SUPER-ČERENKOV RADIATION: A NEW PHENOMENON USEFUL FOR RICH DETECTORS

D. B. ION1,2, M. L. ION3

1 TH Division, CERN, CH-1211 Geneva 23, Switzerland
2 NIPNE-HH, Bucharest P.O. Box MG-6, Romania
3 Bucharest University, Department of Atomic and Nuclear Physics, Bucharest, Romania

October 25, 2006

In this paper the Super-Čerenkov radiation (SCR) as a new phenomenon which includes in a more general and exact form the usual Čerenkov effect is presented. The Super-Čerenkov effect at Čerenkov threshold in the radiators of RICH detectors is investigated. The results on the experimental test of the super-Čerenkov coherence conditions are presented. The SCR-predictions are verified experimentally with high accuracy $\chi^2/n_{dof} = 1.47$ by the data on the Čerenkov ring radii of electron, muon, pion and kaon, all measured with RICH detector. Moreover, it is shown that the Super-Čerenkov phenomenon can explain not only subthreshold ČR but also the observed secondary rings (or anomalous Čerenkov radiation) observed at CERN SPS accelerator. The influence of medium on the particle propagation properties is also estimated and the refractive properties of electrons, muons, pions, in the radiator C4F10Ar are obtained. So, we proved that the refractive indices of the charged elementary particles in medium are also very important for the RICH detectors, especially at low and intermediate energies.

1. INTRODUCTION

The classical theory of the radiation emitted by charged particles moving with superluminal velocities were traced back to Thomson, Heaviside and Sommerfeld. In fact, Heaviside [1] considered the Čerenkov radiation in a nondispersive medium. He considered this topic many times over the next 20 years, deriving most of the formalism [2] of what is now called Čerenkov radiation ($\mathcal{CR}$). In 1904 Sommerfeld [3] considered radiation from a charge moving in a vacuum at a velocity faster than light velocity ($v > c$) and close approached the formulation of the theory of ČR-effect. But Sommerfeld did not applied his results for the particles motion in a refractive (transparent) medium. Moreover, we must add that the ČR have been observed before the Čerenkov and

Vavilov by Mallet and other authors but cannot be regarded as the discovery of ČR since the essential characteristic features of this radiation were not revealed and it was not understood that the observed effect is clearly different from luminescence. Consequently, the realizable case of radiation from a charge moving with a constant velocity greater than the phase velocity of light in a dielectric medium was discovered independently in an experiment in 1934 by Čerenkov [4]. So, doing justice [2] to Heaviside and Sommerfeld, we must recall that the classical theory of the ČR phenomenon in a dispersive medium was first formulated by Frank and Tamm [4] in 1937. This theory explained all the main features of the radiation observed by Čerenkov [4]. The quantum theoretical approach to the ČR-problem was developed by many authors (see refs. in [5]).

The remarkable properties of the Čerenkov radiation find wide applications in practice especially in high energy physics where it is extensively used in experiments for counting and identifying relativistic particles in the fields of elementary particles, nuclear physics and astrophysics. A short review of Čerenkov radiation and its use for particle identification with threshold and differential counters is presented in Refs. [6]. In the last decades the Čerenkov radiation (ČR) is the subject of many studies related to extension to the nuclear media [7–9] as well as to other coherent particle emission via Čerenkov-like mechanisms [10]. The generalized Čerenkov-like effects based on four fundamental interactions has been investigated and classified recently in ref. [11]. In particular, this classification includes the nuclear (mesonic, γ, weak boson)-Čerenkov-like radiations as well as the high energy component of the coherent particle emission via (baryonic, leptonic, fermionic)-Čerenkov-like effects. In 1999, G. L. Gogiberidze, L. K. Gelovani and E. K. Sarkisyan performed the first experimental test [12] of the pionic Čerenkov-like effect in Mg-Mg collisions at 4.3 GeV/c/nucleon and obtained a good agreement with the position and width of the first pionic Čerenkov-like band predicted in ref. [11].

