THE NUMERICAL ANALYSIS
OF TRANSITORY DYNAMIC RESPONSE,
BASED ON THE NON-LINEAR HYDROELASTICITY THEORY,
FOR A BARGE TEST SHIP*

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In this paper there is presented the study of the transitory vertical displacement
response, oscillations and vibrations, of a barge model, under initial imposed fore-
pick displacement conditions. The analysis is carried on with eigen program DYN-
NL, based on the non-linear hydroelasticity theory. The numerical model includes
time domain implicit integration procedure of the motion equations, at zero speed
and still water conditions. For the numerical analyses validation, there are used
experimental results provided by the Bureau Veritas Register Paris, in the frame of
EU-FP6 Marstruct Project. The numerical results are in a good agreement with the
experimental data. The study points out the best approach in order to calculate the
hydrodynamic terms.

Key words: ships hydroelasticity, non-linear numerical analysis, transitory
dynamic response.

1. INTRODUCTION

This study is focused on the time domain analysis of the barge transitory
dynamic response, prismatic ship with small changes in the fore pick, proposed
by the Bureau Veritas Register [5]. The numerical non-linear analysis is carried
on with eigen program pack DYN-NL (module TRANZY).

2. THE THEORETICAL MODEL FOR TRANSITORY SHIP RESPONSE

2.1. THE HYPOTHESES

1) The ship hull girder is modelled using the finite element method (FEM),
with Ne Timoshenko beam elements [2, 3], including bending and shearing
deformations in vertical plane.

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2) The hydrodynamic forces are calculated according to the strip theory Gerritsma & Beukelman [3], with the inclusion of the non-linearities due to the time variation of the hydrodynamic coefficients, at the instantaneous ship-wave position, and the bottom-slamming component.

3) Based on the modal analysis technique, there is considered the ship dynamic response decomposed on the $r = 0, 1$ rigid modes and $r = 2, n$ the first $n-2$ eigen ship elastic girder modes.

4) The hydrodynamic terms on the eigen rigid modes $r = 0, 1$ are calculated at the ship vertical oscillation circular frequency $\omega_{osc}$. The hydrodynamic terms on the eigen vibration modes $r = 2, n$ are calculated for $\omega_2$, the circular frequency of the fundamental vibration mode.

5) The geometry of the ship hull transversal sections is modelled using the conformal multi-parametric transformation [2, 3], including also the possibility of total emersion from water.

6) There is no external excitation wave taken into account, so that no stabilized response occurs.

7) The speed of the ship is zero and the ship-girder initial conditions are non-zero displacements.

### 2.2. THE MOTION NON-LINEAR DIFFERENTIAL EQUATIONS

According the 3-rd and 6-th hypotheses, the ship transitory dynamic response is decomposed in: $x \in [0, L]; n = 4$

$$w_{nl}(x,t) = \sum_{r=0}^{n} w_r(x)p_{nlr}(t); \quad \theta_{nl}(x,t) = \sum_{r=0}^{n} \theta_r(x)p_{nlr}(t);$$

$$\gamma_{nl}(x,t) = \sum_{r=0}^{n} \gamma_r(x)p_{nlr}(t)$$

(1)

where: $w_r(x)$, $\theta_r(x)$, $\gamma_r(x)$ are the displacement, bending and shearing rotations eigen modes form functions (FEM calculated); $p_{nlr}(t)$ are the non-linear principal modal coordinates; $L$ is the ship length.

Based on the 1-st and 3-rd hypotheses, the motion equations system is:

$$[a]\{\dot{p}_{nl}(t)\} + [b]\{\ddot{p}_{nl}(t)\} + [c]\{p_{nl}(t)\} = \{F(t)\}$$

$$[a] = \text{diag}\{a_{ss}\}_{s=0,n}; \quad [b] = \{b_{rs}\}_{r,s=0,n}; \quad [c] = \text{diag}\{c_{ss}\}_{s=0,n}$$

$$a_{ss} = \int_0^L \left[ \mu(x)w_r^2(x) + f_r(x)\theta_r^2(x) \right]dx; \quad \{p_{nl}(t)\} = \{p_{nl0}(t), ..., p_{nln}(t)\}^T$$
where: $\mu(x), j_i(x)$ are the ship mass and inertial mass moment per unit length; $EI(x), GAf(x)$ are the bending and shearing ship rigidity; $\alpha(x), \beta(x)$ are the structural damping coefficients.

