THE INTERACTION OF FREE ELECTRONS
WITH INTENSE ELECTROMAGNETIC RADIATION

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In the first part of the paper, we give a brief presentation of nonlinear Compton scattering, one of the possible results of the electron interaction with a strong electromagnetic field. In the second part, we present our results for the scattered electron energy and angular distributions.

1. INTRODUCTION

In the last 15–20 years the progress in laser technology, electron acceleration and detection have open the way to experimental work on nonlinear processes predicted by QED long time ago. An early review paper on the theoretical work in the domain of electron interaction with strong electromagnetic field is due to Eberly [1]; the recent report of Salamin et al. [2] gives a survey of the progress in the study of relativistic high-power laser-matter interaction.

The new results we present concern the nonlinear Compton scattering, a process predicted by QED and detected in 1996 in an experiment at Stanford Linear Accelerator Center (SLAC), more than thirty years after the first theoretical studies has been published (1964, [3]), soon after the first laser was constructed. Nonlinear Compton scattering was only one of the processes studied after 1964, others were pair creation by a photon in the presence of the laser, pair creation in a combined laser and Coulomb field. The influence of the laser on the muon decay and pion decay were also considered. In order to observe such phenomena very high intensities are needed. The early theoretical investigations were not accompanied by exact or systematic numerical calculations, which is justified by the lack of adequate computing facilities.

The strongest intensity available, \( I = 10^{22} \text{ W/cm}^2 \), was reached in the optical regime (frequency around 1 eV) [4]. The realization of the project ELI (extreme light infrastructure) aims to an intensity of approximately \( 10^{26} \text{ W/cm}^2 \). In a much higher photon energy range (8–12 keV) photon X-ray free electron lasers at DESY and SLAC will reach the intensity of \( 10^{15} \text{ W/cm}^2 \).
The highest intensity in perspective is still far from the intensity of 10^{29} \text{ W/cm}^2 corresponding to \( E_c \), the QED critical electric field, defined as the value of the electric field intensity needed for pair creation due to vacuum polarization by a static field, and given by the condition \( E_c \lambda_C = mc^2 \), with \( \lambda_C \) the electron Compton length, \( m \) the electron mass and \( c \) the velocity of light. This means \( E_c = m^2 c^3 / h = 1.33 \cdot 10^{16} \text{ V/cm} \). The first high intensity quoted above of 10^{22} \text{ W/cm}^2 correspond to an electric field intensity of only 10^{-2} \( E_c \).

In the experiments at SLAC two nonlinear processes have been detected: nonlinear Compton scattering, reported in 1996 [5], and Breit-Wheeler pair production in 1997 [6]. A detailed presentation of the experiments and their conclusion is found in the paper of Bamber et al. [7]. The experiment used a Nd:glass laser, the Final Focus Test Beam at SLAC consisted of terawatt pulses of 1053 nm and 527 nm wavelengths and the peak laser intensity was around 0.5 \cdot 10^{18} \text{ W/cm}^2. The initial electron beam had an energy of 46.6 GeV.

In nonlinear Compton scattering, as a result of the interaction of an electron with an intense monochromatic electromagnetic field of frequency \( \omega_\perp \), the electron is scattered and emits a quanta with energy which can be much larger than \( h\omega_\perp \) (the simultaneous emission of several quanta is also possible, although less probable). The process can be viewed as a multiphoton absorption leading to one-photon emission

\[ e + N\omega_\perp \rightarrow e' + \omega'_\perp, \quad N > 0. \tag{1} \]

The value of \( N \) can be any positive integer. The existing theoretical studies focus on the photon energy and angular distributions and less on the scattered electron. Our results presented here concern the scattered electron.

Section 2 contains general equations in the usual treatment of the process. In Sect. 3 we present several equations relevant for the observation of the electron only. Our numerical results are presented in Sect. 4.

\subsection*{2. GENERAL EQUATIONS}

The presentation here will show only a few of the equations involved in the calculation; atomic units are used. The only existing theoretical approach is a calculation in which the laser is replaced by a monochromatic plane wave with the propagation direction characterized by the unit vector \( \mathbf{n}_\perp \), the frequency \( \omega_\perp \) and an elliptic polarization (see, for instance, LiuLka [8]). The amplitude of the process is proportional to a transition matrix element between two Volkov functions [1]. In the more general case, Volkov functions are solutions of the Dirac equation in the presence of an electromagnetic field with a fixed direction.
of propagation. For a monochromatic electromagnetic plane wave each Volkov solution is characterized by a dressed momentum $q$, a four-vector defined by

$$q = p + \frac{c^2n^2}{2k_L \cdot p}k_L,$$

$$k_L = \frac{\omega}{c}n_L, \quad \eta = \sqrt{\frac{2\Gamma}{c^4\omega^2}},$$

(2)

with the four-vector $n_L$ having the components $(1, n_L)$ (we use $a \cdot b = a_0 b_0 - a \cdot b$). In the absence of the field a Volkov solution reduces to a free electron solution with four-momentum $p$.