In essence, it was revealed (see ref. [4]) by Čerenkov, Tamm and Frank that a charged particle moving in a transparent medium with an refractive index \( n_\gamma \), and having a speed \( v_x \) greater than phase velocity of light \( (v_{\gamma ph} = 1/n_\gamma) \) will emit Čerenkov radiation (ČR) at a polar emission angle \( \theta_c \) relative to the direction of motion given by the relation (we adopted the system of units \( \hbar = c = 1 \))

\[
\cos \theta_c = \frac{v_{\gamma ph}}{v_x} = \frac{1}{n_\gamma v_x} \leq 1
\] (1)
However, by recent experimental observations of the subthreshold [13] and anomalous Čerenkov radiations [14] it was clarified that some fundamental aspects of the ČR can be considered as being still open and that more theoretical and experimental investigations on the ČR are needed. So, these new results stimulated new theoretical investigations [15–16] (using the ČR correct kinematics) leading to the discovery that ČR is in fact only a component (the low energy component) of a more general phenomenon called by us the Super-Čerenkov radiation (SČR) characterized by the Super-Čerenkov coherence condition [16]

$$\cos \theta_{SC} = v_{sph} \cdot v_{\gamma ph} \leq 1$$

(2)

where $v_{sph}$ and $v_{\gamma ph}$ are phase velocities of the charged particle and photon in the medium, respectively.

We must underline that the discovery of the subthreshold ČR as well as experimental observation of the anomalous Cherenkov rings by Vodopianov et al. [14] was of decisive importance for us to formulate the SČR-theory [15–16] which includes the above mentioned new phenomena. Moreover, we can see that the SČR coherence condition (2) is obtained in a natural way from the energy-momentum conservation law when the influence of medium on the propagation properties of the charged particle is taken into account. Therefore, the problem of the experimental test of Super-Čerenkov coherence condition (2) is of great interest not only for the fundamental physics but also for practical applications to the particle detection. Recently such a test was performed [16] by using the Ring Imaging Cherenkov (RICH) detectors.

Now, let us apply the theory of Super-Čerenkov radiation (SČR) to predict some characteristic feature of the subthreshold Čerenkov-like effects in the RICH detectors.

2. THE SUPER-ČERENKOV RADIATION (SČR)

Let start with an electromagnetic decay

$$1 \rightarrow \gamma + 2$$

(3)

in a (dielectric, nuclear or hadronic)-medium (we will work in the system of units $\hbar = c = 1$), described in Fig. 1, where a photon $\gamma$ [with energy $\omega$, momentum $k = \omega Re\ n_\gamma(\omega)$ and refractive index $n_\gamma(\omega)$] is emitted in a (dielectric, nuclear...
or hadronic) medium from an incident charged particle (with charge \( Z_e \), energy \( E_1 \), momentum \( \mathbf{p}_1 = \text{Re} \, n_1(E_1) (E_1^2 - M^2)^{1/2} \), rest mass \( M \), the refractive index \( n_1(E_1) \) that itself goes over into a final particle 2 (with charge \( Z_e \), energy \( E_2 \), momentum \( \mathbf{p}_2 = \text{Re} \, n_2(E_2) (E_2^2 - M^2)^{1/2} \), rest mass \( M \), the refractive index \( n_2(E_2) \)). The refractive index \( n_x(E_x) \) of any particle \( x \) (with the energy \( E_x \), momentum \( \mathbf{p}_x \), rest mass \( M_x \)) in a medium composed from the constituents \( c \) will be described in standard way by the Foldy-Lax formula \([17]\)

\[
n_x^2(E_x) = 1 + \frac{4\pi \rho}{E_x^2 - M_x^2} \cdot C(E_x) \overline{f}_{xc\rightarrow xc}(E_x)
\]  

(4)

where \( \rho \) is the density of constituents \( c \), \( C(E_x) \) is a coherence factor, \( \overline{f}_{xc\rightarrow xc}(E_x) \) is the averaged forward xc-scattering amplitude. \( C(E_x) = 1 \) when the medium constituents are randomly distributed. The phase velocity \( v_{\text{ph}}(E_x) \) of any particle \( x \) in medium is modified as follows:

\[
v_{\text{ph}}(E_x) = \frac{E_x}{p_x} = \frac{v_{\text{ph}}^0(E_x)}{\text{Re} \, n_x(E_x)}
\]  

(5)

Now, using the energy-momentum conservation

\[
E_1 = \omega + E_2, \quad \mathbf{p}_1 = \mathbf{k} + \mathbf{p}_2
\]  

