INVESTIGATION OF THE EFFECT PRODUCED BY THE EFFECTIVE
MASS AND THE DISSIPATION IN THE FISSION CROSS-SECTION

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An investigation of the possible effects due to a variable inertia and of the
modification of the potential barriers is realized in the frame of a modified statistical
model. A symmetric channel associated to the final fragment partition and a very
asymmetric one are selected for this purpose. The barriers are modified by taken into
account theoretical variations obtained within the microscopic-macroscopic theory
due to the channel itself and due to the dissipation effect.

INTRODUCTION

Recently, using the Hartree-Fock-Bogoliubov microscopic equations of
motions it was shown that the isotopic distribution of fission products is ruled by
the dissipated energy along the minimal action trajectory [1] due to redistribution
of energy levels. For different values of the final mass asymmetry, that means
the initial nucleus $A$ decays with different final partitions $(A_1Z_1, A_2Z_2)$ the
trajectories in the configuration space spanned by the three most important
degrees of freedom (elongation, necking-in and mass-asymmetry) encountered in
fission were obtained. The neutron and proton level schemes were determined
along these trajectories. Using the Strutinsky prescriptions the theoretical
adiabatic potential barriers were obtained. The potential barrier for the
asymmetric final partition was found to be lower than the symmetric one, in
accordance with the common knowledge in the field. Unfortunately, calculating
the WKB integral with the effective mass included in the action integral for each
partition, the yield of the asymmetric channel was obtained lower than the
symmetric one despite the good behavior of the barrier. The effective mass is
larger for the asymmetric mass partition than for the symmetric one, especially
for the configurations where the shapes change their mass-asymmetry. By
calculating the energy dissipation for each partition, fortunately it was found that
the dissipated energy is much greater for the symmetric channel. By taking into
consideration the dissipation (added to the adiabatic potential as an excitation),
the known experimental behavior of the mass distribution was restored. In this

work, an investigation of the role of the dissipated energy, and of the effective mass in the frame of the statistical model for fission cross section is intended. The statistical model of fission is re-analyzed here.

**COMPOUND NUCLEUS FORMATION AND FISSION CROSS-SECTION**

The assumptions of our model are different on the existent ones. So the main features are underlined.

The cross-section \( \sigma_c \) for compound-nucleus formation cross-section at excitation energy \( E^* \) by bombarding an \( A-1 \) initial nucleus of spin \( I \) with neutrons is a sum over partial cross-sections for all possible final spin \( J \):

\[
\sigma_c(E^*) = \sum_J \sigma_c(J, E^*) = 
\]

\[
= \pi \lambda^2 \sum_{l=0}^{\infty} \sum_{j=-l}^{l+1/2} \frac{1}{2J+1} \sum_{J=|j-l|}^{j+l} \sum_{\Omega=-J}^{J} \Omega \left| j\Omega0\right| J\Omega^2 (2l+1)(1-|\Omega|^2)
\]

where \( T_{n,l}(E_n) = (2l+1)(1-|\Omega|^2) \) represents the neutron transmission for an angular momentum \( l \) and initial neutron energy \( E_n \). \( \left\langle j\Omega0\right| J\Omega \) are the Clebsh-Gordon coefficients and \( \Omega \) is the projection on the symmetry axis of the spin \( J \). This formula is deduced without taking into account the parity and considering, for simplicity, that the probabilities to obtain systems with \( I \pm 1/2 \) are equals. The cross-section for a particular value of \( J \) can be also obtained using the previous formula. Usually \( E^* = E_n + Q \), \( Q \) being the neutron binding energy. The main channels for the de-excitation of the compound-nucleus are the neutron emission, the gamma de-excitation and the fission. The fission cross-section can be obtained within the detailed balance principle:

\[
\sigma_f(E^*) = \sum_J \sigma_f(J, E^*) \frac{\Gamma_f(J, E^*)}{\Gamma_n(J, E^*) + \Gamma_f(J, E^*) + \Gamma_f(J, E^*)}
\]

