SECOND LAW ANALYSIS OF INCOMPRESSIBLE VISCOUS FLOW THROUGH AN INCLINED CHANNEL WITH ISOTHERMAL WALLS

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In this paper, the second law analysis of a laminar flow of a viscous incompressible fluid through an inclined channel with isothermal walls is investigated. Based on some simplifying assumptions, analytical solutions for the fluid velocity and temperature are constructed. The expressions for the entropy generation rate and irreversibility ratio are obtained and the results are presented graphically and discussed quantitatively for several values of group parameter \( Br\Omega^{-1} \).

**Key words:** laminar flow, inclined heated channel, entropy generation, irreversibility ratio.

1. INTRODUCTION

The study of laminar flow of a viscous incompressible fluid through an inclined heated channel has many significant applications in thermal engineering and industries. Starting from petroleum drilling equipment to various industrial exchanger systems, this type of geometry can be observed. Meanwhile, the improvement in thermal systems as well as energy utilization during the convection in any fluid is one of the fundamental problems of the engineering processes, since improved thermal systems will provide better material processing, energy conservation and environmental effects, [7]. One of the methods used for predicting the performance of the engineering processes is the second law analysis. The second law of thermodynamics is applied to investigate the irreversibilities in terms of the entropy generation rate. Since the entropy generation is the measure of the destruction of the available work of the system, the determination of the active sites motivating the entropy generation is also important in upgrading the system performances. This method was introduced by Bejan [1–3] and followed by many other investigators e.g. [4, 7–11].

Recently, Saouli and Aiboud-Saouli [11] investigated second law analysis of laminar falling liquid film along an inclined heated plate. They considered the upper surface of the liquid film free and adiabatic and the lower wall is fixed.
with constant heat flux. Their results show that entropy generation number decreases transversely and increases for all values of group parameter. Likewise, they found that the irreversibility ratio decreases in the transverse direction and increases as the group parameter increases. Their results also revealed the possibility of irreversibility ratio $\phi \geq 1$ for some group parameter values i.e. the fluid friction irreversibility dominates over heat transfer irreversibility.

In this present study, special attention has been given to the combined effects of viscous dissipation and inclined channel uniform temperature on the entropy generation and irreversibility ratio. In the following sections, the problem is formulated, analysed, solved and discussed.

2. MATHEMATICAL FORMULATION

The geometry of the system under consideration in this present study is shown schematically in Fig. 1. It consists of an inclined channel with fluid flowing steadily downstream in the $x$-direction. The channel half-width is $h$ in the $y$-direction. Other physical properties of the fluid like viscosity and density are taken as constant.

The governing equations with the associated boundary conditions are given by

$$\mu \frac{d^2 u}{dy^2} + \rho g \sin \theta = 0,$$

Fig. 1. – Problem geometry.
\[ u \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}, \]  
\( (2) \)

No slip condition

\[ u(0) = 0, \]  
\( (3) \)

Symmetric condition

\[ \frac{du}{dy}(0) = 0, \]  
\( (4) \)

Inlet condition

\[ T(0, y) = T_0, \]  
\( (5) \)

Inclined channel walls temperature

\[ T(x, h) = T_w, \]  
\( (6) \)

Symmetric temperature condition

\[ \frac{\partial T}{\partial y}(x, 0) = 0, \]  
\( (7) \)

where \( T \) is the absolute temperature and \( \alpha \) the thermal diffusivity, \( T_0 \) is the inlet fluid temperature, \( T_w \) is the channel walls temperature, \( \rho \) the fluid density, \( \mu \) the fluid dynamic viscosity, \( g \) the acceleration due to gravity, \( u \) the fluid axial velocity and \( \theta \) is the inclined angle.

The following dimensionless variables and parameter are introduced,

\[ \tilde{u} = \frac{u \mu}{\rho gh^2 \sin \theta}, \quad \tilde{y} = \frac{y}{h}, \quad \tilde{x} = \frac{x \alpha \mu}{\rho gh^4 \sin \theta}, \quad \tilde{T} = \frac{T - T_0}{T_w - T_0}, \]  
\( (8) \)

Neglecting the bar symbol for clarity, the dimensionless governing equations are obtained as

\[ \frac{d^2 \tilde{u}}{dy^2} = -1, \]  
\( (9) \)

\[ u \frac{\partial \tilde{T}}{\partial x} = \frac{\partial^2 \tilde{T}}{\partial y^2}, \]  
\( (10) \)

\[ u(1) = 0, \]  
\( (11) \)

\[ \frac{du}{dy}(0) = 0, \]  
\( (12) \)

\[ T(0, y) = 0, \]  
\( (13) \)

\[ T(x, 1) = 1, \]  
\( (14) \)
Solving Eq. (9) with Eqs. (11)–(12), we obtain axial velocity as

\[ u(y) = \frac{1}{2} - \frac{y^2}{2}. \] (16)

The problem is now to solve Eq. (10) subject to the conditions Eqs. (13–15). We employed the analytical method of separation of variables and we obtain

\[ T(x, y) = \frac{24x}{24x + 5 - 6y^2 + y^4} \] (17)

### 3. ENTROPY GENERATION RATE

According to Mahmud and Fraser (2002), the entropy generation rate is defined as

\[ E_G = \frac{k}{T_0^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T_0} \left( \frac{\partial u}{\partial y} \right)^2, \] (18)

where \( k \) is the thermal conductivity.