(6)

we obtain

\[
\cos \theta_1 = v_{\text{ph}}(E_1) v_{\text{ph}}^0(\omega) + \frac{1}{2p_1k} [-D_1 + D_2 - D_\gamma]
\]  

(7)

\[
\cos \theta_2 = v_{\text{ph}}(E_1) v_{\text{ph}}(E_2) + \frac{1}{2p_1p_2} [-D_1 - D_2 + D_\gamma]
\]  

(8)
where $D_x$, $x = B_1, B_2, \gamma$, are the mass shell relations in medium for $x$-particle and are given by

$$D_x = E_x^2 - p_x^2$$

We note that the second terms from the right side of Eqs. (7)–(8) can be considered as quantum correction to the first one. So, the semiclassical angles are given by:

$$\cos \theta_{1\gamma} = v_{1ph}(E_1)v_{\gamma ph}(\omega), \quad \cos \theta_{12} = v_{1ph}(E_1)v_{2ph}(E_2),$$

respectively.

(a) **Semiclassical Theory of SČR.** A semiclassical theory of SČR can be developed step by step in a similar way with the classical theory of ČR [4] and here we present only some final results for the case of a transparent nondispersive medium.

We note that, if the influence of medium on the propagation properties of the charged particles is neglected, then from Eq. (2) we get Eq. (1), since according to the duality relation we have $v_x^{-1} = v_{xph}$. Therefore, the SČR-coherence condition (2) include in a more general and exact form the ČR-condition (1). So, it is easy to see that the classical intensity of the Čerenkov radiation can now be written in a more general and form

$$\frac{dN}{d\omega}(SČR) \approx Z^2\alpha L \sin^2 \theta_{1\gamma} = Z^2\alpha L \left[1 - v_{1ph}^2(E_1)v_{\gamma ph}^2(\omega)\right],$$

where $\frac{dN}{d\omega}(SČR)$ is the number of photons emitted in the energy interval: $(\omega, \omega + d\omega)$, and $L$ is the length of path. Hence, if the influence of medium on the propagation properties of charged particles is taken into account (or equivalently when $Re n_1(E_1)$), then at the usual Čerenkov threshold we have

$$\frac{dN}{d\omega}(SČR)_{ČRth} \approx Z^2\alpha L \left[1 - \left(\frac{1}{Re n_1(E_1)}\right)^2\right]$$

Therefore, the existence of the subthreshold Čerenkov radiation can be obtained for $Re n_1(E_1) > 1$.

(b) **Quantum Theory of SČR [16].** Now, we start with a two-body spin $(1/2^+ \rightarrow \gamma + 1/2^+)$ decay in a (dielectric, nuclear, or hadronic) medium where the propagation properties of all three particles (see Fig. 1) are changed according to the eqs. (4)–(5). Next, for simplicity we consider that the same interaction Hamiltonian as in the ČR-theory with some modifications of the source fields in medium can describe the coherent $\gamma$-emission in all SČR-sectors. Then, we obtain that the intensity of the *Super-Čerenkov radiation* for transparent (nonabsorbent) media can be written in the following general form
\[
\frac{d^2 N_{SCR}}{dtd\omega} = \frac{\alpha Z^2}{v_1 |n_{B_1}|^2 |n_{B_2}|^2 |n_r|^2} \frac{k \, dk}{\omega \, d\omega} S \cdot \Theta(1 - \cos \theta_{SC})
\]

where \( N_{SCR} \) is the total number of the SCR -photons, \( \Theta(1 - \cos \theta_{SC}) \) is Heaviside step function, while the spin factor \( S \) is given by

\[
S = \frac{(E_1 + M)(E_2 + M)}{4E_1E_2} \cdot \left[ \frac{p_1^2}{E_1 + M} + \frac{p_2^2}{E_2 + M} + 2 \left( \hat{e}_k \cdot \hat{p}_1 \right) \left( \hat{e}_k \cdot \hat{p}_2 \right) - \left( \hat{e}_k \times \hat{p}_1 \right) \left( \hat{e}_k \times \hat{p}_2 \right) \right] (13)
\]

