DETERMINATION OF DECAY CONSTANTS DIRECTLY FROM EXPERIMENTS

P. DITĂ
Institute of Physics and Nuclear Engineering
P.O. Box MG-6, RO-077125 Bucharest-Magurele, Romania,
E-mail: dita@zeus.theory.nipne.ro

Received August 30, 2008

The aim of this paper is to provide a method for the determination of the CKM matrix moduli, as well as of the decay constants, $f_P$, and of various form factors $f_P(0)$, directly from experimental data.

The consistency problem of experimental data with unitarity constraints was recently solved, and a procedure for recovering the CKM matrix elements from error affected data was provided in [1]. These unitarity constraints say that the four independent parameters $s_{ij}$ and $\cos \delta$, in standard notation, should take physical values, i.e. $s_{ij} \in (0,1)$ and $\cos \delta \in (-1,1)$, when they are obtained from the equations:

$$V_{ud}^2 = c_{12}^2 c_{13}^2, \quad V_{us}^2 = s_{12}^2 c_{13}^2, \quad V_{ub}^2 = s_{13}^2$$
$$V_{cb}^2 = s_{23}^2 c_{13}^2, \quad V_{tb}^2 = c_{13}^2 c_{23}^2$$
$$V_{cd}^2 = s_{12}^2 c_{23}^2 + s_{13}^2 s_{23}^2 c_{12}^2 + 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta,$$
$$V_{cs}^2 = c_{12}^2 c_{23}^2 + s_{13}^2 s_{23}^2 - 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta,$$
$$V_{td}^2 = s_{13}^2 c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 - 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta,$$
$$V_{ts}^2 = s_{13}^2 c_{12}^2 c_{23}^2 + c_{12}^2 s_{23}^2 + 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos \delta$$

where $V_{ij} = |U_{ij}|$, are the measured moduli, and $U_{ij}$ are the entries of the CKM matrix. It was shown in [1] that if the physical quantities also depend upon CKM matrix moduli, the reconstruction of a unitary matrix from such data is essentially unique, and in the following our independent parameters are $V_{ij}$. The consistency condition between data and CKM matrix unitarity says that $\cos \delta$ obtained from (1) should be the same for all possible choices of four independent moduli $V_{ij}$. Because there are 58 groups of four independent moduli, one gets
165 different expressions for $\cos \delta$ and all have to take (roughly) the same value when computed from Eqs. (1). If we use the CKM moduli values provided by PDG2008 group [2], values obtained from a fit, not the experimental measured values given in [2], one gets

$$\langle \cos \delta \rangle = 0.468 - 0.005 i, \quad \langle \sigma_{\cos \delta} \rangle = 0.244 + 0.010 i$$

Numerically $\cos \delta \in (0.1798, 1.613)$, with six values greater than 1., and three imaginary values. By consequence to obtain a good fit the $\chi^2$-function has to contain two kinds of terms: the first has to impose the fulfillment of unitarity constraints, i.e. conditions as $-1 \leq \cos \delta_i = \cos \delta_j \leq 1, \ i \neq j = 1, 2, \ldots$, and the second should take into account the physical measured quantities.

The experimental data we consider are those coming from super-allowed $0^+ \rightarrow 0^+$ nuclear beta decays, and from leptonic and semileptonic decays. In the standard model the decay rate for purely leptonic decay is given by

$$\Gamma(P \rightarrow l\nu) = \frac{G_F^2}{8 \pi} |U_{qq'}|^2 f_p^2 M_p m_l^2 \left(1 - \frac{m_l^2}{M_p^2}\right)^2$$

