rho\mathbf{v}$ ; velocity  $\mathbf{v}$ )

## Solution

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \text{grad} \Phi, \quad \mathbf{H} = \text{curl} \mathbf{A}; \quad \text{Lorentz gauge } \text{div} \mathbf{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t} = 0$$

$$\text{Wave equations } \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} - \Delta \Phi = 4\pi \rho, \quad \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} - \Delta \mathbf{A} = \frac{4\pi}{c} \mathbf{j}$$

$$\text{Kirchhoff: } R = |\mathbf{r} - \mathbf{r}'|$$

$$\Phi(\mathbf{r}, t) = \int d\mathbf{r}' \frac{\rho(\mathbf{r}', t - R/c)}{R}$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{1}{c} \int d\mathbf{r}' \frac{\mathbf{j}(\mathbf{r}', t - R/c)}{R}$$

**Two regimes:** Retarded; Non-retarded (approximate):  $\omega/c = 1/\lambda \ll (\text{body size})^{-1}$

## **Comment on solvability**

Four Maxwell equations plus two other equations (continuity and Lorentz gauge) = six equations; only four unknown:  $\mathbf{E}$ ,  $\mathbf{H}$ ,  $\rho$ ,  $\mathbf{j}$

Actually **only two independent Maxwell equations**

The only absolute input:  $c$

## Maxwell Equations in Matter

$$\mathit{div}\mathbf{D} = 4\pi\rho_0 \text{ (Gauss)}$$

$$\mathit{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \text{ (Faraday)}$$

$$\mathit{div}\mathbf{B} = 0 \text{ (Gauss)}$$

$$\mathit{curl}\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j}_0 \text{ (Ampere)}$$

$\rho_0$  and  $\mathbf{j}_0$  external charge and current;  $\mathbf{D}$  and  $\mathbf{H}$  created by external sources

## Comment on solvability

Two new unknowns: electric displacement  $\mathbf{D}$ , magnetic induction  $\mathbf{B}$  and yet another: the diffusive current  $\mathbf{j}$  in matter (no more  $\mathbf{j} = \rho\mathbf{v}$ ).

**Same scheme of solution**, plus phenomenological relations

$$\mathbf{D} = \varepsilon\mathbf{E} , \mathbf{B} = \mu\mathbf{H} , \mathbf{j}_0 = \sigma\mathbf{E} \text{ (Ohm)}$$

Dielectric function  $\varepsilon$ , Magnetic permeability  $\mu$ , electric conductivity  $\sigma$

## Our em equations in matter

$$\mathit{div}\mathbf{E} = 4\pi\rho_0 - 4\pi\mathit{div}\mathbf{P}$$

$$\mathit{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}, \quad \mathit{div}\mathbf{H} = 0$$

$$\mathit{curl}\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi}{c}\frac{\partial\mathbf{P}}{\partial t} + 4\pi\mathit{curl}\mathbf{M} + \frac{4\pi}{c}\mathbf{j}_0$$

Maxwell equations:  $\mathbf{H} \rightarrow \mathbf{B}$ ,  $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$ ,  $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$

**Reasoning:** polarization  $P$ , polarization charge, by cont eq the polarization current  $\frac{\partial\mathbf{P}}{\partial t}$

In addition:  $\mathit{div}\mathbf{j}$  in the cont eq admits an additional current (magnetization current)  $c \cdot \mathit{curl}\mathbf{M}$  ( $\mathit{div} \cdot \mathit{curl} = 0$ )

## Solvability: Magnetic phenomena

Instead of  $\mathbf{D}$  and  $\mathbf{H}$  now we have  $\mathbf{P}$  and  $\mathbf{M}$ ; what's the gain?

The gain is that we have an additional equation of motion for  $\mathbf{M}$ :

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{H} \times \mathbf{M}$$

Gyromagnetic factor  $\gamma = \mu/\hbar$

**Comment:** Model Ampere molecular currents +spin (quantum relativist) magnetic moments (Bohr magneton  $\mu_B = e\hbar/2mc$ )

Commonly weak magnetism ( $v^2/c^2$ ): disregard

Except for ferromagnetism and related phenomena: special treatment

Still  $\mathbf{j}_0 = \sigma \mathbf{E}_{tot}$ ; disregard external sources

Have we another equation for  $\mathbf{P}$ ? Had we have  $\rightarrow$  **SOLVABILITY**

## We have another equation for P

Weak displacement field  $\mathbf{u}$ , **density disturbances**  $\delta n = -n \operatorname{div} \mathbf{u}$ , polarization charge  $\rho = -e\delta n = n e \operatorname{div} \mathbf{u}$  (as for electrons), polarization  $\mathbf{P} = -ne\mathbf{u}$  and polarization current  $\mathbf{j} = -ne \frac{d\mathbf{u}}{dt}$

