A wreath astroidal undulator model for an free-electron current laser is proposed. The proposed model is represented by a wreaths stack having axial symmetry relative to the $z$ axis. The projection of each wreath in a plane perpendicular on $z$ axis is an astroid (90 degree symmetry). The main feature of the model comes from the alternating dipoles created by each wreath. So the principal component of the magnetic field is mainly transversal on the symmetry $z$ axis. Therefore, the model will allow the study of non-linear dynamics of the electrons accelerated by the undulator’s magnetic fields.

1. INTRODUCTION

In free-electron laser (FEL) research and development one of the main trends is the elaboration of the compact devices [1–3]. The undulator is the principal component where the phenomenon of coherent radiation take place. This can be tuned over a broad range of wavelengths [4]. By a relativistic electron beam injected in a periodic magnetic field which is transverse to the direction of incoming beam ($Z$-direction) the coherent radiation is obtained. The field is produced by a undulator (or wiggler) which is a spatially periodic magnetic structure formed by permanent magnets or currents.

The electrons are forced to oscillate in $Y$ direction by the $B_x$ component of the magnetic field. So we obtained a coupling between this transverse motion and the transverse electric field $E_x$ of the electromagnetic wave propagating in the $Z$-direction. This effect of transverse coupling implies an axial bunching of the electrons which forces them to oscillate and lose energy. As a result a

coherent radiation is generated in the Z-direction. This radiation has a wavelength which is related to the spatial period of the magnetic field of the undulator and its maximum intensity.

2. UNDULATOR MODEL

In this new structure the current is circulating in alternating directions in a stack of wires. The projection of each wreath in a plane perpendicular on z axis is an astroid. The astroid is described by the following equations [5]:

$$x = R \cos \theta^3, \quad y = R \sin \theta^3,$$

where $R$ is the undulator radius. The undulator which is described by an astroid in $xy$ plane can be seen in Fig. 1.

![Fig. 1. – Transversal undulator projection.](image)

The magnetic field was computed by Biot-Savart law:

$$\vec{B} \approx \int_0^l \frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3}, \quad \theta_1 = 2\pi$$

where the wire element is $d\vec{l}$ and $\vec{r}$ represents the distance between a point on the wire and the one on the undulator axis $(X = 0, Y = 0, Z \neq 0)$.

The $B_z$ magnetic field component was numerically computed in SI units. In Fig. 2 the expression $B_z \approx B_z/(\mu_e \cdot \mu_0/I)$ vs. $z$ was represented in arbitrary units.
Fig. 2. – $B_{z_1}$ wreath astroid component.

Where $\mu_r = 1$ is the relative permeability; $I$ is the current (Amper); $\mu_0 = 4 \pi \cdot 10^{-7} \frac{H}{m}$ is the vacuum permeability and we have the following parameters: $(R = 0.02, \text{undulator step} = R)$.

In Fig. 3 the $x$ FFT of the normalized magnetic component in arbitrary units was represented.

Fig. 3. – The FFT of the normalized $x$ magnetic component.

In Fig. 4 the $y$ FFT of the normalized magnetic component in arbitrary units was represented.
In Fig. 5 the $z$ FFT of the normalized magnetic component in arbitrary units was represented.

From Figs. 3–5 we noticed that the principal component was the $y$ one, which was 3 times bigger than the $z$ one, which was also six hundred times bigger than the $x$ one.

3. CONCLUSIONS

In this preliminary paper a new transversal electromagnetic undulator or wiggler configuration for insert devices was described. The magnetic field is
easily adjusted with the current. The versatility of this structure permits new developments in the undulator design. From Fig. 2 (for $B_z$) results that the middle magnetic field aspect is mainly transversal. From FFT we also noticed that $x$ and $z$ magnetic components have smaller magnitudes than $y$ one. This preliminary data suggest to apply sequence-to-sequence transformation $T_{rm}$ to speed up the computation.

REFERENCES