where the vector \( \hat{e}_k \) is the photon polarization vector for a given photon momentum \( \vec{k} \). As in the usual case of ČR-theory, for a given vector \( \vec{k} \) we choose two orthogonal photon spin polarization directions, corresponding to a polarization vector \( \hat{e}_k \) perpendicular and parallel to the decay plane \( \vec{Q} \) given by the vectors \( \vec{p}_1 \) and \( \vec{k} \), respectively. Then, from eq. (13) we get the following expressions of the spin factors \( S_{\perp} \) and \( S_{\parallel} \):

\[
S_{\perp} = \frac{(E_1 + M)(E_2 + M)}{4E_1E_2} \left[ \frac{\vec{p}_1}{E_1 + M} - \frac{\vec{p}_2}{E_2 + M} \right]^2 (14)
\]

\[
S_{\parallel} = v_1 \text{Re} n_1 \, v_2 \text{Re} n_2 \, \sin \theta_{1\gamma} \, \sin \theta_{2\gamma} (15)
\]

where \( v_i, i = 1, 2 \) are the corresponding particle velocities in vacuum.

Now, one can see that Heaviside function \( \Theta(1 - \cos \theta_{SC}) \) is 1 at least in two physical \( \gamma \)-energy regions defined by the inequality (see Fig. 2): \( \cos \theta_{2\gamma} \approx v_{\gamma ph}(\omega)v_{\gamma ph}(E_2) \leq 1 \). Hence, the low \( \gamma \)-energy sector is that where \( \theta_{SC} \approx \theta_{1\gamma} \), while the high \( \gamma \)-energy sector is that where \( \theta_{SC} \approx \theta_{2\gamma} \). Therefore, the limiting low \( \gamma \)-energy SCR-sector can be identified as extended \( \gamma \)-Čerenkov domain where the condition: \( v_{\gamma ph}(\omega)v_{\gamma ph}(E_2) \leq 1 \) is fulfilled, while limiting high \( \gamma \)-energy SCR-sector can be identified as extended source-Čerenkov-like domain, in the sense that the charged particle spontaneously decays into a high \( \gamma \)-energy photon fulfilling a generalized Čerenkov-like relation: \( v_{\gamma ph}(\omega)v_{\gamma ph}(E_2) \leq 1 \).

Of course, in the limit \( n_1(E_1) \to 1 \) these physical regions are going (see Fig. 3) in the corresponding ČR-like domains where the respective conditions: \( v_{\gamma ph}(\omega) \leq v_1 \) and \( v_{\gamma ph}(E_2) \leq v_1 \), are satisfied. In conclusion the Super-Čerenkov effect
Fig. 2. – Schematic description of the Super-Čerenkov radiation as a two-body decay with two main limiting components.

Fig. 3. – Unified description of the two Čerenkov-like limits of the Super-Čerenkov radiation (SČR) when $n_1(E_1) = 1$. 
cannot be confused with the usual Čerenkov radiation since the SČR can be considered as a continuous two-body decay in medium which includes not only two generalized Čerenkov-like phenomena but also their interference effects. The relations (12)–(15) includes in a general and unified way all the main predictions of the Super-Čerenkov radiation from which the results from Table 1 are obtained as two particular limiting cases.

Table 1

<table>
<thead>
<tr>
<th>Name</th>
<th>SČR-Low γ-Energy sector (LE)</th>
<th>SČR-High γ-Energy sector (HE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 SČR emission condition</td>
<td>$v_{1ph} \cdot v_{2ph} \leq 1$</td>
<td>$v_{1ph} \cdot v_{2ph} \leq 1$</td>
</tr>
<tr>
<td>2 SČR-decay angle</td>
<td>$\cos \theta_{1} \approx v_{1ph} \cdot v_{2ph}$ since $\theta_{SC} = \theta_{2r} \approx \theta_{1}$</td>
<td>$\cos \theta_{12} \approx v_{1ph} \cdot v_{2ph}$ since $\theta_{SC} = \theta_{12}$</td>
</tr>
<tr>
<td>3 SČR-threshold velocity</td>
<td>$v_{thr}(SČ) = \frac{1}{n_{1}n_{2}}$</td>
<td>$v_{thr}(SČ) = \frac{1}{n_{1}n_{2}}$</td>
</tr>
<tr>
<td>4 Maximum SČR-emission angle</td>
<td>$\theta_{\gamma} = \arccos \left( \frac{1}{n_{1}n_{2}} \right)$</td>
<td>$\theta_{\gamma} = \arccos \left( \frac{1}{n_{1}n_{2}} \right)$</td>
</tr>
<tr>
<td>5 SČR-spectrum</td>
<td>$\frac{dN_{\gamma}}{d\omega} = \alpha Z^2 L \sin^2 \theta_{1}$</td>
<td>$\frac{dN_{\gamma}}{d\omega} = \alpha Z^2 L/2$</td>
</tr>
<tr>
<td>6 SČR-photon polarization</td>
<td>100% $\hat{\epsilon}_{\gamma} \parallel Q$</td>
<td>100% $\hat{\epsilon}_{\gamma} \perp Q$</td>
</tr>
</tbody>
</table>

