ASYMMETRIC TWO-OUTPUT QUANTUM PROCESSOR
IN ANY DIMENSION

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We propose two different implementations of an asymmetric two-output probabilistic quantum processor. One of them is constructed by combining asymmetric telecloning with a quantum gate array. We analyze the efficiency of this processor by evaluating the fidelities between the desired operation and the ones generated by the processor and show that the two output states are the same as the ones produced by the optimal universal asymmetric Pauli cloning machine and have a success probability of 1/2. We show further that we can perform the same one-qubit operation with unity probability at the cost of using nonlocal operations. We finally generalize the two schemes for $D$-level systems and find that the local ones are successful with a probability of $1/D$ and the nonlocal generalized scheme is always successful.

Key words: quantum cloning, quantum processor.

1. INTRODUCTION

Quantum computers are machines that employ quantum phenomena, such as quantum interference and entanglement, to solve a particular problem. The computers have to execute a program, which is built with the help of a precise set of instructions, in order to give the desired solution. A specification of this set of instructions is called an algorithm. There are problems for which the number of elementary steps for obtaining a solution scales exponentially or at least polynomially better on a quantum computer than on a classical one [1], the Deutsch-Jozsa algorithm for solving the oracle problem [2–4], the Shor algorithm for factoring large integers [5], and the Grover algorithm for searching unsorted databases [6]. Thus, on a large enough quantum computer one could solve problems with a quantum computer which are basically unsolvable with a classical one.

The most important component of the computer architecture is the processor. One crucial property of the classical processor is that we keep the same circuit regardless of the instructions that we want to perform. One may
then ask how to construct a universal quantum processor, a fixed device that implements any desired program on the information stored in quantum systems? This problem was originally investigated by Nielsen and Chuang [7], where they proposed a model of the quantum processor, which consists of a quantum gate array $G$ acting on the data state $|\psi_d\rangle$ and on the program state $|P\rangle$. The dynamics of the quantum gate array is

$$G[|\psi_d\rangle|P_U\rangle] = U(|\psi_d\rangle)|P'_U\rangle,$$

where $U$ is a particular unitary operator implemented by the processor. Nielsen and Chuang found two important results [7]: (i) the state $|P'_U\rangle$ is independent of the data register $|d\rangle$ and (ii) no deterministic universal quantum gate array exists. Therefore they showed how to construct an one-output probabilistic quantum processor, whose operating principle is that of quantum teleportation [8]. The outcome of a Bell measurement tells us when the desired operation succeeded.

In the last few years, much progress has been made on generalizations and applications of the probabilistic quantum processor. Huelga et al. found a generalization of the method of teleporting a quantum gate from one location to another [9]. In this protocol, a device that performs the transformation $|\psi\rangle \rightarrow U|\psi\rangle$ was constructed, $|\psi\rangle$ being an arbitrary state and $U$ an arbitrary unknown unitary operator. Two more proposals of probabilistic quantum processors were considered by Preskill [10] and Vidal et al. [11]. Vidal et al. analyzed a probabilistic gate, which performs an arbitrary rotation around the $z$ axis of a spin-$1/2$ particle with the help of an $N$-qubit program state [11]. More complex probabilistic programmable quantum processors have been proposed by Hillery et al. [12, 13] by investigating the case when an arbitrary linear operation $A$ is performed. They have built the network for this processor by using a quantum information distributor [14, 15] for qubits and then for $D$-level systems (quDit). Hillery et al. have analyzed several classes of quantum processors, which execute more general operations, namely completely positive maps, on quantum systems [16]. In addition, they have found two important results: one can build a quantum processor to perform the phase-damping channel and that this is not possible in the case of the amplitude-damping channel [16].