According to the 2-nd hypothesis, the hydrodynamic terms and the wet sectional areas are decomposed in still and instantaneous ship-water position components:

$$m_{33}(x,t) = m_{330}(x) + m_{33nl}(x,t); \quad N_{33}(x,t) = N_{330}(x) + N_{33nl}(x,t);$$

$$A(x,t) = A_0(x) - b_0(x)w_{nl}(x,t) + A_{nl}|w_{nl}(x,t)|$$

Based on the 2–7 hypotheses, the generalized hydrodynamic force has the expression:

$$\{F_h(t)\} = \{F_{h0}(t)\} + \{F_{h1}(t)\}$$

$$\{F_{h0}(t)\} = -[A_h]^0_{osc}\{\tilde{p}_{nl}(t)\} - [B_h]^0_{osc}\{\tilde{p}_{nl}(t)\} - [C_h]^0_{osc}\{\tilde{p}_{nl}(t)\}$$

$$A_{hrs} = \int_0^L m_{330}(x)w_r(x)w_s(x)dx; \quad B_{hrs} = \int_0^L N_{330}(x)w_r(x)w_s(x)dx$$

$$C_{hrs} = \int_0^L \rho gb_0(x)w_r(x)w_s(x)dx; \quad F_{hl1}(t) = \int_0^L F_{h1}(x,t)w_s(x)dx$$

$$F_{h1}(x,t) = -m_{33nl}|w_{nl}|^2 + [N_{33nl}|w_{nl}| + \rho g A_{nl}|w_{nl}|_w +$$

$$+ K_{imp}|w_{nl}|^2$$

where $K_{imp}$ is the impact bottom-slamming coefficient.

From relations (2), (4) the motion equations system at non-linear dynamic analysis becomes:

$$[A]\{\ddot{p}_{nl}(t)\} + [B]\{\dot{p}_{nl}(t)\} + [C]\{p_{nl}(t)\} = \{F_{h1}(t,\{p_{nl}\},\{\tilde{p}_{nl}\},\{\ddot{p}_{nl}\})\}$$

$$[A] = [a] + [A_h]^0_{osc}; \quad [B] = [b] + [B_h]^0_{osc}; \quad [C] = [c] + [C_h]^0_{osc}$$
2.3. THE TRANSITORY SHIP GIRDER DYNAMIC RESPONSE

Because \( F_{hi}(t) \) is function of the dynamic response \( p_{nl}(t) \), it is necessary to use a integration in time domain method (6), based on the \( \beta \)-Newmark (\( \beta = 1/2 \)) algorithm, in order to solve system (5).

The simulation time is \( T_s = 8 \) s with a time step \( \delta t = 0.001 \) s and the triggering frequency \( f_{es} = 1000 \) Hz. There are obtained 8001 values into a time record file at each transversal section.

Obs. Applying the spectral analysis with the Fast Fourier Transformation (FFT) [3] to the calculated time records, there are obtained the amplitude spectral functions of the dynamic response.

### Methodology

\[
\text{step } t = 0: \{ p_{nl}(0) \} \neq 0; \{ \dot{p}_{nl}(0) \} = 0 \Rightarrow \\
\Rightarrow \{ \ddot{p}_{nl}(0) \} = \left[ A^{-1} \right] \left( \{ F_{hi}(0) \} - \{ C \} \{ p_{nl}(0) \} \right) \\
\text{step } t: \{ p_{nl}(t) \}; \{ \dot{p}_{nl}(t) \}; \{ \ddot{p}_{nl}(t) \} \\
\text{step } t + \delta t: \text{It will be solved the linear equation system in } \{ \ddot{p}_{nl}(t + \delta t) \}:
\]

\[
\begin{align*}
\left\{ A + \frac{B}{2}\delta t + \left[ C \left( \frac{\delta t}{2} \right)^2 \right] \right\} \{ \ddot{p}_{nl}(t + \delta t) \} &= \{ F_{hi}(t + \delta t) \} - \{ F_{hi}(t) \} + \\
&+ \left[ A - B \delta t + \left[ C \left( \frac{\delta t}{2} \right)^2 \right] \right] \{ \ddot{p}_{nl}(t) \} - \left\{ C \delta t \right\} \{ \ddot{p}_{nl}(t) \} \\
\{ \dot{p}_{nl}(t + \delta t) \} &= \{ \ddot{p}_{nl}(t) \} + \left[ \{ \ddot{p}_{nl}(t) \} + \{ \ddot{p}_{nl}(t + \delta t) \} \right] \delta t; \\
\{ p_{nl}(t + \delta t) \} &= \{ p_{nl}(t) \} + \left[ \{ p_{nl}(t) \} + \{ p_{nl}(t + \delta t) \} \right] \delta t + \\
&+ \left[ \{ p_{nl}(t) \} + \{ p_{nl}(t + \delta t) \} \right] \delta t \left( \frac{\delta t}{2} \right)^2 \\
\end{align*}
\]