where the ratio in the right hand of the equation is the probability that the system decays through fission. The energy widths of the three channels, labeled with the corresponding subscripts, depend on the density of states \( \rho(J, E, A) \) and the transmissions. Usually, the transmission integrals are computed by using densities associated to a particular de-excitation way: the fundamental density in the case of gamma de-excitation, the saddle-point density for fission and the residual nucleus density for emission of one neutron. Also, all levels included in the saddle-point density are considered to have practically the same probability
or realization. In the case of sub-barrier transmissions, the transmissions for fission were considered as barrier penetrabilities alone from fundamental states. In our formalism, we consider that the saddle-point density is an implicit quantity, being determined by the fundamental density through a correspondence allowed by the high of each barrier constructed on fundamental levels with good quantum numbers and a probability of realization of the state associated to these numbers. The same fundamental density can be used for sub-barrier and supra-barrier transmissions. The single-particle excitations must be known. These quantities can be obtained from shell model calculations. This feature allows us to unfold the initial state versus these quantum numbers, so that, the states and the associated transition levels will be assigned by a probability in term of Clebsh-Gordon coefficients. In order to obtain a mean energy width which takes into account the fact that the spin $J$ of the nucleus is given by the coupling of the collective angular momentum of the core $L$ and the spin of the intrinsic states $J_f$, the initial states $J$ are unfolded through Clebsh-Gordon coefficients. For one initial state, several channels become possible depending on $L$ (orbital momentum of the core), $J_f$ (the spin of the non-rotational nucleus) or $\Omega$ (projection on the nuclear symmetry axis). In the case of the neutron emission, the transmission is written by taking into account an additional channel due to the orbital momentum $l$ carried by the neutron. The energy widths are assumed to have the following forms in our formalism:

$$
\Gamma(J, E^*) = \frac{1}{2\pi \rho(J, J, E^*, A)} \sum_{L=0}^{L_{\max}} \sum_{J-L=\Omega=M}^{J+L} \left\langle J_f L \Omega 0 | J \Omega \right\rangle^2 \sum_{J_f - |J_f|}^{J_f + |J_f|} \frac{1}{(2J+1)(L_M + 1)} \int_{E-E_L-E(J_f-\Omega)}^{E-E_L-E(J_f-\Omega)} \rho(J_f, \Omega, E^* - E_L - E(J_f - \Omega) - \varepsilon, A) d\varepsilon,
$$

$$
\Gamma_n(J, E^*) = \frac{1}{2\pi \rho(J, J, E^*, A)} \sum_{L=0}^{L_{\max}} \sum_{J-L=\Omega=M}^{J+L} \sum_{l=0}^{l_{\max}} \frac{1}{(2J+1)(L_M + 1)(l_l + 1)} \left\langle J_f L \Omega 0 | J \Omega \right\rangle^2 \int_{E^* - E_l - E(J_f-\Omega)}^{E^* - E_l - E(J_f-\Omega)} T_{nJ}(\varepsilon) \frac{\tilde{\rho}(J_f - 1/2, \varepsilon) + \tilde{\rho}(J_f + 1/2, \varepsilon)}{2} d\varepsilon
$$

$$
\Gamma_f(J, E^*) = \frac{1}{2\pi \rho(J, J, E^*, A)} \times
$$
\[ \times \sum_{L=0}^{L_M} \sum_{J_f=J-L}^{J_f+L} \sum_{\Omega=-M}^{M} \frac{\langle J_f L \Omega | \Omega \rangle^2}{(2J+1)(L_M+1)} \times \]
\[ E - E_L - E(J_f - \Omega) \]
\[ \times \int_{0}^{2} 2T_{J_f L \Omega}(\varepsilon) \rho(J_f, \Omega, E^* - E_L - E(J_f - \Omega) - \varepsilon, A) d\varepsilon, \]

where

\[ \rho(J, \varepsilon) = \rho(J, J, E_n - E_L - E(J_f - \Omega) - \varepsilon, A) \]

and

\[ (2J + 1)(L_M + 1) \approx \sum_{L=0}^{L_M} \sum_{J_f=J-L}^{J_f+L} \sum_{\Omega=-M}^{M} \langle J_f L \Omega 0 | J \Omega \rangle^2 \]