All cross-sections of the process have a multiphoton structure, being a series of terms each characterized by an integer $N$. The most differential cross-section corresponds to a distribution of the scattered electron in its dressed momentum $q'$ and a distribution in emitted photon momentum $k'$. Each term contains a $\delta$-function,

$$d^4\sigma = \sum_{N=1}^{\infty} \sigma_4^{(N)}(q', k') \delta(q' + k' - q - Nnk_L) dq' dk',$$

(3)

expressing a generalized momentum and energy conservation. We refer to the case of unpolarized initial electrons and no detection of the final electron or emitted photon polarization. Each of the functions $\sigma_4^{(N)}$ contains information about the result of an experiment in which the scattered electron and the emitted photon are detected in coincidence. The analytic expression of the function $\sigma_4^{(N)}$ depends on the laser polarization. The simplest is the case of circular polarization, when each term is a combination of three Bessel functions with the variable dependent on the electromagnetic field intensity.

If the electron is not detected, a cross-section describing the emitted photon distribution in frequency and direction is obtained by integrating over the vector $q'$ and by performing elementary operations on the argument of the remaining $\delta$-function

$$d^2\sigma_q = \sum_{N=1}^{\infty} \sigma_2^{(N)}(k') \delta(\omega' - \omega_N) dk'.$$

(4)

The frequencies allowed by the $\delta$-function under the summation sign are compactly written, using scalar products of four-vectors and using the four-vectors $n_L$ defined before and $n'(1, n')$ with $n'$ the unity vector along the emitted photon direction, as

$$\omega_N = N\omega_L \frac{n_L \cdot q}{n' \cdot (q + Nk_L)}.$$

(5)

Ehlotzky and coworkers [9] have published several papers on the emitted photon distributions.
3. THE ELECTRON DISTRIBUTIONS

In what follows we are interested in the cross-sections relevant for the detection of the scattered electron, obtained by integrating (3) over the vector $k'$. The structure of the most differential cross-section regarding the electrons is

$$d^2\sigma_e = \sum_{N=1}^{\infty} \sigma_2^{(cN)}(q') \delta(E + N \omega_L - E' - \tilde{\omega}_N) dq' .$$  \hspace{1cm} (6)

We have used the simpler notation $E$ and $E'$ for the incident and scattered dressed electron energy, respectively. The value of the frequency $\tilde{\omega}_N$, imposed by the integration over the emitted photon momentum,

$$c \tilde{\omega}_N = |Q_N - q'| , \quad Q_N \equiv q + N k_L ,$$  \hspace{1cm} (7)

is $N$-dependent.

According to the theoretical approach adopted here, if the energy and the direction of the electron would be detected in coincidence, to each final electron energy $E'$ would correspond several scattering angles, one for each value of $N$ in (7); a simple expression is valid for the angle $\beta_N$ made with that of the vector $Q_N$ by

$$\cos \beta_N = \frac{q' \cdot Q_N}{|q'||Q_N|} .$$  \hspace{1cm} (8)

The possible values for this angle are given by

$$\cos \beta_N = \frac{E'(E + N\omega_L) - c q' \cdot Q_N}{c |q'||Q_N|} .$$  \hspace{1cm} (9)

The condition $|\cos \beta_N| \leq 1$ imposes a finite energy range for the scattered electron.

Integration of (7) over the electron direction leads to a distribution in energy

$$d\sigma_e = \int_{|q'|} d^2\sigma_e ,$$  \hspace{1cm} (10)

which does not contain $\delta$-functions anymore, but keeps the multiphotonic structure of the previous cross-section. This is one of the quantity we have evaluated numerically in different conditions.

Integration of (7) over electron energy leads to the scattered electron angular distribution.
4. NUMERICAL RESULTS AND DISCUSSION

For illustration we present here numerical results for the final electron energy distribution corresponding to the following conditions: initial energy $E = 866.2$ MeV, circularly polarized laser with frequency $\omega_L = 0.056$ au, intensity $I = 3.51 \cdot 10^{18}$ W/cm$^2$ and two geometries: in the first case the electron and the laser field are counterpropagating [Fig. 1(a)], and in the second case they move in the same sense [Fig. 1(b)].

The contributions from different number of photons ($N = 1, \ldots, 5$) and their sum (which is the observable quantity) are represented. One can see that at this intensity the main contribution is that of $N = 1$ (note the logarithmic scale). In the first case the final electron energy $E'$ is smaller than the initial one, while for the second configuration the electron gains energy from the laser field. We also note that the spreading in the electron energy increases with $N$.

In Fig. 2 we present, as polar diagrams, angular distributions of the scattered electron for the case of initial electron at rest and laser parameters: circular polarization, frequency $\omega = 0.056$ au and intensity $I = 3.51 \cdot 10^{12}$ W/cm$^2$ in Fig. 2(a) and $I = 1.4 \cdot 10^{13}$ W/cm$^2$ in Fig 2(b). The laser propagation direction
Fig. 2 – Polar diagrams of the scattered electron angular distribution (in au) for the case of initial electron at rest, laser frequency $\omega_L = 0.056$ au and two intensities (a) $I = 3.51 \cdot 10^{12}$ W/cm$^2$ and (b) $I = 1.4 \cdot 10^{13}$ W/cm$^2$, for $N = 1$ (left) and $N = 2$ (right).

is a symmetry axis for the problem. In both cases the dominant contribution comes from $N = 1$, also the contribution of $N = 2$ is shown. The scattering angle of the electron takes values between 0 and 40° for the lower intensity and only between 0 and 8° in the second case.

Our future work will include a systematic study of the influence of the initial scattering geometry on the process.

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