where $G_F$ is the Fermi constant, $M_p$ and $m_l$ are the masses of the decaying meson, and, respectively, of the final lepton, $U_{qq'}$ is the corresponding CKM matrix element, and $f_p$ is the decay constant. The physical observable for semileptonic decays, that depends on $|U_{qq'}|$ and $f(q^2)$, is a little more complicated, see [3], and we omit it. The experimenters provide numerical values for products of the form $|U_{qq'} f_p(0)|$, and in this paper we will use these numerical values. It is clear that from such measurements one cannot find two unknown quantities, let’s say, $|f(0)|$ and $|U_{ij}|$, if we have no supplementary constraints. Our point of view is that the unitarity constraints, which depend only on $|U_{ij}|$ moduli, provide the necessary tool for the separation of moduli, and $f(0)$, or $f_p$. Concerning the moduli values we make one natural assumption, namely: the numerical values for all the measured moduli, $|U_{ij}|$, must be the same irrespective of the physical processes used to determine them. The other parameters, such as the decaying constants $f_p$, form factors $f_q(0)$, $g_A/g_V$, etc., which parametrize the data from one given experiment, are considered free parameters to be found from fit.

The $\chi^2$-function has the form
\[ \chi^2 = \sum_{j=u,e,t} \left( \sum_{i=d,s,b} V_{ji}^2 - 1 \right)^2 + \sum_{j=d,s,b} \left( \sum_{i=u,e,t} V_{ij}^2 - 1 \right)^2 + \sum_i \left( \frac{d_i - \hat{d}_i}{\sigma_i} \right)^2 + 
\]

\( + \sum_{i<j} (\cos \delta^{(i)} - \cos \delta^{(j)})^2, \quad -1 \leq \cos \delta^{(i)} \leq 1 \) 

where \( \cos \delta^{(i)} \) are obtained by solving Eqs. (1) for different choices of four independent moduli, \( d_i \) are the theoretical functions one wants to be found from fit, while \( \hat{d}_i \) is the numerical matrix that describes the corresponding experimental data, and \( \sigma \) is the error matrix associated to \( \hat{d}_i \).

Experimental data on \( |U_{ud}| \) comes mainly from three different sources:

a) super allowed, \( 0^+ \rightarrow 0^+ \), nuclear beta decays, [4], and [5],
b) neutron beta decay, \( n \rightarrow p e^+ \nu \), see [6–14],
c) and pion beta decay \( \pi^+ \rightarrow \pi^0 e^+ \nu \), [15]. Data on the decay constants \( f_{\pi}, f_{K}, f_{B}, f_{D}, \) and \( f_{D^*} \) are from papers [16–18], and, respectively, from [19–23]. Numerical results on \( |U_{ud}| \) are from papers [24–36], those upon \( |f_{e^+(0)}^{K+}| \) come from [37] and [38]. The papers [39–48] provide data on \( |F(1)U_{cb}| \), and [40] and [49–51] provide values for \( |G(1)U_{cb}| \).

The central values and uncertainties used in fit are those published in the above papers, and we combined the statistical and systematic uncertainties in quadrature when experimenters provided both of them.

According to [5], the super-allowed beta decays between \( T = 1 \) analog \( 0^+ \) states, together with the conserved vector current (CVC) hypothesis, lead to the conclusion that the \( ft \) values should be the same irrespective of the nucleus, \( i.e. \)

\[ ft = \frac{K}{|G_{\nu}|^2 |M_F|^2} = \text{const}, \]

where \( K \) is the vector coupling constant for semi-leptonic weak interactions, \( f \) is the statistical rate function, and \( t \) is the partial half-life. Because the above relation is only approximately satisfied, one defines a “corrected” \( \mathcal{F}t \) value, which should be “constant”, as

\[ \mathcal{F}t \equiv ft(1 - \delta_\xi)(1 - \delta_C) = \frac{K}{2|G_\nu|^2 (1 + \Delta_K^V)} \]

