EINSTEIN PRESCRIPTION FOR ENERGY LOCALIZATION

BRINDUŞA CIOBANU, IRINA RADINSCHI
Department of Physics, “Gh. Asachi” Technical University, Iasi, 700050, Romania,
bcioanu2003@yahoo.com, radinschi@yahoo.com

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This paper is devoted to an important and challenging problem, the localization of energy by using the Einstein energy-momentum complex. It is obviously that energy-momentum complexes satisfy some requirements which make them powerful tools for energy-momentum localization. In this connection, we use the Einstein prescription and make an investigation of some energy distributions presented in the literature in the last years and which are related to this prescription. Also, we make a discussion of the results that the Einstein prescription furnished. This prescription turns out to be a good tool for energy-momentum localization for various physical systems.

Key words: Einstein prescription, energy-momentum localization.

1. INTRODUCTION

One of the most interesting issues in General Relativity is the subject of energy-momentum localization. This problem of defining in an acceptable manner the energy-momentum density hasn’t got a generally accepted answer yet. The localization of energy implies the use of various energy-momentum complexes, including the prescriptions of Einstein [1–7], Landau and Lifshitz [2], Papapetrou [3], Bergmann [4] and Weinberg [5]. All of these prescriptions have a drawback: the calculations are restricted to quasi-Cartesian coordinates. On the other hand, Möller [6] proposed an energy-momentum complex which enables one to calculate the energy distribution in any coordinate system and not only in quasi-Cartesian coordinates. Hence, an interesting question is whether any of the aforementioned prescriptions provides the best option for energy-momentum localization and, of course, under what conditions. Also, the subject of the localization of energy continues to be an open one since Einstein has given his important result of the special theory of relativity that mass is equivalent to energy.

The problem of energy-momentum localization by using the energy-momentum complexes was re-opened in the recent years. Bondi [8] gave his
opinion that “a nonlocalizable form of energy is not admissible in general relativity, because any form of energy contributes to gravitation and so its location can in principle be found”. Misner et al [9] sustained that to look for a local energy-momentum means that is looking for the right answer to the wrong question. Also, they concluded that the energy is localizable only for spherical systems. On the other hand, Cooperstock and Sarracino [10] demonstrated that if the energy is localizable in spherical systems then it is also localizable in any space-times. Also, Chang, Nester and Chen [11] showed that the energy-momentum complexes are actually quasilocal and legitimate expression for the energy-momentum. They concluded that there exist a direct relationship between energy-momentum complexes and quasilocal expressions because every energy-momentum complexes is associated with a legitimate Hamiltonian boundary term. In his recent paper Virbhadra [12] gave an important result. He showed that different energy-complexes (ELLPW) yield the same result for a general non-static spherically symmetric metric of the Kerr-Schild class. Also, it is very important that these definitions (ELLPW) comply with the quasi-local mass definition of Penrose for a general non-static spherically symmetric metric of the Kerr-Schild class. However, these prescriptions disagree in the case of the most general non-static spherically symmetric metric. Also, the plethora of interesting results recently obtained by many researchers point out that the energy-momentum complexes are powerful tools for obtaining the energy and momentum in a given space-time. Important works were done with the energy-momentum complexes in 2- and 3-dimensional space-times. In this context, very important is the Cooperstock hypothesis [13] which states that in a curved space-time energy and momentum are confined to the regions of non-vanishing energy-momentum tensor. This implies that gravitational waves do not carry energy and momentum in vacuum.

In our paper we point out the results obtained in the problem of localization of energy with the Einstein energy-momentum complex. We make a classification of the papers dedicated to the Einstein prescription, by years. Also, we make two statistics plotting of the number of the papers written by each mentioned author and plotting of the number of the papers about the Einstein energy-momentum complex in function of the year when the paper was published or put on the gr-qc archive.

An important point for our discussion is that in the recent years many satisfactory results were obtained with the Einstein prescription. We make an investigation of the various results presented in the literature and related to this topic.

2. EINSTEIN PRESCRIPTION AND LOCALIZATION OF ENERGY

This section sketches a few ideas about the importance of the energy-momentum complexes in energy-momentum localization, and particularly about
the roll that the Einstein prescription is playing in this context. Every energy-
momentum complex has to satisfy some requirements which make it a good tool
for energy-momentum localization. We presented these requirements in a
previous work [14]. Also, we pointed out that the Einstein energy-momentum
complex satisfies all these requirements and allows obtaining satisfactory results
for energy and momentum of a general relativistic system. So, it is obviously that
the Einstein prescription is an efficient tool for energy-momentum localization.

We give the definition of the Einstein energy-momentum complex. The
Einstein energy-momentum complex [1] is given by

\[ \Theta^k_i = \frac{1}{16\pi} H^{bl}_i, \]

where

\[ H^{bl}_i = -H^k_i = \frac{g^{lm}}{\sqrt{-g}} [-g^{kn} g^{ln} g^{km})]_{,m}. \]

\[ \Theta^0_0 \] and \[ \Theta^0_{\alpha} \] are the energy and, respectively, the momentum density components.