So, a mean transmission weighted by Clebsh-Gordon coefficients is obtained for the three main disintegration ways. Here, \( M \) is the lower value between \( J \) and \( J_f \). \( L_M \) is a maximal collective angular momentum used in the calculations (6 in this work) and \( l_m \) a maximal value for the angular momentum carried by the neutron. For simplicity, it is supposed that for a given \( l \), the neutron transmissions in states \( J_f + \frac{1}{2} \) and \( J_f - \frac{1}{2} \) are equal and that for a given initial spin \( J_f \) all values of \( l \) are obtained with the same probability. Up to this point the parity \( \pi \) is obsolete, because the formulas are given for a specific parity and the density values for positive and negative parities are considered equals. The density belonging to the fission transmission is doubled because the fission channels loose the reflection symmetric configuration and levels with both parities must be taken into account. As it can be remarked, the density used in this work depends on the non-rotational nuclear spin \( J_f \) and on its projection \( \Omega \). Usually, in fission the density of states is considered only for nuclei with \( J_f = \Omega \) (intrinsic density), but in the previous formula (1), this definition applies to intrinsic density of states that are not coupled to additional excitations. In such intrinsic densities, only energy levels given by the sum of individual particle energies are taken into account. By coupling a collective vibration excitation of energy \( E_{vib} \) and spin \( I \) to a nuclear state \( J_f \), the density \( \rho(J_f, J_f, E, A) \) can be transformed in a density \( \rho(J_f + 1, J_f, E + E_{vib}, A) \). Due to the additional energy \( E_{vib} \), the densities with \( J_f \neq \Omega \) appears at larger excitation energies and, therefore, their contributions in the sums of the previous equations are small. In these circumstances, all terms in the previous summations with \( J_f \neq \Omega, \Omega + 1 \) will be neglected. A value \( E_{vib} = 0.5 \text{ MeV} \) is added to the state in the last case and denoted \( E(J_f - \Omega) \) in the formula. In the case of rotations, a quantity \( E_L = L(L + 1)\hbar^2/(2J) \) is lost to permit the rotational collective motion. It is
important to stress again that in the above formula, calculating the transmission for the fission process, the fundamental density of states is used and not the saddle-point one as customary. This procedure is possible due to the fact that a one to one correspondence between levels in the ground state and levels in the saddle point configuration is ensured by our treatment. This procedure is coherent with the Fermi golden rule where the initial density of states and the transition matrix element intervene in calculations of probabilities. It is important to remind that the density of states for the fission channel is doubled because during the tunneling process the nuclear shapes are not longer reflection symmetric and states with both parities participates to disintegration. To simplify the calculations, in the fission channel, the $J$ spin density of states can be shared between a discrete component for low intrinsic excitation energies and a continuum one for greater intrinsic energies:

$$
\rho(J, J = \Omega, E, A) = \begin{cases} 
\sum_i \delta(E - \epsilon_{\Omega i}) & \text{for } E < E_0 \\
\rho_{GC}(J, J, E, A) & \text{for } E \geq E_0
\end{cases}
$$

Here, $\rho_{GC}$ is given by the Gilbert and Cameron approximation [2] and $E_0$, in principle, must be equal to the neutron binding energy. $\epsilon_{\Omega i}$ are all intrinsic energies characterized by a spin $J$ that can be obtained with all single-particle configurations at energy $E$. In practice, during this work, a value of $E_0$ corresponding to an energetic interval which includes several levels $n$ of same spin is used, that means,

$$
\int_0^{E_0} \rho_{CG}(J, J, E, A)\,dE = n
$$

only some discrete levels of same spin were taken into consideration. The number $n = 4$ in the following. In this small energetic interval, the excitations are primary obtained by the transition of the unpaired nucleon from one level to another. This property leads to a simplification of the problem. Therefore, the integral which intervenes in the fission energy width can be written:

$$
\int_0^{E_0 - E(J_f - \Omega) - E_L} T_{f,L,\Omega}(\epsilon)\rho(J_f, J_f, E^* - E_L - E(J_f - \Omega) - \epsilon, A)\,d\epsilon =
$$

