SPECTRAL ANALYSIS FOR HADRONS PRODUCED IN HEAVY ION COLLISIONS

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The problem of heavy ion collision is viewed in mechanisms that may describe the reaction as a superposition of either folding a single nucleon-nucleon interactions or the interaction of projectile clusters with the target. The experimental features of the heavy ion reactions at wide range of energies for different target and projectile nuclear sizes provide us with information that reveals the picture of the interaction mechanism as well as the structure of the nucleus. In this work we concentrate our investigation upon two important measurements, the angular and the multiplicity behavior of the produced particles in the final state. Both the two methods predict that the formation of proton, alpha and carbon clusters during the interaction of Mg with the target emulsion.

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1. INTRODUCTION

The study of decay of heavy nuclei is one of the gold plated channels for heavy ion studies. It looks very interesting from the physical point of view. It presents several advantages related to the dynamics of these decays, characterized by proper-time-dependent angular distributions, which can be described in terms of bilinear combinations of transversity amplitudes [1]. Physical properties, such as masses and life-times, of very short-lived, and hence very rare, nuclei are important ingredients that determine element production mechanisms in the universe. Given that present and future nuclear structure research facilities will open significant territory into regions of medium-mass and heavier nuclei, it becomes important to investigate theoretical methods that will allow for a description of medium mass systems that are involved in such element production. One of these methods is the coupled cluster method [2] in systems involving many single-particle degrees of freedom. The subject was revisited only recently by Bishop et al. [3], for further theoretical development, and by Mihaila and Heisenberg [4, 5] for coupled cluster calculations using realistic two- and three-nucleon bare interactions and expansions in the inverse particle-hole energy spacings. In the present work we present two statistical
methods to analyse the data of multi-particle production that produced in heavy ion collision. The analysis aims to reveal the internal cluster structure of the projectile nucleus. The first method depends on the angular analysis of the knocked on nucleons (grey particles) which depend on the number of collisions made by the projectile nucleons. The second method deals with the fast created particles (shower particles). The multi-peripheral model is used to estimate the particle production due to the expected cluster structure of the projectile nucleus. The paper is organized so that Section 2 deals with data acquisition where two methods are present concerning angular and multiplicity analysis respectively. Conclusive remarks are given in Section 3.

2. DATA ACQUISITION

1. ANGULAR ANALYSIS

The geometric distribution of high-energy hadrons in shower cores measured with the nuclear emulsion is analyzed. The data are checked for sensitivity to hadronic interaction features and indications of new physics as discussed in the literature. The angular correlation of the most energetic hadrons and in particular the fraction of events with hadrons being aligned is quantified by means of some statistical parameter.

Different methods have been applied to analyze the angular distribution of the fast created particles in heavy ion collisions in terms of the particle production due to multiple collisions through the same reaction. It is known that the projectile makes multiple collisions inside the target nucleus. Each collision is associated with the production of newly fast created particles appear as showers (s). The number of the produced showers is energy dependent; consequently it rapidly decreases with the collision order due to the slowing down of the projectile energy. In nuclear emulsion terminology, the number of collisions inside the target nucleus (ν) is measured by the number of encountered nucleons which appears as gray particles (g). The analysis of the whole angular distribution may throw light about the evolution of projectile nucleons through target nucleus and the relative percentage of showers produced in each collision. The average number of encountered nucleons from the target by an incident hadron [6, 7] is a good measure of the number of collisions inside a target nucleus. This is defined as,

$$\nu = A \frac{\sigma_{hN}}{\sigma_{hA}}$$

Where, $\sigma_{hN}$ and $\sigma_{hA}$ are the total inelastic cross section of (N-N) and (N-A) respectively. By analogy, the average number of binary (N-N) collisions inside the (A-A) collision may be defined as,
$$v_{NN} = A_p A_t \frac{\sigma_{NN}}{\sigma_{AA}}$$

(2)

Following now a geometric approach which assumes that the nuclear radius is linearly proportional to the one third of the power of its mass number, then Eqs. (1–2) become [8],

$$R_t \propto A_t^{\frac{1}{3}}$$

$$v_{NN} = \frac{A_p A_t}{A_p^{\frac{2}{3}} + A_t^{\frac{2}{3}} + 2A_p^{\frac{1}{3}}A_t^{\frac{1}{3}}}$$

(3)

$$v_{NA} = \frac{A_p A_t^{\frac{1}{3}}}{A_p^{\frac{2}{3}} + A_t^{\frac{2}{3}} + 2A_p^{\frac{1}{3}}A_t^{\frac{1}{3}}}$$

Let \(F(\theta)\) be the angular distribution of \(s\) particles produced in Mg-Em, and \(f_g(\theta)\) is the angular distribution of \(s\) particles produced in association with \(g\) of gray particles (knocked on target particles). It is assumed that \(F(\theta)\) is a superposition of \(f_g(\theta)\) for all possible values of \(g\).

$$\sum_{g}^N f_g(\theta_i) a_g = F(\theta_i) \quad i = 1, 2, \ldots N$$

(4)