The Einstein energy-momentum complex satisfies the local conservation
laws

\[ \frac{\partial \Theta^k_i}{\partial x^k} = 0. \]

The energy and momentum in the Einstein prescription are given by

\[ P_i = \iiint \Theta^0_i \, dx^1 \, dx^2 \, dx^3. \]

Using the Gauss theorem we obtain

\[ P_i = \frac{1}{16\pi} \int \int H^{0\alpha} n_\alpha \, dS, \]

where \[ n_\alpha = \left( \frac{x}{r}, \frac{y}{r}, \frac{z}{r} \right) \] are the components of a normal vector over an
infinitesimal surface element \[ dS = r^2 \sin \theta \, d\theta \, d\varphi. \]

In the following we present the most important results obtained with the
Einstein prescription in the recent years. Also, we make a classification of the
papers dedicated to the Einstein prescription by years and two graphs. In the first
graph we give the distribution of the results plotting of the number of the papers
written by each mentioned author. In the second graph we plot the number of the
papers about the Einstein energy-momentum complex in function of the year
when the papers were published or put on the gr-qc archive. We obtain a statistic
of the papers written on this topic of energy-momentum localization with the
Einstein prescription.

We give now our results obtained with the Einstein prescription. A paper is dedicated to the study of the energy distribution of the Bianchi VI0 universe [26]. The total energy is found to be zero. A study is dedicated to the energy distribution of a charged tachyon [27]. In other paper, one of the authors calculated the energy distribution of a charged regular black hole (ABG) [28]. The energy distribution is given by (6)

$$E(r) = M - \frac{q^2}{2r} + \frac{q^6}{24r^3 M^2} - \frac{q^{10}}{240r^5 M^4} + O\left(\frac{1}{r^6}\right).$$ (6)

The first two terms correspond to the energy distribution in the Reissner-Nordström space-time obtained in the Penrose prescription and evaluated by Tod. This result supports the Virbhadra conclusion [12] that “the Einstein energy-momentum complex is in a better bill of health” and gives acceptable results for many space-times.

In other paper [29] we investigated the difference of energy between the Einstein and Møller prescriptions, and compare it with the energy density of these black hole solutions. We found out a special relation between the difference of energy between the Einstein and Møller prescriptions and the energy density for the considered black hole solutions.

Another paper is the subject of the study of the energy distribution of the dual solution in the string frame that is known as the magnetic stringy black hole solution [30]. The energy distribution of the magnetic black hole is given by
The energy distribution depends on the mass $M$ and charge $Q$ of the black hole. Also, our results are satisfactory and recommend the Einstein energy-momentum complex as a powerful tool for energy localization. The mentioned results are satisfactory from the physical viewpoint and, also, some of them are the same as those obtained using other prescriptions. Also, in some cases there were obtained the same results for a given space-time using the Einstein definition in both General Relativity and tele-parallel gravity. Because of these considerations the Einstein energy-momentum complex is a powerful concept for energy-momentum localization.

With this plethora of results, we make a classification of the papers about the localization of energy with the Einstein energy-momentum complex in function of the year of publication or when the paper has been put on the gr-qc archive. We obtained the next results that we present in the table below.

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In Fig. 1 we give a statistics plotting of the number of the papers written by each aforementioned author. We denote: A-Aguirregabiria, AO-Aydogdu, BA-Banerjee, B-Bringley, C-Chamorro, HA-Havare, SI-Sůkeník, RO-Rosen, V-Virbhadra, H-Hsu, L-Lee, F-Fatima, ST-Salti, KO-Korunur, VA-Vagenas, S-Sharif, R-Radinschi, SE-Sen, YE-Yeh, Y-I-Ching Yang, X-Xulu, G-Gad, P-Patashnick.

Also, in Fig. 2 we make a statistics plotting of the number of the papers about the Einstein energy-momentum complex in function of the year when the papers were published or put on the gr-qc archive. In Fig. 2 we denote the years of publication by ex. 1997–97 and 2005-5.

We conclude that there is an increasing interest for the energy-momentum localization by using the energy-momentum complexes and, also, particularly by using the Einstein prescription.

3. CONCLUSIONS

The subject of the localization of energy continues to be one of the most interesting and challenging problem of the General Relativity. Also, this issue is an open one since Einstein has given his significant result of the special theory.
of relativity that mass is equivalent to energy. About the energy-momentum localization using several energy-momentum complexes (ELLPW) and Møller we point out that it has many adepts but there was, also, much criticism related to the use of these prescriptions. The main lack of these prescriptions is that most of these restrict one to calculate in quasi-Cartesian coordinates. Only the Møller prescription enables one to make the calculations in any coordinate system.

In this paper we point out the results obtained in the last years in energy-momentum localization with the Einstein energy-momentum complex.

The aforementioned satisfactory results obtained using the Einstein energy-momentum complex recommend it as a powerful tool for energy-momentum localization.

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REFERENCES

18. M. Súkeník, and J. Šima, gr-qc/0101026.