$$
= \sum_i T_{f,L}(E^* - E_L - E(J_f - \Omega) - \epsilon_{\Omega i}) + \int_0^{E_0 - E(J_f - \Omega) - E_L} T_{f,L,\Omega}(\epsilon) \times \rho_{GC}(J_f, \Omega = [J_f, J_f - 1], E^* - E_L - E(J_f - \Omega) - \epsilon, A)\,d\epsilon
$$
in terms of discrete transitions for some low energy initial states and a statistical integral. The summation over \( i \) involves all intrinsic levels of same spin located in the energy interval \([0, E_0]\). Each barrier can be obtained by knowing the fundamental barrier (the barrier of lowest excitation energy) and the single particle excitations energies. The single-particle energies will be determined within a two-center shell model \([3]\). Using some simple assumptions, the double well fundamental barrier can be approximately described with three smoothed joined harmonic segments \([4]\). Finally, the calculation of the transmissions must be realized. Several points need to be elucidated. The problem of saddle-point density for a double humped barrier is somewhat ambiguous. Solving exactly the Schrödinger equation to obtain the penetrability for both barriers, sometimes the highest barrier saddle point is used. When the barrier are treated separately, two saddle point densities are used. When the beta excitation spectra in the second well is treated approximately, the density in the second well is used. In general, the quantity generically known as saddle point density is treated in different way in order to fit better the experiment with simulations. Input parameters as enhancement factors, temperature, variation of the constants in the density, are usually introduced in this context. Such modifications of the level density are not necessary in the present treatment because each barrier constructed on each intrinsic state is studied in the low energy regime. The use of saddle point densities is required only in the statistical treatment implied by the second term in the right hand of the previous equation. However, the main contribution in the integral is determined by the first term and the influence of the saddle point density becomes of second order. In this context, many authors claim that the fundamental and intrinsic densities of states are rather similar, the difference being simulated by introducing some factors. A development similar of that displayed by the previous equation can be found in Ref. \([5]\).

For the neutron transmissions, the “cloudy crystal-ball” model is used \([6]\) with a real potential of 42 MeV and an imaginary one of 8.4 MeV. The relative amplitude of the outgoing wave with angular momentum \( l \) is given as:

\[
\eta_l = \frac{f_l - S_l + i P_l}{f_l - S_l - i P_l} \exp(2i\xi_l)
\]

where \( f_l \) is the logarithmic derivative evaluated at the nuclear surface, \( P_l \) is the penetration factor, \( S_l \) is the shift factor and \( \xi_l \) is the phase constant (obsolete in the transmission estimation). The shift and penetration factors can be obtained by solving the Schrödinger equation for the radial wave function with conditions imposed on the nuclear surface (at the inter-nuclear distance \( R \)):

\[
S_l = R \left[ \frac{G_l(r)G_l'(r) + F_l(r)F_l'(r)}{G_l^2(r) + F_l^2(r)} \right]_{r=R}
\]
\[ P_I = R \left[ \frac{G_I(r)G'_I(r) - F_I(r)F'_I(r)}{G^2_I(r) + F^2_I(r)} \right]_{r=R} \]

where the solutions \( G_I \) and \( F_I \) are expressible in term of Bessel functions:

\[ F_I(r) = \left( \frac{\pi kr}{2} \right)^{1/2} J_{l+1/2}(kr) \]
\[ G_I(r) = -\left( \frac{\pi kr}{2} \right)^{1/2} N_{l+1/2}(kr) \]

The logarithmic derivative for Bessel function interior solutions is

\[ f_I = R \left[ \frac{K_{j_I}(KR)}{J_{j_I}(KR)} \right] + 1 \]

where for \( l = 0 \) the solution is (avoiding a mistype of [6]):

\[ f_0 = (X_1 + iX_2) \left[ \frac{1 - i \tan X_1 \tanh X_2}{\tan X_1 + i \tanh X_2} \right] \]

with \( X_1 = \text{Re}(KR) \) and \( X_2 = \text{Im}(KR) \), \( K = [2m(E - V)/\hbar^2]^{1/2} \) is the imaginary wave number, \( E \) is the energy of the nucleon and \( V \) the complex potential. For other values of \( l \), the logarithmic derivative can be obtained using the recurrence relations for Bessel functions:

\[ f_I = \frac{X^2}{l - f_{I-1}} - l \]

In Fig. 1 the total cross section obtained by bombarding \(^{235}\text{U} \) with neutron is plotted and compared with experimental values to give an estimation of the accuracy of our calculations. The compound nucleus cross section is plotted together the partial \( J \) cross sections. Due to the fact that the target has the spin \( 7/2 \), the partial cross-sections for \( J = 3 \) and \( 4 \) are larger at low energy. Deviations of 30% between the experimental and theoretical values are obtained. However, in the present work, we are not interested in an exact reproduction of the cross section, some deviations being introduced by the model used for neutron cross sections and neutron transmissions. The interest is mainly focussed on an investigation of the dissipation effect and effective mass on the cross section.

