Bone is a living medium which is regularly destroyed and reconstruct according various physical, chemical and biological phenomena strongly depending on mechanical behavior through several complex coupling process.

We present a new model allowing the computation of the physical properties of the human cortical bone considered as strongly heterogeneous medium, knowing the bony architecture and the physical properties of the two basic components: the collagen and the hydroxyapatite (Hap). The numerical simulations are based on the mathematical theory of homogenization. This method allows us to compute, firstly at the macroscopic level, the physical properties of bone and secondly at the microscopic scale, various mechanical information (stresses, strains or energy). Such a two-way investigation tool is well adapted to describe the process of information transport between structural levels.

Key words: piezo elasticity, homogenisation theory, multi scale, mechanical properties, cortical bone, collagen, hydroxyapatite.

1. INTRODUCTION

Bone is a living tissue which is always evolving, it changes its mass during the growth, it can change locally its architecture and its mechanical properties during the bone remodeling (as for instance in the case of consolidation of bone after fracture) and finally it changes its morphology with age. It optimizes simultaneously its structure and its physical properties in order to be well adapted to applied loads.

The description of the simultaneous elaboration of the architecture and the mechanical properties is necessary: unhappily, this mechanism is extremely complex. The structure of human cortical bone contents seven structural levels: macroscopic, osteonal, lamellar, fibrous, fibrillous, microfibrillous and tropocollagenous. The remodeling is made through a destruction phase


(osteoclasts destroy existing bone), a organic reconstruction phase (osteoblasts build the organic structure of next osteon) and a mineralization phase (mineral ions are apposed on this organic architecture).

With some developed mathematical tools, some answers are given to the understanding of this process. The main result that we obtained is to note that some properties are true only at a given scale:
1) the knowledge of physical properties of collagen and hydroxyapatite allows the computation of properties for cortical bone and the simulation, from a macroscopic information, of information in the osteonal structure.
2) the flow of viscous fluid inside a net of channels (Havers et Volkman) induces a visco-elastic behavior law with a long memory term.
3) the strong heterogeneity existing between collagen and Hap is responsible of a visco-elastic behavior with a short memory term.
4) introduction of piezo-elasticity for collagen fibers doesn’t change physical properties of cortical bone (which isn’t piezo) but points out transport of mineral ions according a specific process in resorption lacunae.

2. ARCHITECTURAL ORGANIZATION OF CORTICAL BONE

Cortical bone has an osteonal structure made on osteons and on interstitial system which is also made on old surmineralised osteons. Each osteon is composed by concentric lamellae, each lamella being made on oriented collagen fibbers embedded in hydroxyapatite crystals.

The orientations of the collagen fibbers can differ between two consecutive lamellae. The collagen fibbers are a set of fibrils, each fibril being composed of micro fibrils. A micro fibril is an helicoidal arrangement of three tropocollagen components.

Four levels have been used in order to determine macroscopic mechanical properties of human cortical bone: macroscopic (bone level), microscopic (osteonal level), mesoscopic (lamellar level) and nanoscopic (components level). Our new model introduce the fifth level (fibril’s level) which allows us to introduce the transport of mineralised ions and the mineralisation itself.
3. PIEZOElasticity PHenomenon

We assume that collagen is piezo-elastic medium and our aim is to compute the piezo-elastic properties at the cortical level. This phenomenon is modelled by the two following conservative laws:

\[ -\sum_{j} \frac{\partial}{\partial x_j} (\sigma_{ij}) = b_i, \quad i = 1, 2, 3 \]

\[ \sum_{i} \frac{\partial}{\partial x_i} (D_i) = 0, \quad i = 1, 2, 3 \]

with the classical notations for \( \sigma_{ij} \) and \( D_i \).

These equations are coupled by the two behaviours laws:

\[ \sigma_{ij} = \sum_{k,l} C_{ijkl} \cdot e_{kl} - \sum_{k} G_{kij} \cdot E_k \]

\[ D_i = \sum_{k,l} G_{ikl} \cdot e_{kl} - \sum_{k} a_{ik} \cdot E_k \]

where \( E \) and \( e_{kl} \) are respectively the electrical field and the strain tensor. The electrical field is classically linked to a potential by:

\[ E_k = -\frac{\hat{\partial} \phi}{\partial x_k}, \quad k = 1, 2, 3 \]

It is assumed that all elastic, piezo-electrical and dielectrical coefficients have properties of symmetry, positivity and are bounded.

4. PROCESS OF DETERMINATION OF THE HOMOGENIZED COEFFICIENTS

We consider that the Hydroxyapatite HAp is characterized by the longitudinal Young’s modulus (along the length of the osteon) \( E_1 \), the transverse Young’s modulus \( E_3 \) (along circumferential direction) and the Poisson’s ratio \( \nu \). For the collagen, a Young’s modulus \( E_{coll} \) and a Poisson’s ratio \( \nu_{coll} \) are also considered.

For these numerical values and with a nanoporosity which can be done, we can obtain by homogenisation the mechanical properties of the lamella \( (Q_{lam}) \).

Then, using classical rotations equations, one can compute the stiffness coefficients of the lamella \( (Q_{lam, \phi}) \) after rotation by an angle \( \phi \). In this way, we will have the mechanical properties of the lamella for collagen sticks alignment of \( \phi \) degrees from the long axis of the bone \( (Q_{lam, \phi}) \).
The orientation of the collagen fibers can differ between two consecutive lamellae. We consider two orientations $\varphi_1$ and $\varphi_2$ of the collagen fibers and we will calculate for these orientations the stiffness coefficients for the osteon ($Q_{ost}$). The possibility to take into account caniculae and lamellar interface is kept.

Then, considering four types of osteons (or the osteons of the same type), the interstitial system, and taking into account the macro porosity given by the Haversian and Volkman channels, we will finally find the physical properties of the cortical bone ($Q_{bone}$) at the macroscopic level.

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5. RESULTS AND DISCUSSION

Cortical bone, at macroscopic level, has only elastic properties. Indeed, numerical results for the piezo-elastic and dielectrical coefficients are very low ($\sim 10^{-16}$).

Substructures present piezo-elastic properties; we are able to get, from a macroscopic information on the stress field, the drawing of the potential field in an area of a lamella.

One can transform the macroscopic information on stresses on information on stresses at the osteonal level.

We also study the influence of the ratio longitudinal Young’s modulus / transverse Young’s modulus on the ratio $Q_{33}/Q_{11}$ (see Fig. 1). We can deduce an interval of physical coherence for the ratio $E_3/E_1$. An open problem is to explain the origin of this ratio.

Also, we can see, the effect of orientation of collagen fibers in lamellae (Fig. 2 and Fig. 3).
6. CONCLUSIONS

Knowing the physical properties of collagen and hydroxyapatite allows computation of properties for cortical bone and simulation, from macroscopic information, of information in the osteonal structure.

Piezo-elasticity of collagen fibbers doesn’t change physical properties of cortical bone (which isn’t piezo) but points out transport of mineral ions according a specific process in resorption lacunae.

The anisotropy of cortical bone is not induced, at macroscopic scale, by the only one haversian structure, but it has to be related, for the largest part, to the mechanical properties of $H_{ap}$ crystals at the nanoscopic scale.

REFERENCES