Further extensions have been developed. In [17, 18] a quantum processor, which executes SU(N) rotations was considered, and was found that the probability of success for implementing the operation is increased if conditionated loops are used. Recently Brazier et al. have investigated the case when we have access to many copies of the program state [19]. They have shown that the probability of success cannot be increased and that it is the same as the one obtained using two different schemes: VMC [11] and HZB [17].
Positive-operator-valued measures (POVM) are the most general measurements allowed by quantum mechanics [20, 1]. Therefore it would be interesting to study the possibility of realizations of POVMs on quantum processors. This problem was investigated by Ziman and Bužek [21], where they showed how to encode a POVM into a program state. Another important class of operations is the one of generators of Markovian dynamics, which are relevant in the context of quantum decoherence. Koniorczyk et al. have recently proposed a scenario for the simulation of the infinitesimal generators of the Markovian semigroup on quantum processors [22]. Approximate processors, i.e., processors which implement a set of unitary operators with high precision, have been introduced by Hillery et al. in [23]. The accuracy of the processor is given by the process fidelity, which was shown to be maximum if one chooses the program state to be the eigenvector corresponding to the largest eigenvalue of a certain operator. This operator depends on some operators $A_{jk}$, which characterize the quantum processor, and the desired unitary operator $U$ to be implemented. We emphasize that all the processors described above generate only one output state. A two-output processor was proposed by Yu et al. [24] by combining symmetric telecloning of qubits [25] with a programmable quantum gate array. In this scheme, the sender has a priori information regarding the unitary operation, which has to be performed. This is necessary for building the program state. Therefore, the protocol is different of the one of [9], where the sender has no such information.

In this paper we present two different schemes for obtaining a two-output quantum processor for $D$-level systems. Our schemes are generalizations of the Yu et al. protocol, where we analyze a processor which performs probabilistically any unitary operation. Suppose the following scenario: an observer Peter has to teleport the result of a unitary operation on an unknown data state, to two distant parties, Alice and Bob. We show how this task can be accomplished using a shared entangled state, local operations, and classical communication (LOCC) by proposing two schemes. We initially restrict our study to the class of diagonal unitary operations but show that with a straightforward extension any unitary operation can be transmitted optimally. In the first scheme, described in Section 2, we extend Yu et al.’s scheme by considering asymmetric telecloning of qubits as well as a more general unitary operator. This gives Peter the power to decide on the information sharing, with the two extremes being that one party gets all information, while the other gets none. The symmetric protocol is a special case of our more general study. The program state consists of a four-particle entangled state shared between Peter, Alice, Bob, and another observer Charlie, who holds an ancillary system. (As Charlie’s particle is only used as a resource, and is discarded at the end, Peter and Charlie may be the same party in those schemes that do not need nonlocal operations between Alice’s, Bob’s and Charlie’s particles.) Peter measures the
data qubit and his particle $P$, which is included in the program register, in the standard Bell basis, and then communicates the outcome. With a probability of 1/2, Alice and Bob are able to recover the desired mixed output states, whose fidelities are identical with the ones obtained by the optimal universal asymmetric Pauli cloning machine. We also discuss that if Alice, Bob, and Charlie may use nonlocal operations, the protocol will always be successful with preserved fidelities. Further we propose an alternative local protocol, where the desired unitary operation is encoded in a modified Bell basis.

In Section 3 we present the generalizations of these protocols for quDits. The two local generalized schemes are successful with a probability equal to $1/D$, while unit probability of success again requires nonlocal operations. Finally, in Section 4 we summarize our conclusions.

2. ASYMMETRIC TWO-OUTPUT QUANTUM PROCESSOR FOR QUBITS

2.1. ARBITRARY DIAGONAL UNITARY OPERATOR

In this section we present two schemes for performing the following scenario: we start with an arbitrary qubit state

$$|\psi_d\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle,$$

(2)

where $|\alpha_0|^2 + |\alpha_1|^2 = 1$. We want to obtain two optimal universal asymmetric clones of a diagonal unitary computational operator described by:

$$D_\theta = \begin{pmatrix} e^{i\theta_0} & 0 \\ 0 & e^{i\theta_1} \end{pmatrix},$$