The transmission through the fission barrier needs the knowledge of the shape of the barrier and the effective mass. Using some simple assumptions, the double well barrier can be approximately described with three smoothed joined harmonic segments giving rise to the so-called phenomenological barrier in this text:
Fig. 1. – Comparison between the total experimental neutron cross section of $^{235}$U and the theoretical one as function of the neutron incident energy. The compound nucleus cross section is plotted with dashed line. The partial cross-sections for the formation of compound nucleus with a given spin are plotted with thin full lines. Two spins are marked on the plot.

For this schematic case a solution of the Schrödinger equation can be written in term of parabolic cylinder functions [4] if the effective mass is a constant. On the other hand it is possible to solve numerically the Schrödinger equation by $n = 500$ constant potential steps. In this case, the transmission coefficient is [7]

$$V(\varepsilon) = \begin{cases} E_1 - \frac{1}{2} M \omega_1^2 (\varepsilon - \varepsilon_1)^2, & \varepsilon \leq a, \\ E_2 + \frac{1}{2} M \omega_2^2 (\varepsilon - \varepsilon_2)^2, & a < \varepsilon < b, \\ E_3 - \frac{1}{2} M \omega_3^2 (\varepsilon - \varepsilon_3)^2, & b \geq \varepsilon \end{cases}$$
\[ T_f = 1 - \left| \frac{c_n}{b_n} \right|^2 \]

where \( c_n \) and \( b_n \) are the solutions in the \( n \)-th interval of the equations

\[
\frac{b_{i+1}}{c_{i+1}} = \frac{b_i}{c_i} (k_i + k_{i+1}) \exp[i(k_i - k_{i+1})\varepsilon_i] + (k_{i+1} - k_i) \exp[-i(k_i + k_{i+1})\varepsilon_i]
\]

and

\[
\frac{b_i}{c_i} = \frac{k_1 + k_0}{k_1 - k_0} \exp(-2ik_1\varepsilon_0)
\]

with

\[ k_i = \sqrt{2(E - V_i)B / h^2} \]

where \( E \) is the excitation energy of the nucleus. The numerical solution will be used for the discrete transitions while the exact solution is more appropriate for the statistical integral because it is not time consuming in the numerical evaluations.

Some simplifying assumptions are introduced in this work. Firstly, the excited states are considered in a statistically way. A state \( I \) is present when the next equation is fulfilled:

\[
\int_0^{E_i} \rho_{GC}(J, J, e, A) de = I
\]

and this value of \( E_i \) is constant for any elongation. The excited states are evaluated up to \( I = 4 \) in relation (3). These excited states give the discrete contribution in the fission cross-section, indifferently of the shape or kind of the potential barrier. The statistical integral becomes (the right term of right hand of equation (4)) a small quantity because its integration domain is diminished. Therefore, we can use the exact solution in the integral because the errors introduced in this way become of second order. We also neglect all the terms with \( J \neq \Omega \), because these terms are small compared with those of \( J = \Omega \).

The effective mass divided by the reduced mass \( \mu \) of the fissioning system was obtained along the optimal action path in Ref. [1]. In order to use this result in the frame of our model, it is necessary to transform the effective mass along the elongation \( \mu B(R) \) in an effective mass along the dimensionless coordinate \( B(\varepsilon) \)

\[ B(\varepsilon) = \mu B(R) \left( \frac{dR}{d\varepsilon} \right)^2 \]
where \( \mu = 0.0239225 A_1 A_2 \) in unity \((\hbar^2 \text{ MeV}^{-1} \text{ fm}^{-2})\). \( B(\varepsilon) \) is obtained in unity \((\hbar^2 \text{ MeV}^{-1})\). Usually, in most cross-section evaluations a constant effective mass \( B_c = 0.054 A^{5/3} \) \((\hbar^2 \text{ MeV}^{-1})\) [4] is introduced. For our comparisons, both kind of effective masses will be used. The effective mass (theoretical or as a constant value) intervenes in equation (5).