Equation of motion? **Newton's Law:**

$$m \frac{d^2 \mathbf{u}}{dt^2} = -e\mathbf{E} - e\mathbf{E}_0 \quad (\text{plasma, ext el field } E_0)$$

or

$$m \frac{d^2 \mathbf{u}}{dt^2} = -e\mathbf{E} - m\omega_0^2 \mathbf{u} - e\mathbf{E}_0 \quad (\text{dielectrics, large } \omega_0)$$

or

$$m \frac{d^2 \mathbf{u}}{dt^2} = -e\mathbf{E} - m\omega_0^2 \mathbf{u} - m\gamma \frac{d\mathbf{u}}{dt} - e\mathbf{E}_0 \text{ (loss)}$$

or

$$m \frac{d^2 \mathbf{u}}{dt^2} = -e\mathbf{E} - m\omega_0^2 \mathbf{u} - m\gamma \frac{d\mathbf{u}}{dt} - e\mathbf{E}_0 + \frac{1}{c} \frac{d\mathbf{u}}{dt} \times \mathbf{H}_0 \text{ (ext magn field } H_0)$$

or... whatever else!

**Comment:** discard Lorentz force (relativistic effects); elementary dispersion theory

## Another comment

From  $m \frac{d^2 \mathbf{u}}{dt^2} = -e\mathbf{E} - e\mathbf{E}_0$  and  $\mathbf{E} = -4\pi\mathbf{P} = 4\pi n e \mathbf{u}$  we get the dielectric  $\varepsilon$  function

$$E_0 = \varepsilon E_{tot} = \varepsilon(\mathbf{E} + \mathbf{E}_0), \quad \varepsilon = 1 - \frac{\omega_p^2}{\omega^2}, \quad \omega_p^2 = \frac{4\pi n e^2}{m}$$

And by using  $\mathbf{j} = -ne \frac{d\mathbf{u}}{dt}$  we get the conductivity

$$\mathbf{j} = \sigma \mathbf{E}_{tot}, \quad \sigma = \frac{i\omega_p^2}{4\pi\omega}$$

All these for jellium-like plasma; similar for dielectrics, loss included, etc

## Our Scheme of Calculation

Eq of motion

$$m \frac{d^2 \mathbf{u}}{dt^2} = -e\mathbf{E} - e\mathbf{E}_0$$

Kirchhoff potentials

$$\Phi(\mathbf{r}, t) = \int d\mathbf{r}' \frac{\rho(\mathbf{r}', t-R/c)}{R}, \quad \mathbf{A}(\mathbf{r}, t) = \frac{1}{c} \int d\mathbf{r}' \frac{\mathbf{j}(\mathbf{r}', t-R/c)}{R}$$

Electric field  $\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \text{grad} \Phi$

Charge and current

$$\rho = ne \text{div} \mathbf{u}, \quad \mathbf{j} = -ne \frac{d\mathbf{u}}{dt}$$

Compute  $\mathbf{A}$  and  $\Phi$  using  $\rho$  and  $\mathbf{j}$  expressed by  $\mathbf{u}$  inside the body, compute  $\mathbf{E}$ , eliminate  $\mathbf{E}$  between the resulting equation and the equation of motion

So we get (coupled) **INTEGRAL EQUATIONS** for  $\mathbf{u}$

Once solved, compute  $\mathbf{A}$  and  $\Phi$  and  $\mathbf{E}$  outside, for the reflected, refracted, transmitted, diffracted field

Do the same for the non-retarded regime, where the equation of motion is simple

$$m \frac{d^2 \mathbf{u}}{dt^2} = ne^2 \text{grad} \int d\mathbf{r}' \frac{\text{div} \mathbf{u}}{|\mathbf{r} - \mathbf{r}'|} - e\mathbf{E}_0$$

We can see explicitly its character of integral equation

## Comment

Eigenmodes of the Integral Eq in the non-retarded regime give the **plasmons**

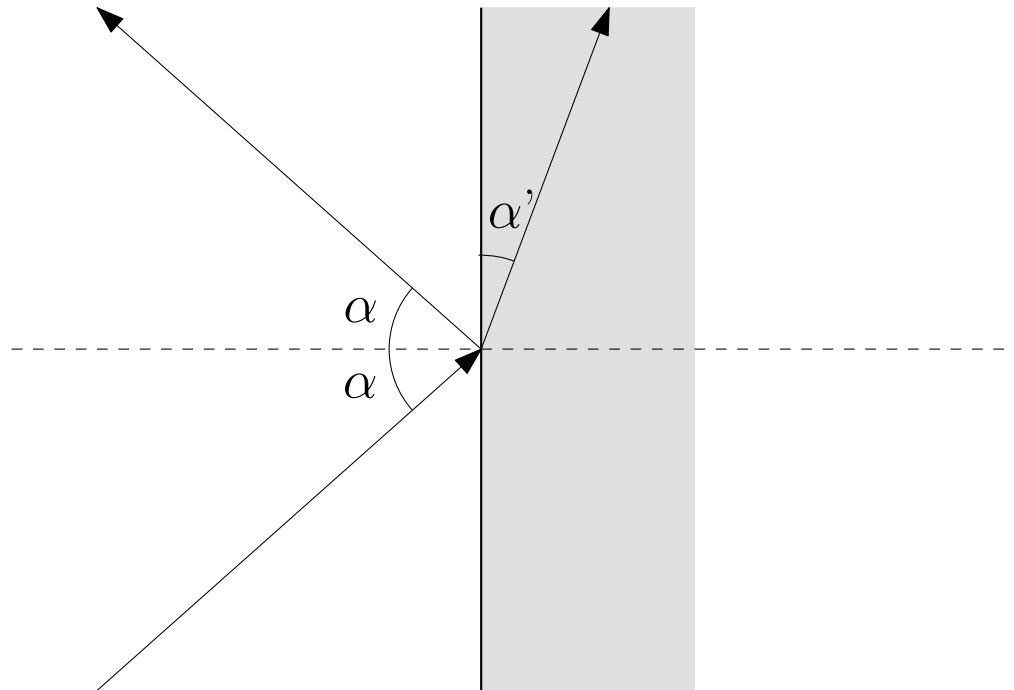
Its full solution gives the **dielectric response**