The dimensionless entropy generation number may be defined by the following relationship:

\[ N_s = \frac{h^2 T_0^2 E_G}{k(T_w - T_0)^2} \] (19)

In terms of the dimensionless velocity and temperature, the entropy generation number becomes

\[ N_s = \frac{1}{Pe^2} \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \frac{Br}{\Omega} \left( \frac{\partial u}{\partial y} \right)^2 = N_x + N_y + N_f, \] (20)

where the dimensionless parameters \( Pe = \mu g h^3 \sin \theta / \alpha \mu \), \( Br = \rho^2 g^2 h^4 \sin^2 \theta / \mu k(T_w - T_0) \) and \( \Omega = (T_w - T_0) / T_0 \) are the Peclet number, Brinkman number and temperature different parameter respectively, \( N_x \) and \( N_y \) are the entropy generation by heat transfer due to both axial and transverse heat conduction respectively and \( N_f \) is the entropy generation due to fluid friction.

In convective problem, both fluid friction and heat transfer contribute to the rate of entropy generation. In order to have an idea which of the fluid or heat transfer dominates, a criterion known as the irreversibility ratio is defined by the following equation
\[ \phi = \frac{N_f}{N_x + N_y}. \] (21)

For \( 0 \leq \phi < 1 \) implies that heat transfer entropy generation dominates the irreversibility ratio, fluid friction dominates when \( \phi > 1 \) and \( \phi = 1 \), implies that both heat transfer and fluid friction have equal contribution of entropy generation to irreversibility ratio.

4. RESULTS AND DISCUSSIONS

The above mathematical analysis is valid for an incompressible viscous fluid flowing through an inclined channel with uniformly heated walls (i.e. with incline angle \( \theta > 0 \)). However, the model can be easily adapted to study the entropy generation rate in a plane Poiseuille flow. For numerical validation of our analytical results, we have taken \( Pe = 7.1, Br\Omega^{-1} = 0, 0.1, 0.4, 0.8, x = 0.1, 0.3, 0.5 \). The velocity profile is plotted in Fig. 2. It can be observed that the fluid velocity is parabolic with maximum value along the channel centerline and minimum value at the walls. In Fig. 3 we observed that the fluid temperature increases transversely with maximum value at the heated walls and minimum along the channel centerline. It is interesting to note that the fluid temperature generally increases in the longitudinal direction downstream. Figs. 4 and 5 show the distribution of entropy generation number. For all group parameters \( (Br\Omega^{-1}) \), the entropy generation rate increases in the transverse direction with maximum value at the heated walls and minimum value along the channel centerline due to zero velocity gradient at \( y = 0 \). Meanwhile, a general increase in the entropy generation rate is noticed towards the heated wall with an increase in the magnitude of the group parameter, while a decrease in the entropy generation rate is observed in the longitudinal direction downstream. This clearly implies that viscous dissipation as well as angle of inclination of the channel enhance
Fig. 3. – Temperature profiles for $x = 0.1$; $x = 0.3$; $x = 0.5$.

Fig. 4. – Entropy generation rate for $x = 0.3$; $\text{Pr} = 7.1$; $Br^{-1} = 0$; $Br^{-1} = 0.4$; $Br^{-1} = 0.8$.

Fig. 5. – Entropy generation rate for $Br^{-1} = 0.4$; $\text{Pr} = 7.1$; $x = 0.1$; $x = 0.3$; $x = 0.5$.

Fig. 6. – Irreversibility ratio for $x = 0.3$; $\text{Pr} = 7.1$; $Br^{-1} = 0.1$; $Br^{-1} = 0.4$; $Br^{-1} = 0.8$. 
entropy generation rate. In Figs. 6 and 7, the graph of irreversibility ratio is plotted as a function of transverse distance (y) for different values of group parameter and axial distance. The group parameter is an important dimensionless number for irreversibility analysis. It determines the relative importance of viscous effects to that of temperature gradient entropy generation. Irreversibility ratio profile increases gradually in the transverse direction from zero value along the channel centerline to its maximum value, then decreases towards the heated channel walls with heat transfer irreversibility dominating. It is very interesting to note that the heat transfer irreversibility dominates fluid friction irreversibility along the channel centerline, however, the value of irreversibility ratio may increase gradually above the unit value as the group parameter increases as well as in the longitudinal direction far downstream with fluid friction irreversibility dominating over the heat transfer irreversibility.

Fig. 7. – Irreversibility ratio for $Br\Omega^{-1} = 0.4$; $Pr = 7.1$;

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5. CONCLUSION

This paper presents the application of second law of thermodynamics to laminar flow of an incompressible viscous fluid through an inclined channel with isothermal walls. The velocity and temperature profiles are obtained analytically and used to compute the entropy generation number and irreversibility ratio for several values of group parameter ($Br\Omega^{-1}$) and axial distance. Generally, our results show that the heat transfer irreversibility dominates along the channel centerline while an increase in group parameter may cause fluid friction irreversibility to dominate near the channel heated walls.

REFERENCES