INFLUENCE OF ELECTRIC FIELD AT ELECTRON ENERGY SPECTRUM IN CYLINDRICAL QUANTUM WIRE WITH TWO QUANTUM DOTS★

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The energy spectrum of electron is investigated in complicated nanoheterosystem consisting of two cylindrical semiconductor quantum dots placed into semiconductor quantum wire. Quantum dots are separated by barrier-layer, which is under the influence of constant electric field. The dependences of electron energies on geometric parameters of quantum dots and electric field intensity are analyzed.

Key words: quantum dot, quantum wire, energy spectrum.

1. INTRODUCTION

The optical and transport properties of nanoheterosystems containing semiconductor quantum dots are intensively researched during last decades. It is connected with wide perspectives of their utilization in the devices of modern opto- and nanoelectronics. The special attention is paid at the systems containing two quantum dots (QDs) separated by thin layer (several monoshells) of other crystal, creating a potential barrier for the quasiparticles (electrons, holes).

The nanosystems with double quantum wells can be used as the source of laser radiation in middle infrared region of spectrum (λ = 5–20 micrometers). In this case, the laser generation is achieved by interband transitions of electrons (holes) between energy levels splitting due to the anti crossing effect [1, 2].

The nanohetrosystems containing two quantum dots are also interesting because here one can guide and control the electron density, carrying it from one quantum dot to the other with the help of electric field. Such system creates the unitary quabit which can be used as the basic element of quantum calculations [3, 4].

The nanosystems with two quantum dots are already produced experimentally [5]. Therefore, it is necessary to establish the theory of electron spectrum and wave functions in such systems.

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In this paper we are going to study the stationary energy states of electron in nanoheterosystem consisting of two semiconductor quantum dots (with different sizes) embedded into the quantum wire (QW). The thin barrier-layer separating quantum dots is under the influence of the constant electric field with fixed intensity $(\vec{F})$, directed along the axial axis of nanohetrosystem (Fig. 1).

2. THE THEORY OF ELECTRON ENERGY SPECTRUM IN CYLINDRICAL QUANTUM WIRE WITH TWO QUANTUM DOTS INSIDE

The complicated semiconductor cylindrical quantum wire ("0"), containing two quantum dots of the same material ("1"), separated by thin layer of the other material ("0") is under research. The radius of QW (ρ_0), heights of QDs (h_1 and h_2), respectively and thickness of the layer separating QDs (Δ) are assumed as fixed (Fig. 1).

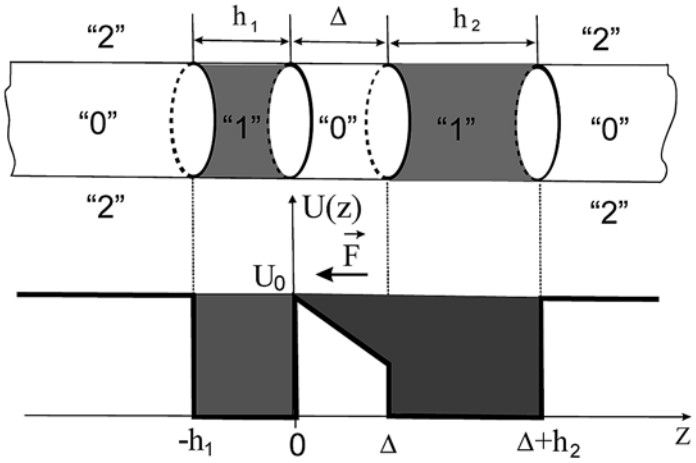

Fig. 1 – Geometrical scheme of nanosystem and dependence of electron potential energy on z variable.

The complicated quantum wire is placed into the external medium forming the infinite potential barriers for the electron. The electron effective masses are different in the different parts of nanosystem

$$\mu(z) = \begin{cases} \mu_0, \text{ medium "0"} \\ \mu_1, \text{ medium "1"} \end{cases} \tag{1}$$

It is also assumed that the lattice constants a_0 and a_1 of the media "0" and "1" are to be close over the magnitude. For example, the computer calculations are performed for the nanosystem created at the base of $\beta - HgS$ and $\beta - CdS$

crystals where the lattice constants are so close that $(a_1 - a_0)/a_0 \le 1\%$. Since, the interfaces between different parts of nanosystem are rather straight, it is possible to use the approximation of rectangular potential energies for the electron.

Taking into the account the latter and the fact that the constant electric field with intensity $\vec{F}$ is applied to the barrier-layer ("0") in such a way as it is shown in Fig. 1, the potential energy can be written in the form

$$U(\rho, \varphi, z) = \begin{cases} U_0, & z < -h_1 \text{ and } z > \Delta + h_2 \\ 0, & -h_1 \le z \le 0 \quad i \quad \Delta \le z \le \Delta + h_2, \\ U_0 - eFz, & 0 \le z \le \Delta \end{cases} \tag{2}$$

where e – electron charge, $U_0 = V_0 - V_1$, V_0, V_1 – electron potential energies in the corresponding media "0" and "1", taken respectively vacuum.