Eigenmodes of the Integral Eqs in the retarded regime give the **polaritons**

Their full solutions give the **refracted, reflected, transmitted, diffracted fields**

## PART II: APPLICATIONS

### Semi-infinite body (half-space)



$$\mathbf{u} \rightarrow \mathbf{u}\theta(z) , \operatorname{div}\mathbf{u} \rightarrow \operatorname{div}\mathbf{u} \cdot \theta(z) + u_3(z=0)\delta(z)$$

**Non-retarded:**

$$\omega^2 v = \frac{1}{2} k \omega_p^2 \int_0^\infty dz' v e^{-k|z-z'|} + \frac{1}{2k} \omega_p^2 \int_0^\infty dz' \frac{\partial v}{\partial z'} \frac{\partial}{\partial z'} e^{-k|z-z'|} - \frac{iek}{m} \Phi$$

$v = \mathbf{k}\mathbf{u}/k$  , in-plane  $\mathbf{k}$

**Dielectric response**

$$\frac{e}{m\omega^2} E_{tot\perp} = v = \frac{iek\omega_p^2}{m} \frac{\Phi_0}{(\omega^2 - \omega_p^2)(2\omega^2 - \omega_p^2)} e^{-kz} - \frac{iek}{m} \frac{\Phi}{\omega^2 - \omega_p^2} ,$$

$$\frac{e}{m\omega^2} E_{tot\parallel} = u_3 = -\frac{ek\omega_p^2}{m} \frac{\Phi_0}{(\omega^2 - \omega_p^2)(2\omega^2 - \omega_p^2)} e^{-kz} - \frac{e}{m} \frac{\Phi'}{\omega^2 - \omega_p^2}$$

Bulk plasmons  $\omega^2 = \omega_p^2$ , **surface plasmons**  $\omega^2 = \omega_p^2/2$

**No proper dielectric function:** surface contribution

**Energy loss** of an energetic particle

$$P_b = \left(-e^2\omega_p^2/v\right) \ln(vk_0/\omega_p)$$

$$P_s = -\frac{e^2\omega_p}{vt} \left(\sqrt{2} \sin \omega_{pt}/\sqrt{2} - \sin \omega_{pt}\right)$$

**Retardation:**  $\omega = cK = c\sqrt{k^2 + \kappa^2}$ , incidence angle  $\alpha$ , polarization  $\beta$ ,  $k = K \cos \varphi$

$$\omega^2 v_1 = -\frac{i\omega_p^2 \kappa}{2} \int_0 dz' v_1(z') e^{i\kappa|z-z'|} - \frac{\omega_p^2 k}{2\kappa} \int_0 dz' u_3(z') \frac{\partial}{\partial z'} e^{i\kappa|z-z'|} + \frac{e}{m} E_{01} e^{i\kappa z}$$

$$\omega^2 v_2 = -\frac{i\omega_p^2 \omega^2}{2c^2 \kappa} \int_0 dz' v_2(z') e^{i\kappa|z-z'|} + \frac{e}{m} E_{02} e^{i\kappa z}$$

$$\begin{aligned} (\omega^2 - \omega_p^2) u_3 = & \frac{\omega_p^2 k}{2\kappa} \int_0 dz' v_1(z') \frac{\partial}{\partial z} e^{i\kappa|z-z'|} - \frac{i\omega_p^2 k^2}{2\kappa} \int_0 dz' u_3(z') e^{i\kappa|z-z'|} + \\ & + \frac{e}{m} E_{03} e^{i\kappa z} \end{aligned}$$

## Refracted field

$$\frac{e}{m\omega^2} E_{tot1} = v_1 = \frac{2eE_{01}}{m\omega_p^2} \cdot \frac{\kappa'(\kappa - \kappa')}{\kappa\kappa' + k^2} e^{i\kappa'z}$$

$$\frac{e}{m\omega^2} E_{tot2} = v_2 = \frac{2eE_{02}}{m\omega_p^2} \cdot \frac{\kappa(\kappa - \kappa')}{K^2} e^{i\kappa'z}$$

$$\frac{e}{m\omega^2} E_{tot3} = u_3 = \frac{2eE_{03}}{m\omega_p^2} \cdot \frac{\kappa(\kappa - \kappa')}{\kappa\kappa' + k^2} e^{i\kappa'z}$$

$$\kappa' = \sqrt{\kappa^2 - \omega_p^2/c^2} = \frac{1}{c} \sqrt{\omega^2 \cos^2 \alpha - \omega_p^2}$$

