NUCLEAR STRUCTURE CALCULATIONS WITH SKYRME FORCES

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Starting from an effective Skyrme interaction we present a method to take into account the coupling between one- and two-phonon terms in the wave functions of excited states. The approach is a development of a finite rank separable approximation for the quasiparticle RPA calculations proposed in our previous work. As an application of our method we present results of calculations for the isoscalar giant quadrupole resonance properties in the $^{132}\text{Sn}$.

1. INTRODUCTION

A study of new exotic nuclei stimulates a development of the nuclear models to describe properties of nuclei away from the stability lines. The quasiparticle random phase approximation (QRPA) with the self-consistent mean-field derived by making use of the effective nucleon-nucleon interaction is one of the most popular way to investigate the nuclear structure. When the residual interaction is separable, the QRPA problem can be easily solved no matter how many two-quasiparticle configurations are involved. Starting from an effective interaction of the Skyrme type, a finite rank separable approximation was proposed [1] for the particle-hole (p-h) residual interaction. Such an approach allows one to perform structure calculations in very large particle-hole spaces. Thus, the self-consistent mean field can be calculated within the Hartree-Fock method with the original Skyrme interaction whereas the RPA solutions would be obtained with the finite rank approximation to the p-h matrix elements. This approach was extended to include the pairing correlations within the BCS approximation [2]. Recently, we generalized our approach to take into account a coupling between the one- and two-phonon components of wave functions [3]. An extension of our approach to take into account the particle-particle residual interaction has been done in [4]. As an application of our method we present results of calculations for the isoscalar giant quadrupole resonance properties in the $^{132}\text{Sn}$.
2. METHOD OF CALCULATIONS

The starting point of the method is the HF-BCS calculation [5] of the ground states, where spherical symmetry is imposed on the quasiparticle wave functions. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic oscillator basis [6]. We work in the quasiparticle representation defined by the canonical Bogoliubov transformation:

\[ a_{jm}^+ = u_j \alpha_{jm}^+ + (-1)^{j-m} v_j \alpha_{j-m}, \]  

where \( jm \) denote the quantum numbers \( nljm \). The effective Skyrme interaction [7] in the p-h channel and the surface peaked density-dependent zero-range force in the particle-particle (p-p) channel are used. A strength of the p-p interaction is fixed to reproduce the odd-even mass difference of nuclei in the studied region [4].

The residual interaction in the p-h channel \( V_{res}^{ph} \) and in the p-p channel \( V_{res}^{pp} \) can be obtained as the second derivative of the energy density functional with respect to the particle density and the pair density, respectively. Hereafter we simplify \( V_{res}^{ph} \) by approximating it by its Landau-Migdal form and the \( l = 1 \) terms are dropped. Thus the residual interaction is given by the following form:

\[ V_{res}^a (r_1, r_2) = N_0^{-1} \left[ F_0^a (r_1) + F_0^a (r_2) (\tau_1 \cdot \tau_2) \right] \delta(r_1 - r_2), \]  

where \( a \) is the channel index \( a = \{ph, pp\}; N_0 = 2k_F m^*/\pi^2 \hbar^2 \). The expressions for \( F_0^{ph} \), \( F_0^{pp} \), \( F_0^{pp} \), \( F_0^{ph} \) can be found in Ref. [1] and in Ref. [4], respectively.

The p-h matrix elements and the antisymmetrized p-p matrix elements can be written as the separable form in the angular coordinates [1, 2, 4]. After integrating over the angular variables one needs to calculate the radial integrals,

\[ I_a^a (j_1 j_2 j_3 j_4) = N_0^{-1} \int_0^{\infty} \left( F_0^a (r) + F_0^a (r) (\tau_1 \cdot \tau_2) \right) x u_{j_1} (r) u_{j_2} (r) u_{j_3} (r) u_{j_4} (r) \frac{dr}{r^2}, \]  

where \( u_j (r) \) is the radial part of the single-particle wave function. Following the method introduced in Ref. [1], the radial integrals (3) can be calculated accurately by choosing a large enough the cut-off radius and using the N-point integration Gauss formula. Thus, the residual interaction can be reduced to a sum of \( N \) separable terms [4].