For the investigation of electron quantum states it is necessary to solve the Schrödinger equation

$$\hat{H}\Psi(\vec{r}) = E\Psi(\vec{r}) \tag{3}$$

with Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2}\vec{\nabla}\frac{1}{\mu(z)}\vec{\nabla} + U(\rho, \varphi, z). \tag{4}$$

Taking into account the cylindrical symmetry it is convenient to take the wave function $\Psi(\vec{r})$ as in ref. [6]

$$\Psi_{n_\rho m}(\vec{r}) = \left(-\pi\rho_0^2 J_{m-1}\left(\chi_{n_\rho m}\right) J_{m+1}\left(\chi_{n_\rho m}\right)\right)^{-1/2} J_m\left(\frac{\chi_{n_\rho m}}{\rho_0}\rho\right) e^{im\varphi}\varphi(z), \tag{5}$$

where $m = 0; \pm 1; \pm 2; \ldots$ – magnetic quantum number, $J_m\left(\frac{\chi_{n_\rho m}}{\rho_0}\rho\right)$ – Bessel function of the whole order, $\chi_{n_\rho m}$ – roots of Bessel function (n_ρ – radial quantum number fixing the number of Bessel function root at the fixed m).

Setting wave function (5) in Schrödinger equation (3), the variables are separated and for z th term of wave function it is obtained the equation

$$\frac{\partial^2}{\partial z^2}\varphi(z) + \varphi(z)\left[\frac{2\mu(z)}{\hbar^2}\left(E - U(\rho, \varphi, z)\right) - \frac{\chi_{n_\rho m}^2}{\rho_0^2}\right] = 0. \tag{6}$$

The solutions of eq. (6) with notations

$$k_0^2 = \frac{2\mu_0}{\hbar^2}E - \frac{\chi_{n_\rho m}^2}{\rho_0^2}, \quad k_1^2 = \frac{2\mu_1}{\hbar^2}(U_0 - E) + \frac{\chi_{n_\rho m}^2}{\rho_0^2}$$

for the different parts of nanosystem are the following

$$\varphi(z)=\begin{cases} A^{+}e^{k_0 z}, & z<-h_1 \\ B^{+}e^{-ik_1 z}+B^{-}e^{-ik_1 z}, & -h_1\le z\le 0 \\ C^{+}Ai\left[-\xi\left(z+z_B\right)\right]+C^{-}Bi\left[-\xi\left(z+z_B\right)\right], & 0\le z\le\Delta \\ D^{+}e^{-ik_1 z}+D^{-}e^{-ik_1 z}, & \Delta\le z\le\Delta+h_2 \\ E^{-}e^{-k_0 z}, & z>\Delta+h_2 \end{cases}, \qquad (7)$$

here, $Ai(z)$, $Bi(z)$ – Airy functions of first and second kind and

$$\xi=\left(\frac{2eFm}{\hbar^2}\right)^{1/3}, \quad z_B=\frac{E}{eF}.$$

Using, further, the conditions of wave function and density of probability current continuity at the media interfaces of nanosystem ($z=-h_1$, $z=0$, $z=\Delta$, $z=\Delta+h_2$) and the condition of wave function normalization

$$\int_{-\infty}^{\infty}\left|\varphi(z)\right|^2 dz=1 \qquad (8)$$

one can obtain the analytical expressions for the coefficients A^{+}, $B^{\pm}$, $C^{\pm}$, $D^{\pm}$, E^{-} (eq. (7)) and dispersion equation for the defining of electron energies in nanoheterosystem (the respective expressions are not presented because they are rather sophisticated). We must note that the electron wave function and its energy is characterized by three quantum numbers: $n_\rho m n_z$ ($\Psi_{n_\rho m n_z}(\vec{r}), E_{n_\rho m n_z}$). The axial quantum number (n_z) numerates the solutions of dispersion equation at the fixed n_ρ and m quantum numbers.

3. DISCUSSION OF RESULTS

The computer calculations of electron energy spectrum is performed for the nanoheterosystem created at the base of semiconductor crystals $\beta-HgS$ ("1") and $\beta-CdS$ ("0").

The results of calculation of electron energy ($E_{n_\rho m n_z}$) (without the electric field) as function of the height of the second QD (h_2) at the fixed $n_\rho = 1$, radius of QW: $\rho_0 = 8a_{HgS}$, height of the other QD: $h_1 = 7a_{HgS}$ and width of the potential barrier: $\Delta = 2a_{CdS}$ are presented in Fig. 2.