(3)

with $\theta_j \in [0, 2\pi)$, $j = 0, 1$, applied on the arbitrary input data state $|\psi_d\rangle$. We have used the notation $\bar{\theta} = (\theta_0, \theta_1)$. We impose the condition that the processor generates two output states, whose fidelities must be independent on the input data state, i.e., independent on $\alpha_0$ and $\alpha_1$. We assume that $D_{\bar{\theta}}$ is known. The desired output state is

$$|\Lambda\rangle = D_{\bar{\theta}} |\psi_d\rangle = \alpha_0 e^{i\theta_0} |0\rangle + \alpha_1 e^{i\theta_1} |1\rangle.$$

(4)

We define a state required in the preparation of the program as

$$|\xi\rangle_{PABC} = \frac{1}{\sqrt{2}} (|0\rangle_P |\phi_0\rangle_{ABC} + |1\rangle_P |\phi_1\rangle_{ABC}),$$

(5)

where

$$|\phi_0\rangle_{ABC} = \frac{1}{\sqrt{2(1 - p + p^2)}} \left[ |000\rangle + p |011\rangle + (1 - p) |101\rangle \right],$$

and
\[ \langle \phi_1 \rangle_{ABC} = \frac{1}{\sqrt{2(1 - p + p^2)}} [ (111) + p|100\rangle + (1 - p)|010\rangle], \]  
\[ \langle \phi_0 \rangle_{ABC} = \frac{1}{\sqrt{2(1 - p + p^2)}} [ (111) - p|100\rangle + (1 - p)|010\rangle], \]  
with \( 0 \leq p \leq 1 \). The two states \( \langle \phi_0 \rangle \) and \( \langle \phi_1 \rangle \) are obtained by applying an optimal universal asymmetric Pauli cloning machine to the states \( |00\rangle \) and \( |10\rangle \) [26]. The data system \( d \) and the first qubit \( P \) of the state \( |\xi\rangle \) belong to an observer Peter, while the qubits \( A, B, C \) are held by other distant parties, Alice, Bob, and Charlie, respectively. This “ownership” of the qubits (or, later, quDits) will hold for all our schemes. Note that, in the case of the “local” schemes, Peter and Charlie may be identically the same, as Charlie’s qubit is discarded at the end.

Peter wants to teleport the result of the desired unitary operator \( T \) to Alice and Bob. Peter encodes the information carried by the unitary operator, in the program state \( |P\rangle_{PABC} \) by locally applying \( D_T \) on the qubit \( P \) (see Fig. 1):

\[ |P\rangle_{PABC} = D_T \otimes I_{ABC} |\xi\rangle_{PABC} = \frac{1}{\sqrt{2}} (e^{i\theta_0} |0\rangle_P |\phi_0\rangle_{ABC} + e^{i\theta_1} |1\rangle_P |\phi_1\rangle_{ABC}). \]  

We write now the input state with the help of the standard Bell basis \( |\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle) \) and \( |\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|01\rangle \mp |10\rangle) \):

\[ |\psi_d\rangle|P\rangle_{PABC} = \frac{1}{2} \left[ |\Phi^+_d\rangle_{dp} \left( \alpha_0 e^{i\theta_0} |\phi_0\rangle + \alpha_1 e^{i\theta_1} |\phi_1\rangle \right) + \right. \]  
\[ + |\Phi^-_d\rangle_{dp} \left( \alpha_0 e^{i\theta_0} |\phi_0\rangle - \alpha_1 e^{i\theta_1} |\phi_1\rangle \right) + \]  
\[ + |\Psi^+_d\rangle_{dp} \left( \alpha_1 e^{i\theta_0} |\phi_0\rangle + \alpha_0 e^{i\theta_1} |\phi_1\rangle \right) + \]  
\[ + |\Psi^-_d\rangle_{dp} \left( \alpha_1 e^{i\theta_0} |\phi_0\rangle - \alpha_0 e^{i\theta_1} |\phi_1\rangle \right). \]  