\(\{a_i\}\) are the weight factors concerning the \(N\) fundamental base functions \(\{f_g(\theta_i)\}\) that are used to analyze the spectrum. \(N\) algebraic equations are formed at \(N\) different angular domains. The whole angular range is divided into domains concerning the forward angle, the medium angle and the backward angle emission. The system of \(N\) algebraic equations are simultaneously solved using the iterative truncation technique [9]. The results are tabulated in Table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
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<tbody>
<tr>
<td>The coefficients ({a_i}) representing the weight factors concerning the (N) collisions inside the target nucleus at different angular domains</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\Delta\theta_i)</th>
<th>Coefficients ((a_i))</th>
<th>Weight factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–45°</td>
<td>3.688</td>
<td>0.230</td>
</tr>
<tr>
<td></td>
<td>9.750</td>
<td>0.609</td>
</tr>
<tr>
<td></td>
<td>2.581</td>
<td>0.161</td>
</tr>
<tr>
<td>45–80°</td>
<td>1.111</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>9.759</td>
<td>0.699</td>
</tr>
<tr>
<td></td>
<td>3.091</td>
<td>0.222</td>
</tr>
<tr>
<td>80–120°</td>
<td>1.314</td>
<td>0.219</td>
</tr>
<tr>
<td></td>
<td>2.768</td>
<td>0.458</td>
</tr>
<tr>
<td></td>
<td>0.1809</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>1.778</td>
<td>0.293</td>
</tr>
</tbody>
</table>
Table 1 shows that the contribution of the first three collisions \((g = 1, 2, 3)\) are predominant in all angular domains. The fourth collision has appreciable contribution only in the backward direction. Contribution of higher order collisions becomes important as the emission angle increases. Moreover, most particles are produced in the second collision as well.

II. MULTIPLICITY ANALYSIS

In this section we use the multiplicity of newly created particles to pursue the evolution of projectile nucleons inside the target. In this phenomenological model it is assumed that a projectile nucleon may interact collectively with a cluster of \(N\) target nucleons upon which the square of the center of mass energy \(s_N\) is explicitly dependent.

Consequently the phase space integral as well as the average produced particles is to be calculated. The phase space integral \(I_n(s)\) of the produced hadrons is a measure of the probability of producing \(n\) particle in the final state at center of mass energy \(\sqrt{s}\). It depends mainly on the volume in phase space and the transition matrix element \(T\). According to the multi-peripheral model (MPM) \([10]\),

\[
I_n(s) = \int \cdots \int \prod_i \frac{d^3p_i}{2E_i} \delta^4 \left( s - \sum_j p_j \right) \left| T \right|^2
\]  

Assuming that equation (5) describes well the hadron-nucleon interactions, then it is easy to proceed for the hadron-nucleus interactions.

On extending the model to the hadron-nucleus collisions, we should consider the possible number of interactions encountered with the nucleons forming the target nucleus \(A_t\). The incident hadron makes successive collisions inside the target. The energy of the incident hadron (leading particle) slows down after each collision, producing number of created hadrons each time that depends on the available energy. The phase space integral \(I_{nA}^{NA}\) in this case has the form,

\[
I_{nA}^{NA}(s) = \sum_v I_{n_v}(s_v)P(v, A_t)\delta \left( n - \sum_i n_i \right)
\]  

Where \(P(v, A_t)\) is the probability that \(v\) nucleons out of \(A_t\) will interact with the leading particle. The delta function in Eq. (6) is to conserve the number of particle in the final state. Treating all nucleons identically, and that \(\chi_{NN}\) is the N-N phase shift function \([11, 12]\) then, according to the eikonal approximation,
\[ P(l, A_i) = -\left( \frac{A_i}{l} \right) \sum_{j=0}^{l} (-1)^j \binom{l}{j} \left[ 1 - \exp(2 \text{Re}(A_i - l + j)\chi_{NN}) \right] \]  

(7)

The working out of this approach is to put the multi-dimension integration of Eq. (5) and the generated kinematical variables into a Monte Carlo subroutine [10].

The extension of the multi peripheral model to the nucleus-nucleus case is more complicated. The number of available collisions is multi-folded due to the contribution of the projectile nucleons. By analogy to the N-A collision, it is possible to define the phase space integral \( I_{nAA} \) in A-A collisions as,

\[ I_{nAA}(s) = \sum_j \sum_k I_{n_{j,k}} P_{AA}(j, A_p, k, A_t) \delta\left(n - \sum_j n_{j,k}\right) \]  

(8)

Where \( I_{n_{j,k}} \) is the phase space integral due to \( j \) knocked on nucleons from the projectile with \( k \) nucleons from the target. The probability that the A-A collision encounters \( \nu_{p} \) collisions from the projectile and \( \nu_{T} \) collisions from the target is treated as independent events. So that,

\[ P_{AA}(\nu_{p}, A_p, \nu_{T}, A_t) = P(\nu_{p}, A_p) \cdot P(\nu_{T}, A_t) \]  

(9)

