In this paper we continue our study about the energy of a regular black hole in
the Landau and Lifshitz prescription. The energy depends on the mass $m$ and charge $e$
of the black hole. We make a discussion of the energy in function of the value of
the mass $m$ and charge $e$ of the black hole. For $e = 0$ or at large distances, the
expression of energy reduces to the energy of the Schwarzschild metric that is given
by $m$. This is the same result as that obtained by Sharif for the charge $e = 0$ or at large
distances. The calculations are performed with the Maple program that has attached
the GrTensor II version 1.50 platform.

1. INTRODUCTION

The subject of the localization of energy continues to be an open one since
Einstein [1] has given his important result of the special theory of relativity that
mass is equivalent to energy. Misner et al. [2] sustained that to look for a local
energy-momentum means that is looking for the right answer to the wrong
question. Also, they concluded that the energy is localizable only for spherical
systems. On the other hand, Cooperstock and Sarracino [3] demonstrated that if
the energy is localizable in spherical systems then it is also localizable in any
space-times. Bondi [4] gave his opinion that “a nonlocalizable form of energy is
not admissible in general relativity, because any form of energy contributes to
gravitation and so its location can in principle be found”.

Related on the method that used the energy-momentum complexes we can
say that there are doubts that these prescriptions could give acceptable results for
a given space-time. This is because most of the energy-momentum complexes
are restricted to the use of particular coordinates. But some interesting results
obtained by several authors [5, 6] demonstrated that the energy-momentum
complexes are good tools for evaluating the energy and momentum in general
relativity.

In this paper we study the energy distribution of a regular black hole in the
Landau and Lifshitz prescription [7]. The energy distribution depends on the
mass $m$ and charge $e$ of the black hole. We make a discussion in function of the value of the mass $m$ and charge $e$ of the black hole. Also, we give the program for the Landau and Lifshitz energy-momentum complex. Through the paper we use geometrized units ($G = 1$, $c = 1$) and follow the convention that Latin indices run from 0 to 3.

2. ENERGY OF THE BLACK HOLE

The solution that we study was given by Bardeen [8] and represents a regular black hole obeying weak energy condition, and it was powerful in shaping the direction of research on the existence or avoidance of singularities. The metric is given by

$$ds^2 = \left(1 - \frac{2mr^2}{(r^2 + e^2)^\frac{3}{2}}\right)dt^2 - \frac{dr^2}{1 - \frac{2mr^2}{(r^2 + e^2)^\frac{3}{2}}} - r^2d\theta^2 - r^2 \sin^2 \theta d\phi^2.$$

(1)

For the charge $e = 0$ the metric reduces to the Schwarzschild metric.

In the Landau and Lifshitz prescription [7] we obtained for the energy distribution [9] the expression

$$E = -\frac{mr^3}{(r^2 + e^2)^\frac{3}{2} + 2mr^2}.$$  

(2)

For $e = 0$ or at large distances, the expression of energy reduces to the energy of the Schwarzschild metric that is given by $m$. This is the same result as that obtained by Sharif for the charge $e = 0$ or at large distances.

From (2) it results that the energy distribution depends on the mass $m$ and charge $e$ of the black hole. If the charge $e$ vanishes we obtain for the energy distribution the value

$$E = -\frac{mr^3}{r^3 - 2mr^2}.$$  

(3)

We give the Maple program, GrTensor version 1.50 that we used for calculating the components of the Landau and Lifshitz energy-momentum complex:

readlib(grii):
grtensor();
grdef(`L3(^m^j^n^k):=-detg*(g^m^n*g^n^k-g^m^k*g^n^n)`):
Created definition for L3(up,up,up):
qload(bardeen);
Calculating ds for bardeen ... Done. (0.000000 sec.)
grcalc(L3(up,up,up,up)):
Calculating detg for bardeen ... Done. (0.000000 sec.) Calculating g(up,up) for 
bardeen ... Done. (0.000000 sec.) Calculating detg for bardeen ... Done. 
(0.000000 sec.) Calculating L3(up,up,up,up) for bardeen ... Done. (0.000000 
sec.)
grdef(`L4^m^j^n:=L3^m^j^n^k,k`):
grcalc(L4(up,up,up)):
Calculating L3(up,up,up,up,pdn) for bardeen ... Done. (1.000000 sec.)
Calculating L4(up,up,up) for bardeen ... Done. (0.000000 sec.)
grdisplay(L4(up,up,up));
In the following, we make a discussion related on the graphic 
representations of the expressions of energy given by (2) and (3). For the 
expression given by (2), first consider the value \( m = 1 \) for the mass of the black 
hole and we choose only positive values for the charge \( e \) of the black hole and, 
also, positive values for the \( r \) parameter. We have \( e = 0 \ldots 0.1 \) and \( r = 0 \ldots 10 \). We 
get the Fig. 1.

![Fig. 1](image_url)

The energy distribution takes negative and, also, positive values.

Lets see some modifications in the graphic representation of the expression 
of energy distribution \( E \). First, we consider that we take \( e = 0.1 \ldots 1 \) and 
\( r = 10 \ldots 100 \). The graphic representation of the energy distribution is given in 
Fig. 2.

If in the first case we have negative values and, also, positive values for the 
energy distribution in the second case we have only positive values for the 
energy of the black hole. The maximum of the energy is less than in the first 
case.

Now, if we take for the charge \( e = 1 \ldots 10 \) and for the \( r \) parameter \( r = 0 \ldots 100 \) 
we get the graphic representation from Fig. 3.
The energy distribution of the black hole takes only positives values. The maximum of the energy is less than in the first case and greater than in the second case.

3. DISCUSSION

One of the most interesting and challenging problems of relativity, the energy and momentum localization remained an open issue because the numerous attempts at constructing an energy-momentum density don’t give a generally accepted expression. As we pointed out above, we can obtain good results related on the localization of energy with several energy-momentum complexes, particularly with the Landau and Lifshitz energy-momentum complex. The importance of the energy-momentum complexes was pointed out by Chang, Nester and Chen [10] who showed that the energy-momentum complexes are actually quasilocal and legitimate expression for the energy-momentum. They concluded that there exist a direct relationship between
energy-momentum complexes and quasilocal expressions because every energy-
momentum complexes is associated with a legitimate Hamiltonian boundary
term.

We use the Landau and Lifshitz prescription to compute the energy
distribution of a regular charged black hole solution and, also, we make a
discussion of the result.

The energy of the regular black hole solution given by Bardeen [9] depends
on the mass $m$ and charge $e$ of the black hole. In the case $e = 0$ or at large
distances, the expression of energy reduces to the energy of the Schwarzschild
metric that is given by $m$. This result is the same with the one obtained by Sharif
for the charge $e = 0$ or at large distances.

We make a comparison between our results for the energy distribution
taking different values for the charge $e$ of the black hole and $r$ parameter. In all
the cases we have the same value for the mass $m$ of the black hole. The energy
distribution takes negative and, also, positive values. The maximum of the
energy is reached in the first case. These results sustain that the Landau and
Lifshitz energy-momentum complex is a powerful tool for obtaining the energy
distribution in a given space-time.

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