Alice and Bob will obtain two mixed states \( \rho_A \), \( \rho_B \) that are implemented by the quantum processor.
Peter performs a measurement in the Bell basis on the data qubit and the first qubit $P$ in the program state as it is shown in Fig. 1. Then he communicates the outcome of the measurement to Alice, Bob, and Charlie. With a probability equal to 1/4 the outcome is $|\Phi^+\rangle_{dp}$ and therefore the output is projected to

$$|\eta\rangle_{ABC} = \alpha_0 e^{i\theta_0} |\phi_0\rangle + \alpha_1 e^{i\theta_1} |\phi_1\rangle. \quad (9)$$

Accordingly, after tracing over Charlie’s qubit, the two final states of Alice and Bob, respectively, are:

$$\rho_A = \text{Tr}_{B,C} |\eta\rangle \langle \eta| = \frac{1}{1-p+p^2} \left[ (1-p)^2 \frac{I}{2} + p |\Lambda\rangle \langle \Lambda| \right];$$

$$\rho_B = \text{Tr}_{A,C} |\eta\rangle \langle \eta| = \frac{1}{1-p+p^2} \left[ p^2 \frac{I}{2} + (1-p) |\Lambda\rangle \langle \Lambda| \right]. \quad (10)$$

with $|\Lambda\rangle$ given by Eq. (4). The efficiency of the quantum processor is evaluated with the help of the fidelities of the output states with respect to the exact data register outputs $D_\theta |\psi_d\rangle$.

$$F_A = \langle \psi_d | D_\theta^\dagger \rho_A D_\theta | \psi_d \rangle = \frac{1+p^2}{2(1-p+p^2)}, \quad \text{and}$$

$$F_B = \langle \psi_d | D_\theta^\dagger \rho_B D_\theta | \psi_d \rangle = \frac{2-2p+p^2}{2(1-p+p^2)}. \quad (11)$$

If the output of the measurement is $|\Phi^-\rangle_{dp}$, then the final state is

$$|\eta\rangle_{ABC} = \alpha_0 e^{i\theta_0} |\phi_0\rangle - \alpha_1 e^{i\theta_1} |\phi_1\rangle. \quad (12)$$

Alice, Bob, and Charlie can transform the state $|\eta\rangle$ to the state $|\eta\rangle$ of Eq. (9) by applying the local unitary operator $V = \sigma_3^A \otimes \sigma_3^B \otimes \sigma_3^C$. Therefore, in this case, Alice and Bob obtain the same final states $\rho_A$ and $\rho_B$ as above. For the other two outcomes, when Peter obtains $|\Psi^+\rangle_{dp}$ and $|\Psi^-\rangle_{dp}$, the result cannot be transformed to the state of Eq. (9) by local operations. Hence there is no chance to obtain the desired mixed outputs $\rho_A$ and $\rho_B$ if we don’t allow for nonlocal transformations. The processor hence succeeds with the probability 1/2 and generates two asymmetric output mixed states. The fidelities of the final states given by Eq. (11) are identical with the ones of the clones emerging from the optimal universal asymmetric cloning machine given in [26, 27, 28]. In the particular case when $p = 1/2$, we recover the result of Yu et al. [24], namely the two output states are identical. Our protocol presents the advantage that Peter can decide whether to send all information to Alice, i.e., $F_A = 1/2$, $F_B = 1/2$ by fixing $p = 1$, or to Bob, giving $F_A = 1/2$, $F_B = 1$ by fixing $p = 0$, or to send two imperfect output states to both receivers, $F_A$ and $F_B$ being given by Eq. (11) for $0 < p < 1$. 
However, if we allow nonlocal operations between Alice, Bob, and Charlie, which either entails their respective particles to interact directly, or via a shared auxiliary entangled state, it is also possible to convert the states $|\Psi^+\rangle_{dp}$ and $|\Psi^-\rangle_{dp}$ to $|\eta\rangle$ and thereby always succeed with the protocol. This is generally true for all our proposed schemes.