We introduce the phonon creation operators

\[ Q_{\lambda, \mu}^\dagger = \frac{1}{2} \sum_{ij} \left( X_{ij}^{j_1} A^+(jj'; \lambda \mu) - (-1)^{\lambda-\mu} Y_{ij}^{j_1} A(jj'; \lambda - \mu) \right). \]
The index $\lambda$ denotes total angular momentum and $\mu$ is its z-projection in the laboratory system. One assumes that the ground state is the phonon vacuum $|0\rangle$. We define the excited states as $Q^\mu_{\lambda \mu} |0\rangle$. Making use of the linearized equation-of-motion approach one can get the QRPA equations [5]:

$$
\begin{pmatrix}
A & B \\
-B & -A
\end{pmatrix}
\begin{pmatrix}
X \\
Y
\end{pmatrix}
=
\begin{pmatrix}
wX \\
wY
\end{pmatrix}.
$$

Solutions of this set of linear equations yield the eigen energies and the amplitudes $X, Y$ of the excited states. The dimension of the matrices $A, B$ is the space size of the two-quasiparticle configurations. The explicit solution of the corresponding QRPA equations is given in Ref. [4]. It is shown that the matrix dimensions never exceed $6N \times 6N$ independently of the configuration space size.

Our calculations [2] show that, for the normal parity states one can neglect the spin-multipole terms of the p-h residual interaction. Using the completeness and orthogonality conditions for the phonon operators one can express bifermion operators $A^+(jj'; \lambda \mu)$ and $A(jj'; \lambda \mu)$ through the phonon ones and the initial hamiltonian can be rewritten in terms of quasiparticle and phonon operators in the following form:

$$
H = h_0 + h_{QQ} + h_{QB}
$$

$$
h_0 = \sum_{jm} \epsilon_j \alpha^+_j \alpha^+_m
$$

$$
h_{QQ} = -\frac{1}{4} \sum_{\lambda \mu \mu'} W^{ij}_{\lambda \mu} (\tau) Q^+_{\lambda \mu} Q_{\lambda \mu'}
$$

$$
h_{QB} = -\frac{1}{2} \sum_{\lambda \mu \tau} \sum_{i j} \Gamma^{ij}_{\lambda \mu} (\tau) \left( (-\hat{\lambda} - \mu) Q^+_{\lambda \mu} + Q_{\lambda - \mu} \right) B(jj'; \lambda - \mu) + h.c.
$$

The coefficients $W, \Gamma$ of the hamiltonian (7) are sums of N combinations of phonon amplitudes, the Landau parameters, the reduced matrix elements of the spherical harmonics and radial parts of the HF single-particle wave function. It is worth to point out that the term $h_{QB}$ is responsible for the mixing of the configurations and, therefore, for the description of many characteristics of the excited states of even–even nuclei [8].

To take into account the mixing of the configurations in the simplest case one can write the wave functions of excited states as:
\[ \Psi_\nu(\lambda \mu) = \left\{ \sum_i R_i(\lambda \nu) Q^+_{\lambda \mu i} + \sum_{\lambda_1 \lambda_2} P^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) \left[ Q^+_{\nu \lambda \mu 1} Q^+_{\lambda_1 \nu \lambda_2 2} \right] \right\} |0\rangle \]  

(11)

with the normalization condition:

\[ \sum_i R_i^2(\lambda \nu) + 2 \sum_{\lambda_1 \lambda_2} (P^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu))^2 = 1 \]  

(12)

Using the variational principle in the form:

\[ \delta \left( \langle \Psi_\nu(\lambda \mu) | H | \Psi_\nu(\lambda \mu) \rangle - E_\nu \left( \langle \Psi_\nu(\lambda \mu) | \Psi_\nu(\lambda \mu) \rangle - 1 \right) \right) = 0, \]  