The results of quantum theory presented here are complete for the Super-Čerenkov radiation (SČR) produced by any spin-1/2 particle [such as $(e^{\pm}, \mu^{\pm}, \tau^{\pm})$-leptons, $(\rho, \Sigma^{\pm}, \Xi^{-}, \Omega^{-})$-baryons etc.] moving in a (dielectric, nuclear or hadronic)-medium, with its phase velocity satisfying the Super-Čerenkov coherence condition (2).

3. EXPERIMENTAL TESTS OF SČR

The problem of the experimental test of Super-Čerenkov coherence condition (2) is of great interest not only for the fundamental physics but also for practical applications to the particle detection. Such a test can be performed by using a Ring Imaging Čerenkov (RICH) detectors (see ref. [18]).

In RICH detectors, particles pass through a radiator, and a spherical mirror focuses all photons emitted at $\theta_{SČ}$ along the particle trajectory at the same radius $r_{SČ} = (R/2)\tan \theta_{SČ}$ on the focal plane. Photon sensitive detectors placed
at the focal plane detect the resulting ring images in the RICH detector. So, RICH-counters are used for identifying and tracking charged particles. Čerenkov rings formed on a focal surface of the RICH provide information about the velocity and the direction of a charged particle passing the radiator. The particle’s velocities are related to the Čerenkov angle $\theta_C$ or to the Super-Čerenkov $\theta_{SC}$ by the relation (1) (or (2), respectively). Hence, these angles are determined by measuring the radii of the rings detected with the RICH. In ref. [18] a $C_4F_{10}Ar(75:25)$ filled RICH-counter read out was used for measurement of the Čerenkov ring radii. Fig. 5a shows the experimental values of the ring radii of electrons, muons, pions and kaons measured in the active area of this RICH-detector. The saturated light produced from electrons was a decisive fact to take an index of refraction $n_e = 1.00113$ for the radiator material. The absolute values for excitation curves of electron, muon, pion and kaon, shown by dashed curves in Fig. 5a, was obtained by using this value of refractive index in formula: $r_C(p) = (R/2)\tan\theta_C(p)$. The solid curves show the individual best fit of the experimental ring radii with eq. $r_{SC}(p) = (R/2)\tan\theta_{SC}(p)$ (see Table 1).

### Table 2

The best fit parameters of experimental ring radii with the Super-Čerenkov prediction

<table>
<thead>
<tr>
<th>Particle</th>
<th>Number of exp. data</th>
<th>$10^3 a^2$ (GeV/c)</th>
<th>$\chi^2/n_{dof}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>6</td>
<td>-0.081 ± 0.101</td>
<td>0.468</td>
</tr>
<tr>
<td>$\mu$</td>
<td>4</td>
<td>1.449 ± 0.098</td>
<td>3.039</td>
</tr>
<tr>
<td>$\pi$</td>
<td>7</td>
<td>2.593 ± 0.167</td>
<td>0.234</td>
</tr>
<tr>
<td>$K$</td>
<td>1</td>
<td>21.140 ± 2.604</td>
<td>$&lt; 10^{-14}$</td>
</tr>
<tr>
<td>All data</td>
<td>18</td>
<td>$\langle a/m\rangle^2 = 0.1211 ± 0.0053$</td>
<td>1.47</td>
</tr>
</tbody>
</table>

For the particle refractive index we used the parametrization

$$n_x^2(p) = 1 + a^2/p^2, \quad v_x = p/\sqrt{p^2 + m^2}$$  \hspace{1cm} (16)$$

where $p$ is the particle momentum in the vacuum. In the paper [16] we fitted all the 18 experimental data on the ring radii from ref. [18] with our Super-Čerenkov prediction formula (see Fig. 4)

$$r_{SC}(p/m) = \frac{R}{2}\tan\theta_{SC}$$  \hspace{1cm} (17)$$

and we obtained the following consistent result (see Fig. 5b). The best fit parameters are as follows: $\langle a/m\rangle^2 = 0.12109 ± 0.00528$ and $\chi^2/n_{dof} = 1.47$, where
Fig. 4. – Schematic description of a RICH detector radiator.