RESULTS AND DISCUSSION

Firstly, the model applied to find the better heights of the barriers for the fission of \(^{235}\text{U}\) bombarded with neutrons by considering a constant effective mass. In Fig. 2 a plot between the best theoretical values of the barrier parameters and the ENDF [8] evaluation is displayed. We deduced after several tries the best

Fig. 2. – Comparison between the theoretical and experimental values of the fission cross-section as function of the incident neutron energy.
parameters for the phenomenological barrier available in our context. The parameters of the barriers are \( E_1 = 5.43 \text{ MeV}, E_2 = 2.1 \text{ MeV}, E_3 = 5.33 \text{ MeV}, \)
\( \hbar \omega_1 = 1.04 \text{ MeV}, \ h\omega_2 = 1 \text{ MeV} \) and \( h\omega_3 = 0.6 \text{ MeV} \). Differences between the theory and ENDF evaluation are of the same order of magnitude as in the case of the neutron cross-section. In this work only the first chance fission is taken into account.

Two channels are analyzed giving rise to two different partitions \(^{118}\text{Pd} + ^{118}\text{Pd}\) and \(^{86}\text{Se} + ^{150}\text{Ce}\). So, a symmetric channel and an asymmetric one are investigated. To simplify the problem, it is considered that the phenomenological barrier characterizes the asymmetric channel. In this way we take benefits from the simplicity of the phenomenological barrier. On the other hand, the asymmetric channel is produced with a large yield in the isotopic distribution of fission products. That means, the barrier for the asymmetric channel has no very large differences in the heights of the barriers towards the phenomenological one. On other words, we assign the parameters of the potential barrier determined previously to the asymmetric channel. The potential barrier for the symmetric channel is obtained by adding an additional energy. This additional energy is obtained theoretically as the difference between the theoretical deformation energy of the symmetric channel and the theoretical deformation energy of the asymmetric channel at every deformation (or elongation). These differences can be obtained from the theoretical potential barriers plotted in Ref. [1]. In this reference, we determined the theoretical deformations energies and the effective masses of the system along the best trajectory in a configuration space spanned by the most important degree of freedom encountered in fission: elongation, necking-in and mass-asymmetry. Moreover, the theoretical barriers that include the dissipation were also determined. So, as a first approximation, the lower energy barrier corresponds to the asymmetric channel. Two different cases can be extracted: deformation energy with and without dissipation. Therefore, the problem is treated in two assumptions. Firstly, the lower energy barrier corresponds to the adiabatic theoretical barrier. That means no dissipation is included. In this particular case, the barrier of the asymmetric channel will be obtained by adding the difference between the theoretical adiabatic barriers of the two channels. Secondly, the lower energy barrier corresponds to the theoretical barrier with dissipation included. In this case, the difference to be added is given by the two barriers modified with their dissipation. A discussion is necessary. The phenomenological barrier is suitable for simulations. The heights of the barriers can be deduced from experimental values and the model will reproduce approximately the experimental data. Therefore, the use of the phenomenological barrier help us to investigate approximately only the effect of the difference in the deformation energy introduced by the difference in energy due to the mass-asymmetry or,
alternatively, the effective mass. From Ref. [1] we can also determine the effective mass obtained in the Wheeler-Wheeler approximation for different values of the elongation. An hybrid model emerges which convolutes the theoretical energy differences and the effective masses and the phenomenological simplicity. The mixing is realized in the very simple way, by using a linear interpolation based on a correspondence between some points \((R, \varepsilon)\) along the \(x\)-axis. \(R\) is the theoretical elongation that characterizes the shape of the theoretical model and \(\varepsilon\) is the dimensionless parameter used in simulating the phenomenological barrier. The correspondence was chosen for the two minimums, the two heights and the exit point. In the regions between these points the correspondence was assumed linear. New barriers for the symmetric channel are constructed. In Fig. 3 the phenomenological barrier corresponding to the asymmetric channel was plotted with full line in the upper part. The two barriers corresponding to the adiabatic assumption and barrier corresponding to the dissipation assumption of the symmetric channel are plotted with point-dashed and dashed lines, respectively. In the lower part of the figure, the effective mass for the symmetric channel is plotted with full line while the effective mass for the asymmetric channel is displayed with point-dashed line. The value of the constant inertia \(B_C\) is shown with a dashed line. The barrier constructed for the symmetric channel is very different from the phenomenological one being characterized by several local minimums. The symmetric channel barrier reaches the zero value (exit point) at \(\varepsilon \approx 0.73\) while the asymmetric channel is characterized by a value \(\varepsilon \approx 0.82\) of the exit point from the outer barrier. It follows that the transmission for the symmetric channel is not sensitive to the larger values of the theoretical effective mass obtained at \(\varepsilon > 0.73\). That explains the fact that the transmission in this case is comparable to that of the asymmetric channel, as it will be see below, despite the fact that the barrier heights of the asymmetric channel are lower. The penetrabilities of these barriers are displayed in Fig. 4 as function of the incident energy of the neutron. In the lower part of the Fig. 4 the penetrabilities of the asymmetric partition are plotted, that is, of the phenomenological barrier. In the case when the theoretical effective mass is used, supra-barrier resonances appear in the transmission of low amplitude. In this last case, the transmission does not reach the unity value for small excitation energy values above the height of the barrier. The transmission for a constant effective mass reaches the unity for very small energies above the height of the barrier. The distance between the sub-barrier resonances seems to not fluctuate strongly by using the constant effective mass or the theoretical one. In the case of the symmetric channel with dissipation and theoretical effective mass, the supra-barrier resonances fluctuate strongly. In the case of the asymmetric channel with theoretical effective mass the supra-barrier resonances are not so pregnant. That means, the supra-barrier resonances depend not only on the effective mass, but also
Investigation of the effect produced by the effective mass