**Extinction theorem**  $\kappa \rightarrow \kappa'$ ; "effective medium theory"?

## Reflected field

$$E_1 = E_{01} \frac{\kappa - \kappa'}{\kappa + \kappa'} \cdot \frac{\kappa \kappa' - k^2}{\kappa \kappa' + k^2} e^{-i\kappa z}$$

$$E_2 = E_{02} \frac{\kappa - \kappa'}{\kappa + \kappa'} e^{-i\kappa z}$$

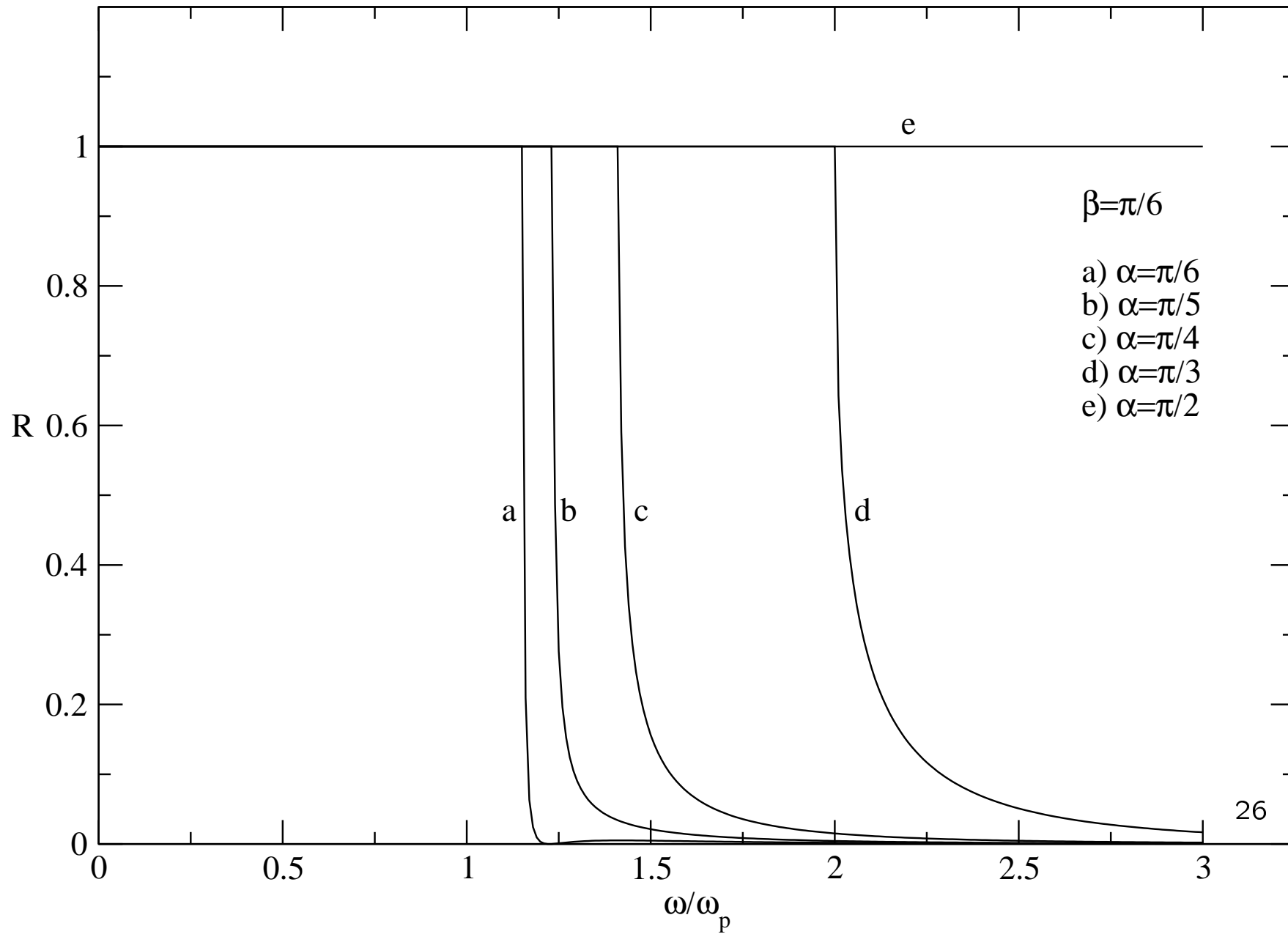
$$E_3 = -E_{03} \frac{\kappa - \kappa'}{\kappa + \kappa'} \cdot \frac{\kappa \kappa' - k^2}{\kappa \kappa' + k^2} e^{-i\kappa z}$$

## Reflection coefficient

$$R = R_1 \left[ \cos^2 \beta \sin^2 \varphi + R_2 \left( \cos^2 \beta \cos^2 \varphi + \sin^2 \beta \right) \right]$$

$$R_1 = \left| \frac{\sqrt{\omega^2 \cos^2 \alpha - \omega_p^2} - \omega \cos \alpha}{\sqrt{\omega^2 \cos^2 \alpha - \omega_p^2} + \omega \cos \alpha} \right|^2$$

$$R_2 = \left| \frac{\cos \alpha \sqrt{\omega^2 \cos^2 \alpha - \omega_p^2} - \omega \sin^2 \alpha}{\cos \alpha \sqrt{\omega^2 \cos^2 \alpha - \omega_p^2} + \omega \sin^2 \alpha} \right|^2$$