Instead of encoding the information of the desired unitary operator into the program state, we can incorporate it into a measurement in a modified Bell basis, which depends on $T$:

$$
|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}} \left( e^{-i\theta} |00\rangle \pm e^{-i\theta} |11\rangle \right),
$$

$$
|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}} \left( e^{-i\theta} |01\rangle \pm e^{-i\theta} |10\rangle \right).
$$

(13)

Thus, as an alternative protocol we use the same channel $|\xi\rangle_{PABC}$ of Eq. (5) initially shared by Peter, Alice, Bob, and Charlie as above, but this time Peter performs a measurement in the above modified Bell basis as is shown in Fig. 2. Alice and Bob obtain the same outputs $\rho_A, \rho_B$ given by Eq. (10). The total success probability is $p = 1/2$. Since this protocol is constructed using only LOCC it is suitable for distant computation at two locations.

2.2. PERMUTED DIAGONAL OPERATOR

Suppose the operator is not diagonal, but that it can be written as a permutation $\mathcal{P}$ of a diagonal operator. In the two-dimensional case, there is only one such operator, namely
considered by Yu, Feng, and Zhan [24]. The target state in this case becomes
\[ |\Lambda\rangle = U_\bar{B} |\Psi'_d\rangle = \alpha_1 e^{i\theta_1} |0\rangle + \alpha_0 e^{i\theta_0} |1\rangle. \]

The protocol becomes: The unknown data state is modified by a unitary basis-permutation
\[ |\Psi'_d\rangle = \mathcal{P} |\Psi'_d\rangle = \alpha_1 |0\rangle + \alpha_0 |1\rangle. \]

Subsequently this state is subjected to the protocol in the previous subsection using the diagonal unitary operator \( \mathcal{D}_{\bar{B}} \). It is trivial to show that Alice’s (Bob’s) final state and associated fidelity \( \langle \Lambda |\rho_{A(B)}|\Lambda\rangle \) become identical to the ones of Eq. (10) and Eq. (11), respectively.

2.3. AN ARBITRARY UNITARY OPERATOR

Suppose that we want that the quantum processor to implement an arbitrary, but known, unitary operator \( U \). Therefore, the desired output state is \( |\Lambda\rangle = U |\Psi_d\rangle \). Since \( |\Psi_d\rangle \) is unknown, the teleportation cannot be accomplished through a classical channel. In contrast, \( U \) is known and can be arbitrarily well be described by information transmitted classically. We always can diagonalize \( U \) with the help of a unitary operator \( U_{tr} \):
\[ U = U_{tr} \mathcal{D} U_{tr}^\dagger, \]
where the diagonal unitary operator will have the form given by Eq. (3). We define a modified data state as \( |\Psi''_d\rangle = U_{tr}^\dagger |\Psi_d\rangle \) and an intermediate target state \( |\chi\rangle = \mathcal{D} |\Psi'_d\rangle \). According to the Eq. (10) Alice and Bob get two intermediate final states:
\[ \tilde{\rho}_A = \frac{1}{1 - p + p^2} \left[ (1 - p)^2 \frac{I}{2} + p |\chi\rangle \langle \chi| \right]; \]
\[ \tilde{\rho}_B = \frac{1}{1 - p + p^2} \left[ p^2 \frac{I}{2} + (1 - p) |\chi\rangle \langle \chi| \right]. \]

Subsequently, the sender Peter communicates to Alice and Bob, by a classical channel, which the transformation operator \( U_{tr} \) is. The final output states are obtained by Alice and Bob by applying this transformation unitary operator, which depends only on the initial desired operation \( U \). Applying this
transformation to the intermediate state they get $\rho_{A(B)} = U_{tr} \bar{\rho}_{A(B)} U_{tr}^\dagger$, respectively. Expressing this state in terms of the identity and the target state $|A\rangle$ we recover expressions Eq. (10) and Eq. (11).

3. ASYMMETRIC TWO-OUTPUT QUANTUM PROCESSOR FOR QUDITS

3.1. AN ARBITRARY DIAGONAL UNITARY OPERATOR

One-output probabilistic programable quantum processors have recently been analyzed by Hillery et al. [12, 17] in the case when the data are encoded on a $D$-level systems (quDits). More precisely, they have investigated the possibility to construct a processor which performs an arbitrary linear operation $A$ [12].