(13)

one obtains a set of linear equations for the unknown amplitudes \( R_i(\lambda \nu) \) and \( P^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) \):

\[ (\omega_i - E_\nu) R_i(\lambda \nu) + \sum_{\lambda_1 \lambda_2} U^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) P^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) = 0 \]  

(14)

\[ \sum_i U^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) R_i(\lambda \nu) + 2(\omega_0 h + \omega_{\lambda_2} - E_\nu) P^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) = 0 \]  

(15)

\( U^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) \) is the matrix element coupling one- and two-phonon configurations [8]:

\[ U^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) = \langle 0 | Q_{\lambda \mu} h Q_{\lambda \nu}^+ Q^+_{\lambda \lambda_1 \nu} Q^+_{\lambda_2 \lambda_2 2} | 0 \rangle. \]  

(16)

The expression of \( U^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) \) is given in [3]. The number of linear equations (14), (15) equals the number of one- and two-phonon configurations included in the wave function (11).

The energies of excited states \( E_\nu \) are solutions of the secular equation

\[ F(E_\nu) \equiv \det \left( \omega_{\lambda_1} - E_\nu \right) \delta_{\nu \nu} - \frac{1}{2} \sum_{\lambda_1 \lambda_2} U^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) U^{\lambda_2 h}_{\lambda_2 \lambda_1}(\lambda \nu) = 0, \]  

(17)

where the rank of the determinant equals the number of the one-phonon configurations. Using Eqs. (14), (15) and the normalization condition (12), one can find the amplitudes \( R_i(\lambda \nu) \) and \( P^{\lambda_1 h}_{\lambda_1 \lambda_2}(\lambda \nu) \).

It is necessary to point out that the equations derived above have the same form as the basic QPM equations [8], but the single-particle spectrum and the p-h residual interaction are determined making use of the Skyrme interactions.
3. DETAILS AND RESULTS OF CALCULATIONS

We apply the present approach to study characteristics of the isoscalar giant quadrupole resonance in the $^{132}$Sn. In this paper we use the parametrization SLy4 [7] of the Skyrme interaction. This parametrization was proposed to describe isotopic properties of nuclei from the $\beta$-stability line to the drip lines. As it was shown in [4] a contribution of the p-p channel is not important for the $^{132}$Sn and one can neglect by the p-p channel terms. We take into account all two-phonon terms that are constructed from the phonons with the multipolarities $\lambda \leq 6$. Using such a set of parameters we can reproduce experimental data for the low-lying vibrational states very well [4]. Results of our calculations for the isoscalar quadrupole strength distribution in the $^{132}$Sn are presented in Fig. 1. The strength functions [8] for the $B(E2)$-values are calculated with an averaging parameter $\Delta = 0.2$ MeV. The dashed curve corresponds to the RPA calculation and the solid curve presents our results for a case when the phonon-phonon coupling is taken into account. As one can see from this figure an inclusion of the two-phonon terms results in an essential increasing of the isoscalar giant quadrupole resonance width $\Gamma$. For the RPA case we get $\Gamma = 1.7$ MeV that must be compared with the value $\Gamma = 4.0$ MeV when one takes into account the phonon-phonon coupling. For the isoscalar giant quadrupole resonance energy the $^{132}$Sn our calculation predicts a value $E_x = 14.5$ MeV. It is worth to mention that for nuclei far from the closed shells effects of phonon-phonon is more strong.

![Fig. 1 – The quadrupole strength distribution in the $^{132}$Sn.](image)

4. CONCLUSIONS

A finite rank separable approximation for the QRPA calculations with Skyrme interactions that was proposed in our previous work is extended to take
into account the coupling between one- and two-phonon terms in the wave functions of excited states. The suggested approach enables one to reduce considerably the dimensions of the matrices that must be diagonalized to perform structure calculations in very large configuration spaces. As an application of the method we have studied the properties of the isoscalar giant quadrupole resonance in $^{132}\text{Sn}$. The inclusion of the two-phonon configurations results in an essential increasing of the isoscalar giant quadrupole resonance width.

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