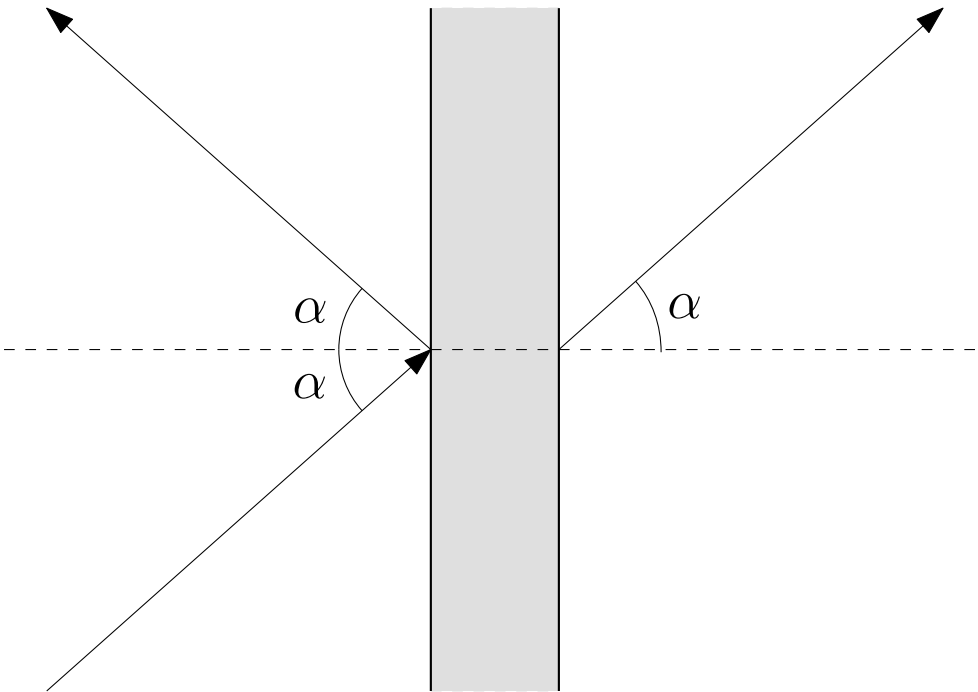


## Surface plasmon-polariton modes

$$\omega^2 = \frac{2\omega_p^2 c^2 k^2}{\omega_p^2 + 2c^2 k^2 + \sqrt{\omega_p^4 + 4c^4 k^4}}$$

Limits  $ck$  and  $\omega_p/\sqrt{2}$ .