..... iteration \( t = T_s \)

3. THE BARGE TEST SHIP MODEL

There is considered the model of the barge, as it is defined in the report [5] (see Fig. 1). The idealization of the input data for the barge model is presented in the Table 1 [4].

The structural damping coefficients, according to Johnson & Tamita [1], are:

\[
\alpha_r(x) \approx \beta_r(x) = 0.001 \cdot \Gamma_r; \quad \Gamma_0 = \Gamma_1 = 0; \quad \Gamma_2 = 1; \quad \Gamma_3 = 0.95; \quad \Gamma_4 = 0.9 \quad (7)
\]

Based on the method of Vugts [1, 3], there are obtained the hydrodynamic mass coefficients \( c_{33} \) and the hydrodynamic damping coefficients \( \lambda_{33} \), at oscillations and vibrations frequencies domains.
Table 1

Barge model characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ne FEM beam elements</td>
<td>38</td>
</tr>
<tr>
<td>ρ [kg/m³] water density</td>
<td>1000</td>
</tr>
<tr>
<td>D.O.F. degrees of freedom</td>
<td>78</td>
</tr>
<tr>
<td>uₜ [m/s] ship speed</td>
<td>0</td>
</tr>
<tr>
<td>Δ [kg] ship displacement</td>
<td>172.53</td>
</tr>
<tr>
<td>Aₛ [m²] shear area</td>
<td>5.00E-04</td>
</tr>
<tr>
<td>Wave excitation</td>
<td>no</td>
</tr>
<tr>
<td>L [m] ship length</td>
<td>2.445</td>
</tr>
<tr>
<td>μ [kg/m] mass per unit length</td>
<td>70.564</td>
</tr>
<tr>
<td>B [m] ship breadth</td>
<td>0.600</td>
</tr>
<tr>
<td>jₛ [kgm²/m] inertial mass / L</td>
<td>1.39E-05</td>
</tr>
<tr>
<td>D [m] ship depth</td>
<td>0.250</td>
</tr>
<tr>
<td>E [N/m²] Young modulus</td>
<td>2.06E+11</td>
</tr>
<tr>
<td>d [m] ship draft</td>
<td>0.12000</td>
</tr>
<tr>
<td>G [N/m²] Transversal module</td>
<td>7.92E+10</td>
</tr>
<tr>
<td>dₐf [m] aft draft</td>
<td>0.11316</td>
</tr>
<tr>
<td>jₛ [kgm²/m] gravity acceleration</td>
<td>9.81</td>
</tr>
<tr>
<td>dₐf [m] fore draft</td>
<td>0.12691</td>
</tr>
<tr>
<td>ρₘ [kg/m³] material density</td>
<td>7.70E+03</td>
</tr>
<tr>
<td>cₜ ship block coefficient</td>
<td>0.98</td>
</tr>
<tr>
<td>dx [m] FEM element length</td>
<td>0.0815 / 0.0545 / 0.019</td>
</tr>
</tbody>
</table>

\[
m_{33} = c_{33}J_n \cdot \rho \pi b^2 / 8 \quad N_{33} = \lambda_{33} \cdot \rho \omega b^2 / 4
\]

\[
c_{33}, \lambda_{33} = f \left( c_T, H, \delta \right) \quad H = b / 2d \quad \delta = \omega^2 b / 2g
\]

where: \( c_T, b, d \) are the section data, \( \omega \) is the circular frequency, \( J_n \) is the Townsin coefficient [1].

In Fig. 2a, b there are presented the hydrodynamic coefficients \( c_{33}, \lambda_{33} \), for a transversal section at the prismatic zone, function to the \( z / d, z \in [0, d] \).

In Table 2 there are presented the eigen circular frequencies and in Fig. 3 the eigen oscillations and vibration modes.