Here we propose a generalization of the quantum processor presented in the Sec. 2.1 for quDits. An arbitrary data quDit-state is described by

$$|\psi_d\rangle = \sum_{k=0}^{D-1} \alpha_k |k\rangle,$$

where $\sum_{k=0}^{D-1} |\alpha_k|^2 = 1$. We will initially analyze the action of a unitary, diagonal operator:

$$D_B = \sum_{j=0}^{D-1} \exp(i\theta_j) |j\rangle\langle j| = \begin{pmatrix}
  e^{i\theta_0} & 0 & \ldots & 0 & 0 \\
  0 & e^{i\theta_1} & \ldots & 0 & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  0 & 0 & \ldots & e^{i\theta_{D-2}} & 0 \\
  0 & 0 & \ldots & 0 & e^{i\theta_{D-1}}
\end{pmatrix}.$$ 

We define a family of states which depends on a parameter $p$, $0 \leq p \leq 1$:

$$|\phi_j\rangle = \frac{1}{\sqrt{1 + (D-1)(2p^2 - 2p + 1)}} \left[ |j\rangle |j\rangle + p \sum_{r=1}^{D-1} |j+r\rangle |\overline{j+r}\rangle + (1-p) \sum_{r=1}^{D-1} |j+r\rangle |\overline{j+r}\rangle \right],$$

where $j = 0, \ldots, D-1$, and $j+r = j+r$ modulo $D$. These states were found by one of us in [26] by considering the action of an optimal universal asymmetric Heisenberg cloning machine on the state $|j\rangle|00\rangle$. In addition, we introduce the state $|\vec{z}\rangle_{PABC}$ as follows:
\[ |\xi\rangle_{PABC} = \frac{1}{\sqrt{D}} \sum_{j=0}^{D-1} |j\rangle_P |\phi_j\rangle_{ABC}. \tag{22} \]

Peter encodes the operation \( D_P \) in the state of a program register \( |P\rangle_{PABC} \) as follows
\[ |P\rangle_{PABC} = D_P \otimes I_{ABC} |\xi\rangle = \frac{1}{\sqrt{D}} \sum_{j=0}^{D-1} \exp(i\theta_j) |j\rangle_P |\phi_j\rangle_{ABC}. \tag{23} \]

We denote by \( |\Phi_{m,n}\rangle \) the standard Bell basis for quDits:
\[ |\Phi_{m,n}\rangle = \frac{1}{\sqrt{D}} \sum_{k=0}^{D-1} \exp\left(\frac{2\pi ikn}{D}\right) |k\rangle |k+m\rangle. \tag{24} \]

The input state can be written by using the standard Bell basis as
\[ |\psi_d\rangle |P\rangle_{PABC} = \frac{1}{D} \sum_{k,m,n=0}^{D-1} \alpha_k \exp\left(i\theta_{k+m}\right) \exp\left(-\frac{2\pi ikn}{D}\right) |\Phi_{m,n}\rangle_{dP} |\phi_{k+m}\rangle_{ABC}. \tag{25} \]

The desired output state should be
\[ |\Lambda\rangle = D_P |\psi_d\rangle = \sum_{j=0}^{D-1} \alpha_j e^{i\theta_j} |j\rangle. \tag{26} \]

A measurement in the standard Bell basis onto the data state and the first qubit \( P \) of the program state is performed by Peter. Then he announces the outcome of the measurement to Alice, Bob, and Charlie. With a probability \( 1/D^2 \) he gets the outcome \( |\Phi_{0,n}\rangle \) (where \( n \) is a fixed number) and at the same time the state of the quDits \( A, B \) and \( C \) becomes
\[ |\eta\rangle_{ABC} = \sum_{k=0}^{D-1} \alpha_k e^{i\theta_k} \exp\left(-\frac{2\pi ikn}{D}\right) |\phi_k\rangle_{ABC}. \tag{27} \]