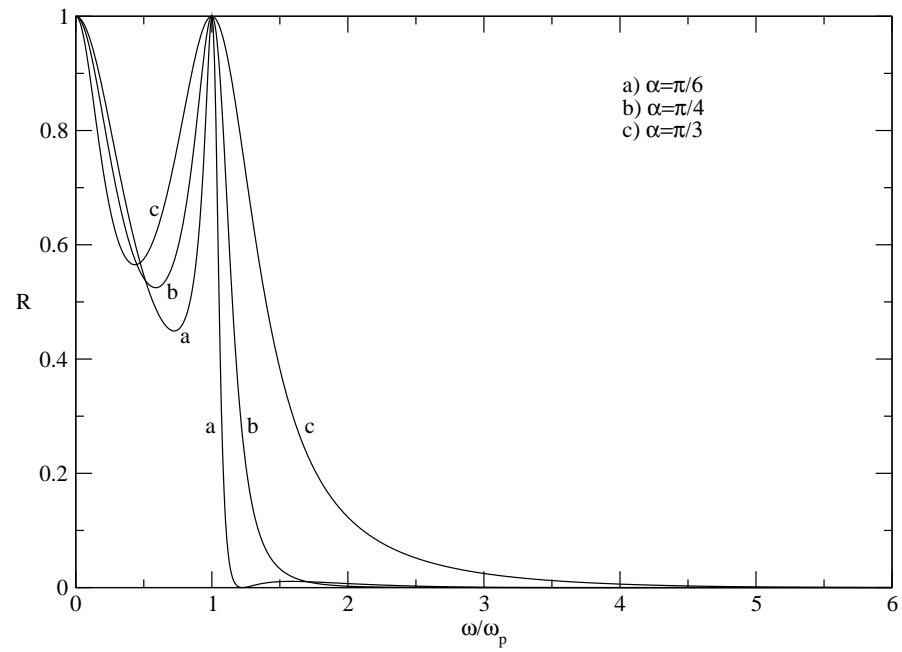
Slab of thickness  $d$



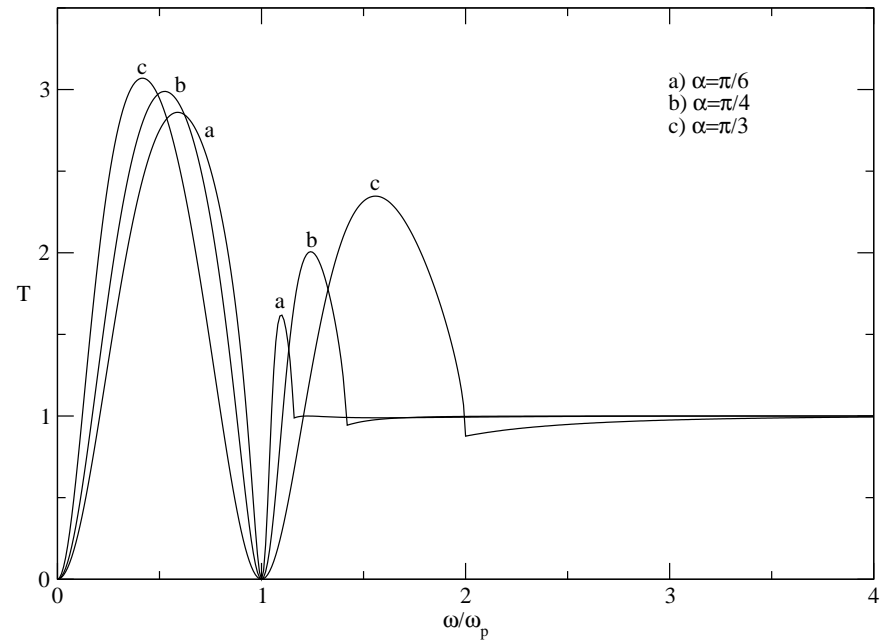
Dielectric response, surface terms, surface plasmons  $\omega^2 = \frac{1}{2}\omega_p^2 (1 \pm e^{-kd})$

Surface energy loss  $\int_0^\infty dt P_s = -\pi (\sqrt{2} - 1) \frac{e^2 \omega_p}{v}$

Transmitted field  $\sim e^{i\kappa z}$ , Reflected field  $\sim e^{-i\kappa z}$ , refracted field  $\sim e^{i\kappa'z}, e^{-i\kappa'z}$

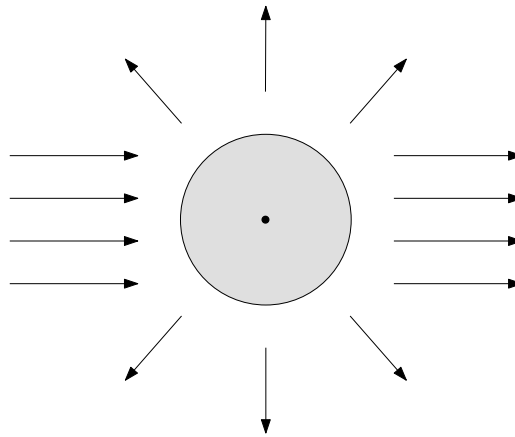


Reflection coefficient (for  $p$ -wave)



Transmission coefficient (for *p*-wave)

## Sphere



(Mie solution, 1908)

**Spherical plasmons** (sphere)  $\omega = \omega_p \sqrt{\frac{l}{2l+1}}$ , (void)  $\omega = \omega_p \sqrt{\frac{l}{2l+1}}$

**Diffraction**  $F^{0,\pm} \sim j_l \mathbf{Y}_{ljm}$ ,  $H^{0,\pm} \sim h_l \mathbf{Y}_{ljm}$ ,  $\mathbf{Y}_{ljm} \sim C - G$ ,  $\mathbf{e}_1 Y_{lm}$

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = \frac{ik}{4\pi} \sum_{lm} F_{lmk}^*(\mathbf{r}<) H_{lmk}(\mathbf{r}>)$$

## Fields

$$\mathbf{E}_0(\mathbf{r}) = E_0 \sum_{l=1}^{\infty} \sum_m \left( a_{lm} \mathbf{F}_{lmk}^0(\mathbf{r}) + b_{lm} \mathbf{F}_{lmk}^+(\mathbf{r}) \right)$$

$$\mathbf{E}_i(\mathbf{r}) = \frac{m\omega^2}{e} u(\mathbf{r}) = E_0 \sum_{l=1}^{\infty} \sum_m \left[ A_l a_{lm} \mathbf{F}_{lmk_1}^0(\mathbf{r}) + B_l b_{lm} \mathbf{F}_{lmk_1}^+(\mathbf{r}) \right]$$

$$\mathbf{E}_s(\mathbf{r}) = \frac{ka^2}{16\pi^2} E_0 \sum_{l=1}^{\infty} \sum_m \left[ A_l a_{lm} f_l^0 \mathbf{H}_{lmk}^0(\mathbf{r}) + B_l b_{lm} f_l^+ \mathbf{H}_{lmk}^+(\mathbf{r}) \right]$$

$$k_1 = \frac{1}{c} \sqrt{\omega^2 - \omega_p^2}$$

## Coefficients

$$A_l = \frac{16\pi^2(-1)^{l+1}c}{\omega a^2} \cdot \frac{1}{k_1 h_l(ka) j_{l+1}(k_1 a) - k h_{l+1}(ka) j_l(k_1 a)}$$

$$B_l = \frac{16\pi^2(-1)^{l+1}c^3 k_1}{\omega_p^2 \omega a^2}$$

$$\frac{1}{\left[ \left( -\frac{\omega^2}{\omega_p^2} + \frac{l}{2l+1} \right) h_{l+1}(ka) + \frac{l+1}{2l+1} h_{l-1}(ka) \right] j_l(k_1 a) + \frac{\omega^2 k_1}{\omega_p^2 k} h_l(ka) j_{l+1}(k_1 a)}$$