Fig. 3. – Upper panel. The phenomenological barrier (which corresponds also to the asymmetric channel) is plotted with a full line as function of the dimensionless elongation parameter. The barrier of the symmetric channel obtained by adding the difference between the theoretical adiabatic deformation energies in the symmetric and asymmetric channels is plotted with dot-dashed line. The symmetric channel barrier obtained by taking into account the dissipation is displayed with dashed line. Lower panel. The effective mass for the symmetric channel is plotted with full line. The effective mass for the asymmetric channel is plotted with dot-dashed line. The constant inertia $B_c$ is given with a dashed line.

on the shape of the barrier. If the barrier shape deviates from that of the phenomenological one (oscillators) as is expected in the realistic case, the supra-
Fig. 4. – Barrier transmissions as function of the incident neutron energy. Upper panel refers to the symmetric channel. Full line: theoretical effective mass and dissipation included in the deformation energy. Dashed line: constant effective mass without dissipation. Dot-dashed line: constant mass with dissipation. Dot line: theoretical effective mass without dissipation. Lower panel refers to the asymmetric channel. Dot-dashed line: constant effective mass and full line theoretical effective mass.

barrier resonances must appear. Fluctuations of the same type can be simulated with triple humped barriers.

In Fig. 5, the partial fission cross-sections are plotted as function of the incident neutron energy for the two analyzed channels and the four different situations. The partial fission cross section of the asymmetric channel is always larger than that of the symmetric channel. The difference between the curves reflects the behaviors of the transmission previously discussed. At low values of \( E_n \), in the vicinity of zero, the difference seems to be of several orders of magnitude. This difference falls for \( E_n \approx 3–4 \) MeV, being larger in the panel 5(a) where the dissipation is included. At neutron incident energies of 40 MeV, it can be expected that the yield given by the symmetric channel surpasses the asymmetric one. This is not the case. That means, the phenomenological treatment
Fig. 5. – Partial fission cross section as function of the neutron incident energy for the four situations. Full line corresponds to the asymmetric channel. (a) Theoretical effective mass in both channels and dissipation included. (b) Constant effective mass in both channels and dissipation included. (c) Theoretical effective mass and without the dissipation. (d) Constant effective mass and without the dissipation.

cannot be used up to very large values of $E_n$ (greater than 10–15 MeV). It is also possible to find resonances in the fission cross section which are due to fluctuations of the effective mass but in the actual theoretical context, attributed to other effects.

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