Let us now define a local operator \( V_n := V_n^A \otimes V_n^B \otimes V_n^C \), where
\[ V_n^X := \sum_{j=0}^{D-1} \exp\left(\frac{2\pi ijn}{D}\right) |j\rangle \langle j|, \quad X = A, B; \tag{28} \]
\[ V_n^C := \sum_{j=0}^{D-1} \exp\left(-\frac{2\pi ijn}{D}\right) |j\rangle \langle j|. \]

Depending on Peter’s outcome, Alice, Bob, and Charlie apply subsequently the local operator \( V_n \) on the output state (27) and obtain
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\[ V_n |\eta\rangle_{ABC} = |\eta\rangle_{ABC} = \sum_{k=0}^{D-1} \alpha_k e^{i\theta_k} |\phi_k\rangle_{ABC}. \]  

Since there are \(D\) equiprobable measurement outcomes \(|\Phi_{0,n}\rangle\) for \(n = 0, \ldots, D - 1\), the total success probability is \(1/D\). The two mixed output states of the quantum processor are, in all the \(D\) cases,

\[ \rho_A = \text{Tr}_{B,C} |\eta\rangle\langle\eta| = \frac{1}{1 + (D-1)(2p^2 - 2p + 1)} \left\{ D(1-p)^2 \frac{I}{D} + 2p \right\} |\Lambda\rangle\langle\Lambda|, \]

\[ \rho_B = \text{Tr}_{A,C} |\eta\rangle\langle\eta| = \frac{1}{1 + (D-1)(2p^2 - 2p + 1)} \left\{ Dp^2 \frac{I}{D} + D - 2(D-1)p + (D-2)p^2 \right\} |\Lambda\rangle\langle\Lambda|. \]

The fidelities of the output states are given by:

\[ F_A = \langle\Lambda|\rho_A|\Lambda\rangle = \frac{1 + (D-1)p^2}{1 + (D-1)(2p^2 - 2p + 1)}, \text{ and} \]

\[ F_B = \langle\Lambda|\rho_B|\Lambda\rangle = \frac{1 + (D-1)(1-p)^2}{1 + (D-1)(2p^2 - 2p + 1)}. \]

They are identical with the ones generated by the optimal universal asymmetric Heisenberg cloning machine given in [26]. In the case when the result of Peter’s measurement is not \(|\Phi_{0,n}\rangle\), Alice, Bob, and Charlie are not able to recover the state \(|\eta\rangle_{ABC}\) given by Eq. (29) only by LOCC, and then the computation fails. However, both in this and the following protocol, the success probability can be boosted to unity if the three parties may use nonlocal operations on their particles.

We can generalize the second scheme given in Section 2.1 by defining a modified Bell basis which depends on \(T\) also for quDits. The total success probability of this scheme is \(1/D\) and the two output states coincide with the ones of Eqs. (30), (31).

3.2. AN ARBITRARY UNITARY OPERATOR

The generalizations of the qubit protocols in Sections 2.2 and 2.3 to quDits is straightforward. The permutation matrix \(P\) and the diagonalization operator \(U_\theta\) become larger and more complex, but the protocols remain identical. Alice’s
and Bob’s output states, expressed in the relevant target states, and the fidelities are given by (30), (31) and (32), respectively.

4. CONCLUSIONS

In this paper we have demonstrated the possibility of building a general two-output programmable processor of qubits, which generates two asymmetric states with a probability of success of 1/2. It is characterized by the same fidelities as the ones generated by the optimal universal asymmetric Pauli cloning machine. We implement these schemes by using only local operations and classical communication, and therefore they are suitable for distributed computation to two spatially separated receivers.

In addition, we have presented the generalizations of these two protocols to $D$-level systems. The generalizations present the advantage of sending the outcomes to different locations and they are achieved with a probability of 1/2.

Finally, we have shown how to increase the success probability to unity by considering nonlocal operations. In this case the two output states are obtained only after the particles A, B, and C have interacted, either directly or through an auxiliary entangled state.

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