## Plasmonic resonances - optical spectroscopy of small spheres

### Cross-section

$$\sigma = \operatorname{Re} \left[ r^2 \frac{Q_s}{|S_0|} \Big|_{r \rightarrow \infty} \right] = \frac{a^4}{16\pi^2} \sum_{l=1}^{\infty} \sum_m \left( |A_l a_{lm} f_l^0|^2 + |B_l b_{lm} f_l^+|^2 \right)$$

## Cylindrical geometries

### Two-dimensional screen

Plasmons  $\omega^2 = \omega_p^2 kd/2$ ; Transmitted, reflected fields, discontinuous

### Green Functions

$$\frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = i \sum_{m=-\infty}^{+\infty} e^{im(\varphi-\varphi')} \int_0^{\infty} k dk J_m(k\rho) J_m(k\rho') \frac{e^{i\kappa|z|}}{\kappa}$$

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \frac{2}{\pi} \sum_m \int_0^{\infty} dk e^{im(\varphi-\varphi')} \cos k(z-z') I_m(k\rho_{<}) K_m(k\rho_{>})$$

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \sum_m e^{im(\varphi - \varphi')} \int_0^\infty dk e^{-k(z > - z <)} J_m(k\rho) J_m(k\rho')$$

$$\frac{e^{i\frac{\omega}{c}\sqrt{r^2 + z^2}}}{\sqrt{r^2 + z^2}} = \sum_{\mathbf{k}} \frac{2\pi i}{\kappa} e^{i\mathbf{k}\mathbf{r}} e^{i\kappa|z|}$$

**Infinite cylindrical hole plasmons**

$$\omega^2 = \omega_p^2 \left[ 1 - \kappa a K_m(\kappa a) I'_m(\kappa a) \right]$$

**Infinite cylindrical rod**

$$\omega^2 = \omega_p^2 \left[ 1 + \kappa a I_m(\kappa a) K'_m(\kappa a) \right]$$

Diffraction: **No**

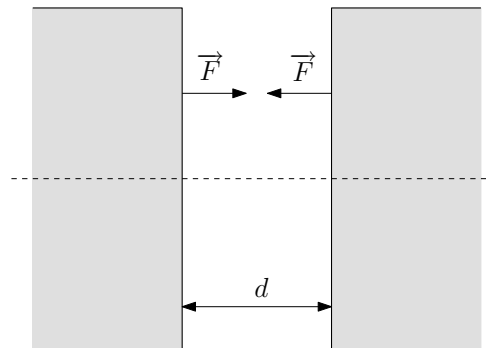
Circular aperture and circular disk: **No**

Finite cylindrical hole and rod: **No**

## PART III

### Electromagnetic Eigenmodes

#### Van der Waals - London - Casimir Forces



Coupled integral equations for the displacement  $\mathbf{u}$

Their eigenmodes  $\Omega_\alpha$

Non-retarded regime

$$\left(\omega^2 - \frac{1}{2}\omega_1^2\right) \left(\omega^2 - \frac{1}{2}\omega_2^2\right) - \frac{1}{4}\omega_1^2\omega_2^2 e^{-2kd} = 0$$

Force

$$F = \frac{\partial}{\partial d} \sum_\alpha \frac{1}{2} \hbar \Omega_\alpha$$

**van der Waals - London** (two identical metals, per unit area)

$$F \simeq \hbar\omega_p/2\pi\sqrt{2}d^3$$

Similar  $d^{-3}$  for any pair of bodies (two molecules  $\sim 1/R^7$ ).

**Retarded regime:** dispersion equations  $\kappa_{1,2} = \sqrt{\kappa^2 - \omega_{1,2}^2/c^2}$

$$e^{2i\kappa d} = \frac{(\kappa_1 + \kappa)(\kappa_2 - \kappa)}{(\kappa_1 - \kappa)(\kappa_2 + \kappa)}$$

$$e^{2i\kappa d} = \frac{(\kappa_1 + \kappa)(\kappa_2 - \kappa)(\kappa\kappa_1 + k^2)(\kappa\kappa_2 - k^2)}{(\kappa_1 - \kappa)(\kappa_2 + \kappa)(\kappa\kappa_1 - k^2)(\kappa\kappa_2 + k^2)}$$

Solutions

$$\Omega_\alpha = c\sqrt{k^2 + \frac{\pi^2 x_n^2}{d^2}}$$

For surface plasmon-polariton modes (in metals) or for identical bodies; bound cds

Standard renormalization procedure leads to **Casimir force** (per unit area)

$$F = \frac{\pi^2 \hbar c}{240 d^4}$$

Similar  $d^4$  for other pairs of bodies, except for distinct dielectrics

**Comment:** original Casimir argument; fluctuation theory

**Point-like particle** and a semi-infinite body  $F = \frac{3\hbar\omega_p}{8\sqrt{2}} \cdot \frac{\alpha a^3}{d^4}$  (non-retarded); no force when retardation included

## Part IV

### Conclusion

Some issues in Classical Electromagnetism:

- 1 Fresnel-Huygens interference in refraction (extinction theorem Ewald-Oseen 1915)?
- 2 Effective medium theory,  $\epsilon$  problem?
- 3 Origin of  $\epsilon$ ?