Fig. 2 proves that at $h_2 = 0$ there are three energy levels in nanosystem, coinciding (as it must be) with the levels arising in QD (HgS) with the height h_1

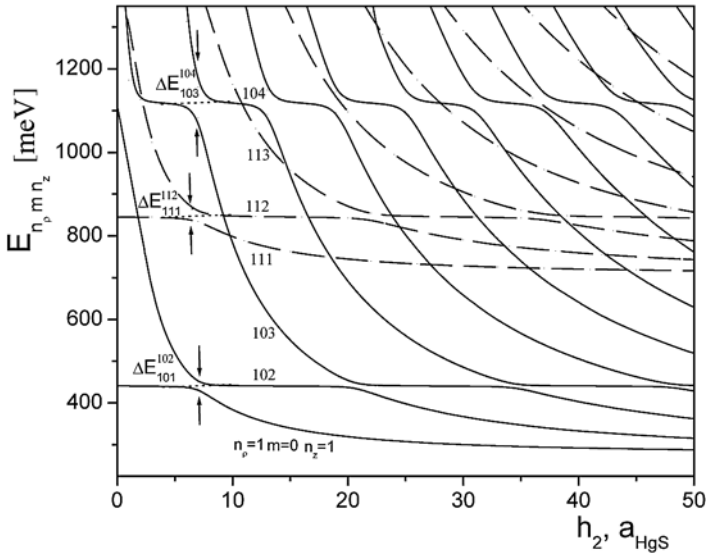

Fig. 2 – Dependences of electron energies on the height of second QD (h_2) at $n_\rho = 1$, $\rho_0 = 8a_{HgS}$, $h_1 = 7a_{HgS}$, $\Delta = 2a_{CdS}$, $F = 0$.

embedded into QW (CdS). The increasing of h_2 height brings to the appearance new energy levels becoming smaller and creating anti crossings. The anti crossings are caused by the splitting of the levels, having the origin of both potential wells, accounting their interaction through the potential barrier with finite height and width.

The applying of the electric field $\vec{F}$ qualitatively does not change the behavior of electron energy spectrum but brings to the small shift of all anti crossings into the region of smaller magnitudes of h_2 and to the varying of the splitting of energy levels with equal symmetry $\Delta E^{n'_\rho m n_z}_{n_\rho m n_z}$. It is shown in Fig. 3, where the dependence of $\Delta E^{n'_\rho m n_z}_{n_\rho m n_z}$ on the intensity of magnetic field $\vec{F}$ at the fixed $h_1 = h_2 = 7a_{HgS}$, $\Delta = 2a_{CdS}$ is presented. From the figure one can see that increasing of intensity $\vec{F}$ causes the increasing of the splitting $\Delta E^{n'_\rho m n_z}_{n_\rho m n_z}$. The latter is bigger and increases faster, the higher are the anti crossings in the energy scale of quantum well. Such behavior of energy spectrum is clear from physical considerations. Really, the increasing of electric field intensity causes the change of potential barrier profile in such a way that, on one hand, its effective thickness becomes smaller, *i.e.* its transparency increases, obviously increasing the interaction between quantum wells and, consequently, bringing to the increasing of the splitting of the respective energy levels. From the other side, the increasing

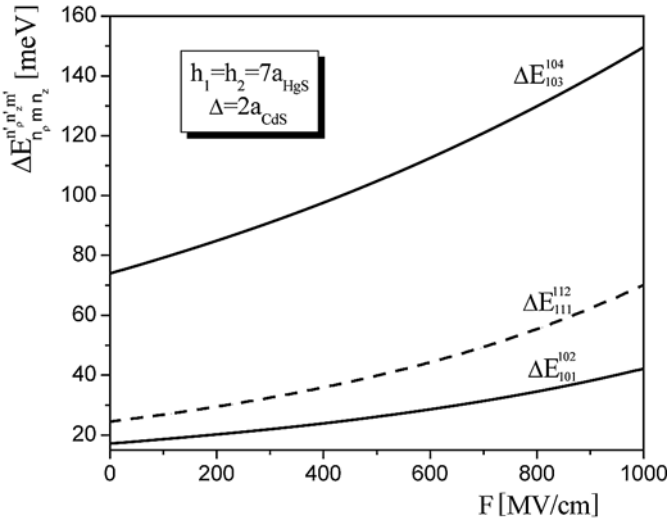

Fig. 3 – Dependence of splitting on the intensity of electric field F at $\rho_0 = 8a_{HgS}$, $h_1 = h_2 = 7a_{HgS}$, $\Delta = 2a_{CdS}$.

of transparency effectively increases the quantum well volume (toned region $0 \le z \le \Delta + h_2$ in Fig. 1), therefore the anti crossing arise at the smaller values of the second quantum dot (h_2).

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