Eq. (9) is applied to calculate the probability that a cluster of \( \nu_{p} \) nucleons from the projectile may participate the reaction. In Table (2) we demonstrate the results of the calculation for the cases of clusters of Proton, Alpha and Carbon \( \nu_{p} = 1, 4, 12 \). On the other hand the average multiplicity due to the interaction of each cluster is calculated using the distribution function defined by Eq. (8). The result also is demonstrated in Table 2. In Fig. 1 we demonstrate the prediction of Eq. (9) which is applied for the interaction of Mg projectile with the average emulsion as a target nucleus. The figure shows clear peaks

**Table 2**

<table>
<thead>
<tr>
<th>Coefficients ( a_i )</th>
<th>( i = 1 ) (proton)</th>
<th>( i = 2 ) (alpha)</th>
<th>( i = 3 ) (carbon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative ratio of projectile clusters (EX)</td>
<td>35%</td>
<td>57%</td>
<td>8%</td>
</tr>
<tr>
<td>Relative ratio of projectile clusters (MPM)</td>
<td>18%</td>
<td>47%</td>
<td>35%</td>
</tr>
<tr>
<td>( &lt;n_{p}&gt;_{EX} )</td>
<td>1.63</td>
<td>3.18</td>
<td>7.98</td>
</tr>
<tr>
<td>( &lt;n_{p}&gt;_{MPM} )</td>
<td>1.70</td>
<td>3.41</td>
<td>6.92</td>
</tr>
</tbody>
</table>
corresponding to the interaction clusters of $A = 1, 4$ and $12$. This returns to the fact that Mg nucleus during the interaction with a target nucleus; it behaves as consisting of clusters. The Alfa fragment ($A = 4$) plays the main role in the interaction.

It is required now to estimate experimentally the weight factor of the target fraction that may participate in the reaction. It is assumed that the yield of particles produced in the reaction is a measure of the available energy in the center of mass energy i.e. it depends on the target size. Let $f_i(n)$ represents the probability distribution function for producing $n$- particles due to the collision with $i$-nucleons from the target, $F(n)$ is the total multiplicity distribution function of the A-A collision. It is assumed that $F(n)$ is a superposition of functions due to different target sizes.

$$F(n) = \sum_{i=1}^{N} a_i f_i(n) \quad n = 1, 2, \ldots$$

where $a_i$ is the coefficient of expansion. It represents the statistical weight factor of the basic function $f_i(n)$. 

Fig. 1. – The probability of interaction of $(i)$ nucleons out of 24 during the interaction of Mg with the emulsion nucleus as a target.
Eq. (10) forms a family of $N$-independent equations corresponding to $N$-values of the multiplicity $n$. Solving these equations simultaneously, it is possible to determine the coefficients $a_i$ algebraically or using the linear regression technique by minimizing the difference between the left and right sides of Eq. (10).

$$\frac{\partial}{\partial a_j} \left[ F(n) - \sum_{i=1}^{N} a_i f_i(n) \right]^2 = 0 \quad (11)$$

It is assumed that the fundamental distribution functions $f_i(n)$ would have the Gaussian form as

$$f_i(n) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(\frac{-(n-\bar{n}_i)^2}{2\sigma_i^2}\right) \quad (12)$$

$\bar{n}_i$ & $\sigma_i$ are the average multiplicity and the dispersion due to reactions of $i$-nucleons with the emulsion target. Their values are found from experiments as given in Fig. 2 [13]. Accordingly, the multiplicity distribution due to a cluster of

![Fig. 2. – The average multiplicity and dispersion of the fast created particles produced by the interaction of a projectile of mass number A with Emulsion target at 4.2 A GeV.](image-url)
Hadrons produced in heavy ion collisions

$i$-nucleons is demonstrated in Fig. 3. The overall multiplicity distribution $F(n)$ is presented in Fig. 4 and compared with the corresponding experimental data.

The coefficients $a_i$ of Eq. (10) are determined and demonstrated in Table 2 and compared with the prediction of the MPM.

All the results of Table (2) lead to support the fact that the magnesium behaves as clusters during its interactions. Although the predictions of the statistical analysis of the multiplicity are much consistent with that of the MPM, the relative ratio of projectile clusters little bit differs in the two methods.

3. CONCLUSIVE REMARKS

The main conclusions of our work are:

– angular and multiplicity measurements are used to probe the cluster structure of target nuclei;
– the high energy interaction of Mg projectile goes through the interaction of its clusters with the target;
– Proton, Alpha and Carbon form the main clusters that participates the interaction of the Mg projectile. A high signal is detected corresponding to the Alpha cluster;
– small size clusters contribute in small angular domains, while higher order clusters become important as the emission angle increases;
– the statistical analysis as well as the prediction of the MPM show very close results.

REFERENCES

13. Particle Data Group, (PDG), www.slac.stanford.edu/spires
Fig. 3. – The multiplicity distributions due to the interaction of clusters of “$I$” nucleon with the emulsion target.

Fig. 4. – The multiplicity distribution of shower particles produced in Mg-Em interactions at 4.5A GeV.