Table 2

<table>
<thead>
<tr>
<th>Mode</th>
<th>( r = 0 ) heave</th>
<th>( r = 1 ) pitch</th>
<th>( r = 2 ) flex1</th>
<th>( r = 3 ) flex2</th>
<th>( r = 4 ) flex3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega ) osc./vib.</td>
<td>5.617</td>
<td>5.617</td>
<td>5.771</td>
<td>17.070</td>
<td>34.810</td>
</tr>
</tbody>
</table>

For this barge test \( \omega_{osc} = 5.617 \) rad/s \( \approx \omega_{vib} = 5.771 \) rad/s and \( \delta \approx 1 \), so that the hydrodynamic damping is non-zero (with high values) and the hydrodynamic mass is between oscillations and vibrations values.

4. THE TIME DOMAIN ANALYSIS OF SHIPS TRANSITORY RESPONSE

According to report [5], the initial displacement conditions are obtained by pulling vertically the barge fore-pick to a prescribed level and then releasing the model by cutting the attached rope. In Table 3 there are presented the initial
displacements from the experimental extinction test [5] and in table 4 there are the calculated initial conditions in terms of modal principal coordinates.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial vertical displacements</strong></td>
<td><strong>Initial modal principal coordinates</strong></td>
</tr>
<tr>
<td>Nr section</td>
<td>$x$ [m]</td>
</tr>
<tr>
<td>1</td>
<td>2.445</td>
</tr>
<tr>
<td>3</td>
<td>2.035</td>
</tr>
<tr>
<td>5</td>
<td>1.625</td>
</tr>
<tr>
<td>7</td>
<td>1.215</td>
</tr>
<tr>
<td>9</td>
<td>0.805</td>
</tr>
<tr>
<td>11</td>
<td>0.395</td>
</tr>
</tbody>
</table>

In Fig. 4 there is presented the initial experimental deformation of the barge girder. In Fig. 5 there is presented the deformations of the barge girder at some time values $t$.

In Fig. 6.1–2a, b there are presented the time records for transitory vertical displacement response, based on the numerical analysis. There are considered two cases for the hydrodynamic terms calculated on modes $r = 2, 3, 4$: $\omega = \omega_{vib}^2$ or $\omega \to \infty$ [4]. In Fig. 7a, b there are presented the FFT amplitude spectrums for the time records at section 1.

In Fig. 8.1–2 there are presented the time records for transitory vertical displacement response, based on the experimental analyses presented in the Bureau Veritas Register report [5].

The time records in Fig. 6, Fig. 8 are non-dimensional, using the maximal value $A = 101.9$ mm.

5. CONCLUSIONS

The numerical time records for transitory response of vertical displacements [4] are in good agreement with the experimental data [5]. The differences that occur have the following main sources: the precision of the input data idealization used in the tests; the structural damping coefficients are based on empiric values; the eigen induced waves are neglected in the theoretical model and at the numerical analyses; method induced differences, because this study it is based on the 2D flow approach (strip theory).

Because $w_{\text{max}} = 101.90$ mm $< d_{\text{fore}} = 126.91$ mm no bottom slamming occurs.

It results that in the case of hydrodynamic terms calculated for $\omega \to \infty$ (Fig. 6b), with zero hydrodynamic damping, the transitory response dose not corresponds to the experimental data.
In conclusion, because the vibration fundamental mode is dominant at the transitory response (see Fig. 7), the hydrodynamic terms must be calculated for \( \omega \rightarrow \omega_2 \) (Fig. 6.a), with non-zero hydrodynamic damping \( (\omega_{osc} \approx \omega_{ vib}^2) \).

Also further investigations have to be carried on for the structural damping terms.

REFERENCES

Fig. 4 – Initial experimental deformation [5].

Fig. 5 – The deformations [4] at $t = 0; 0.4; 0.75; 1.5; 1.9; 3$ s.

Fig. 6.1 – (a) $w/A$ time record, section S1, $\omega = \omega_{v,\infty}$; (b) $w/A$ time record, section S1, $\omega \to \infty$. 
Fig. 7. (a) FFT amplitude spectrum, section S1, $\alpha = 0$.
(b) FFT amplitude spectrum, section S1, $\alpha \rightarrow \infty$.

Fig. 6.2. (a) kA time record, section S3, $\alpha = 0$.
(b) kA time record, section S3, $\alpha \rightarrow \infty$.
Fig. 8.1 w/A time record, section S1, experimental [5].

Fig. 8.2 w/A time record, section S3, experimental [5].