$n_{\text{dof}} = 16$ is the number of degree of freedom (dof). The $r_{SC}(p/m)$ scaling function [16] together with all experimental data on the ring radii of the electron, muon, pion and kaon, are plotted as a function of the scaling variable $(p/m)$ in Fig. 5b. Now combining eqs. (10) and (16) we obtain

$$\frac{dN}{d\omega} (SCR) = Z^2 \alpha L \left[ 1 - \frac{1}{n_f^2} \cdot \frac{p^2 + m^2}{p^* + a^2} \right]$$

Therefore, at ČR threshold we get:

$$\frac{dN}{d\omega} (SCR) |_{\text{ČR thr}} \approx Z^2 \alpha L \left[ \frac{a_s}{m_s} \right]^2 \frac{(n_f^2 - 1)}{1 + \left[ \frac{d_s}{m_s} \right]^2 (n_f^2 - 1)}$$

(18)

or

$$\frac{dN}{d\omega} (SCR) |_{\text{ČR thr}} \bigg|_{\omega = 1} = \left[ \frac{a_s}{m_s} \right]^2 \cdot \frac{n_f^2}{1 + \left[ \frac{a_s}{m_s} \right]^2 (n_f^2 - 1)} = 0.1301 \pm 0.088$$

(19)
Fig. 5. – Experimental test of the SČR condition (2) by using (17) and the Čerenkov ring radii of the electrons, muons, pions and Kaons, obtained by Debbe et al [18] with a RICH detector. (a) The SČR-predictions (solid lines) and ČR-predictions (dashed lines) are compared with the experimental ring radii. (b) The scaling of the SČR-predictions (solid lines) and ČR-predictions (dashed lines) as functions of the $(p_x/m_x)$-variable (see text).

since

$$p_{thr}(\tilde{C})/m = \frac{1}{(n_{\gamma}^2 - 1)^{1/2}} = 21.029$$

$$p_{thr}(S\tilde{C})/m = \frac{1}{(n_{\gamma}^2 - 1)^{1/2}} \left[ 1 - \left( \frac{\alpha}{m_x} \right)^2 n_{\gamma}^2 \right]^{1/2} = 18.477$$ (20)

Moreover, we can estimate the $\theta_{SC}$ and the SČ-ring radius $r_{SC}$ at Čerenkov thresholds and we obtain:
\[ \theta_{SC}(v_{Cthr}) = \arctan \left( \frac{\alpha_x}{m_x} \right) \left( n^2_{\gamma} - 1 \right)^{1/2} = 0.948 \pm 0.021 \text{deg} \quad (21) \]

\[ r_{SC}(\mathcal{C}R - thr) = \frac{R}{2} \left( \frac{\alpha_x}{m_x} \right) \left( n^2_{\gamma} - 1 \right)^{1/2} = (1.466 \pm 0.055) \text{ cm} \quad (22) \]

In Fig. 6 we present the results for S\( \mathcal{C} \)-Yields defined as \( Y_{SCR}(p) = \frac{N_{SCR}(p)}{N_{SCR}(\infty)} \). Also the difference \( D_T(p) = Y_{SCR}(p) - Y_{CR}(p) \) is calculated and given in Fig. 7. What is interesting is that \( D_T(p) \) is maximum at \( \mathcal{C} \)-erenkov threshold (see eq. (20)).