Answered all, understood none!

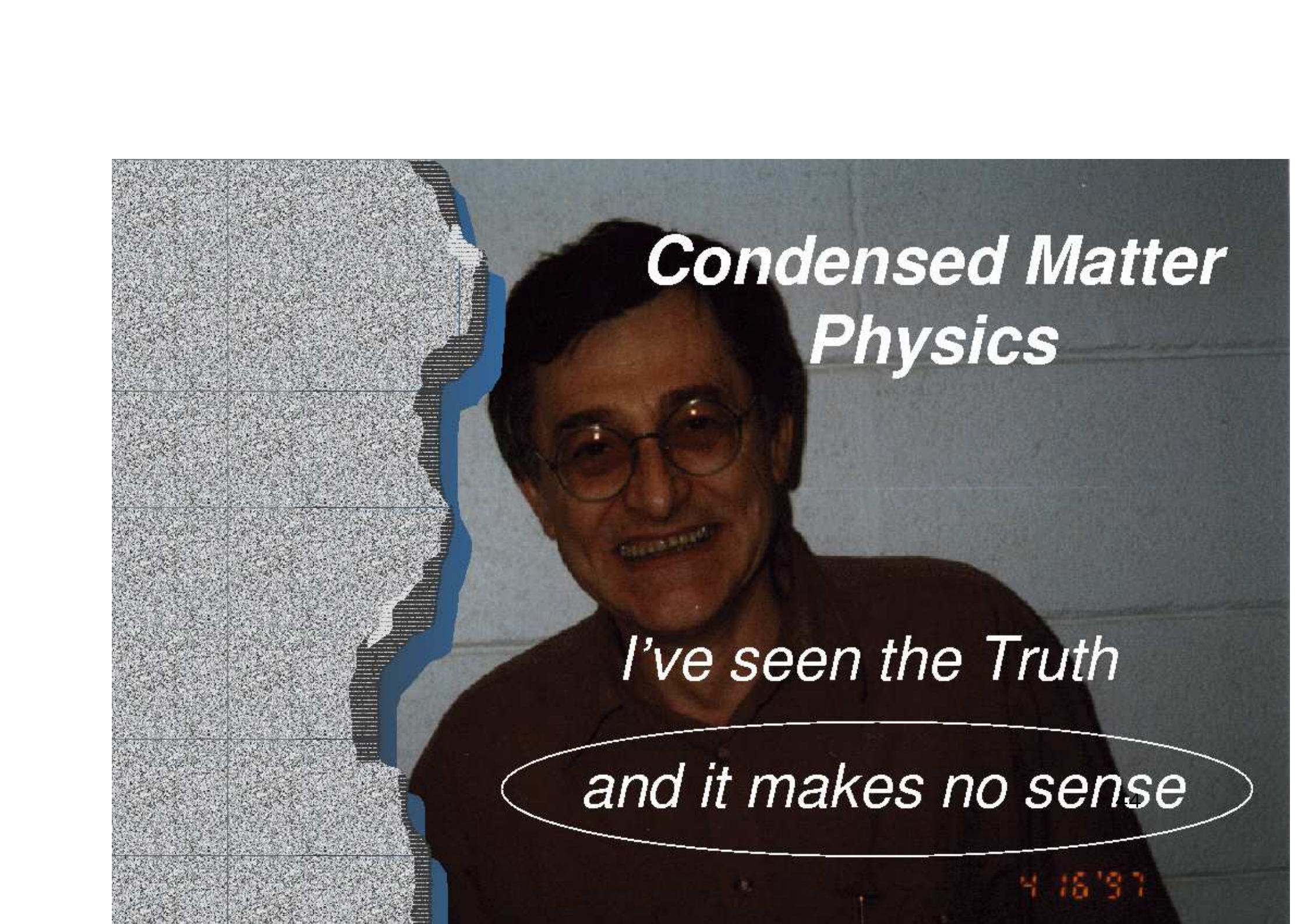
\_ Introduced  $\mathbf{u}$  and its equation of motion (Newton's) (equation of motion for polarization  $\mathbf{P}$ )

\_ similar for magnetization  $\mathbf{M}$

\_ rewriting Maxwell equations in matter with  $\mathbf{u}(\mathbf{P})$  and  $\mathbf{M}$

\_ Fully determined system of eqs, no need for phenomenological  $\epsilon$  and  $\mu$

## **INTEGRAL EQUATIONS**



*Condensed Matter  
Physics*

*I've seen the Truth*

*and it makes no sense*

4 16 '97