### 4. ANOMALOUS ČERENKOV RINGS

The anomalous Čerenkov ring observed recently by Vodopianov et al. [14] at SPS accelerator at CERN Pb-beam can be considered also as experimental signature of the HE-Super-Čerenkov component (see Fig. 4) since both Pb and high-energy \( \gamma \) after spontaneous emission of a photon can produce secondary anomalous ČR-rings. One of the characteristic feature of the predicted by S\( \mathcal{C} \)-R-effect (see Fig. 4) is that the secondary anomalous ČR-rings must have a constant inclination angle \( \alpha \) relative to beam direction given by the relation

\[ \cos \alpha = \cos \theta_{12} = v_{1ph}(E_1) \cdot v_{2ph}(E_2) \quad (23) \]

This first important condition is verified experimentally with high accuracy by four from seven anomalous ČR-rings observed in ref. [14]. Therefore these ČR-anomalous rings can be interpreted as being produced by the photons emitted by secondary Pb (see again Fig. 4). Then, it is easy to show that the Čerenkov angles \( \cos \theta' \) must be given by a relation of form:

\[ \cos \theta' = v_{1ph}(\alpha) \cdot v_{2ph}(E_2) = \frac{1}{n_{\gamma}} \cdot \frac{1}{n_2v_2} \quad (24) \]

Therefore, the velocities higher than unity inferred by Vodopianov et al. [14] from their anomalous ČR-rings, must be divided by the refractive index \( n_2 > 1 \) of the secondary Pb. Consequently, their anomalous ČR-rings [14] cannot be interpreted as being produced by tachyons.
5. SUMMARY AND CONCLUSIONS

In this paper the Super-Čerenkov effect as a new dual coherent particle production mechanism is presented. The main results and conclusions can be summarized as follows:

(i) The Super-Čerenkov phenomenon can be considered as a continuous two-body decay in medium which is possible only in two distinct limiting physical regions where the Super-Čerenkov coherence condition (2) can be fulfilled. One of them is at very low γ-energies (LE) where

\[ v_{\gamma ph}^{-1}(E_i) \geq v_{\gamma ph}(\omega), \quad \text{(extended } \gamma-\text{ČR region)} \quad (25) \]

(see Figs. 2–3 and predictions in Table 1), and, a second region at very high γ-energies (HE) where

\[ v_{\gamma ph}^{-1}(E_2) \geq v_{\gamma ph}(E_1), \quad \text{(extended } 2-\text{Čerenkov-like region)} \quad (26) \]

(see Figs. 2–3 and predictions in Table 1) where \( E_{ai} \) and \( E_{af} \) are the particle energy before and after γ-emission.

(ii) The experimental test of the SCR-coherence condition: \( v_{\gamma ph}(\omega)v_{\gamma ph}(E_2) \leq 1 \), is performed by using the data of Debbe et al. [18] on Čerenkov ring radii of electrons, muons, pions and kaons in a RICH detector (see Fig. 4). The results on this experimental test of the super-coherence conditions are presented. These SCR-predictions are verified experimentally with high accuracy: \( \chi^2/\text{dof} = 1.47 \) (see Fig. 5). The scaling law of the ring radii and Yields (see Fig. 5b) predicted by the SCR-effect are also experimentally confirmed with high accuracy.

(iii) The inferred SCR-yield at just ČR-threshold, is of order of magnitude \( Y(SCR) = 0.1301 \) (see eqn. 19).

(iv) The influence of medium on the particle propagation properties is investigated and the refractive properties of electrons, muons, pions, in the radiator \( \text{C}_4\text{F}_{10}\text{Ar} \) are obtained. The refractive indices for this radiator at \( p_{lab} = 1 \text{ GeV} \) are as follows: \( n_\mu = 1.001449 \pm 0.000098 \), \( n_\pi = 1.0012593 \pm 0.000167 \), \( n_K = 1.0214 \pm 0.0026 \), \( n_p = 1.1066 \pm 0.046 \). So, we proved that the refractive indices of the particles in medium are also very important for the RICH detectors, especially at low and intermediate energies.

(v) We proved that the anomalous Čerenkov rings observed recently by Vodopianov et al. [14] at SPS accelerator at CERN Pb-beam can also be
considered as an experimental signature of the HE-Super-Čerenkov component (see Fig. 3).

Finally, we remark that new and accurate experimental measurements of Čerenkov ring radii, as well as for the anomalous HE-component of SCR effect are needed.

Acknowledgments. We would like to thank Prof. Dr. G. Altarelli for hospitality and fruitful discussions during my stay in TH-Division CERN-Geneva. This work was supported by CERES Projects C2-86-2002, C3-13-2003.

REFERENCES