where \( \delta_C \) is the isospin-symmetry-breaking correction, \( \delta_\xi \) is the transition-dependent part of the radiative correction, and \( \Delta_K^V \) is the transition-independent part. Numerical values for \( \mathcal{F}t \) are given in [4] and [5]. In our fit we use the
above formula with \( G_V \) by supposing \( g_V(0) = 1 \), as CVC requires, and \( |U_{ud}| \) and \( \Delta_{R}^V \) are the free parameters to be obtained from fit. Similarly for the neutron beta decay data we make use of the formula

\[
|U_{ud}|^2 (1 + 3 \lambda^2) = \frac{4908.7(1.9) s}{\tau_n} \tag{7}
\]

see [52], where \( \tau_n \) is the neutron mean life, and the free parameters are \( |U_{ud}| \) and \( \lambda = g_A/g_V \).

The ratio \( g_A/g_V \) also enters in the \( \beta \)-decay asymmetry parameter \( A_0 \), see papers [53–56]. The effect of \( A_0 \)-data was a lowering of \( \lambda \) to the value given in the Table 2, while by using only results from neutron beta decay data the value, \( \lambda = -1.2705 \pm 0.0029 \), is obtained.

The surprising result of our fit was that \( \Delta_{R}^V \) is not transition-independent as it is usually assumed, see Ref. [5]. We remind that the new theoretical estimate given in [5] is

\[
\Delta_{R}^V(new) = (2.361 \pm 0.038)\%
\tag{8}
\]

The fit results on \( \Delta_{R}^V \) are given in Table 1.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>(^{10}\text{C})</th>
<th>(^{14}\text{O})</th>
<th>(^{22}\text{Mg})</th>
<th>(^{34}\text{Ar})</th>
<th>(^{26}\text{Alm})</th>
<th>(^{34}\text{Cl})</th>
<th>(^{38}\text{K})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta_{R}^V ) %</td>
<td>2.319</td>
<td>2.416</td>
<td>2.193</td>
<td>2.489</td>
<td>2.386</td>
<td>2.426</td>
<td>2.412</td>
</tr>
<tr>
<td>Nucleus</td>
<td>(^{42}\text{Sc})</td>
<td>(^{46}\text{V})</td>
<td>(^{50}\text{Mn})</td>
<td>(^{54}\text{Co})</td>
<td>(^{62}\text{Ca})</td>
<td>(^{74}\text{Rb})</td>
<td></td>
</tr>
<tr>
<td>( \Delta_{R}^V ) %</td>
<td>2.429</td>
<td>2.356</td>
<td>2.573</td>
<td>2.579</td>
<td>2.369</td>
<td>2.236</td>
<td></td>
</tr>
</tbody>
</table>

which lead to the mean value

\[
\Delta_{R}^V(fit) = (2.398 \pm 0.108)\%
\tag{9}
\]

The values (8) and (9) are compatible, although our central value is 1. \( \sigma_{new} \) higher than the “constant” \( \Delta_{R}^V \), and our \( \sigma_{fit} = 2.8 \sigma_{new} \). A big discrepancy appears for \(^{50}\text{Mn}\) and \(^{54}\text{Co}\) whose values are bigger than \( \Delta_{R}^V(new) \) with \((5.6–5.7)\sigma_{new} \), and for \(^{22}\text{Mg}\) whose value is lower with \(4.4 \sigma_{new} \). As one conclusion one can say that our approach does not confirm the (approximate) constancy of \( \Delta_{R}^V \), and at the same time it allows a fine structure analysis of all nuclear beta decays,
providing to each experimental group one measure of how far from the ideal situation their measured values are standing. Numerical values for CKM moduli, decay constants and form factors are given in Table 2.

Table 2

<table>
<thead>
<tr>
<th>Para’s</th>
<th>c.v. ± σ</th>
<th>Para’s</th>
<th>c.v. ± σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>V_{ud}</td>
<td>0.974021 ± 4. \times 10^{-7}</td>
<td>δ_{c}</td>
<td>(3.104 ± 0.005)%</td>
</tr>
<tr>
<td>V_{us}</td>
<td>0.22645 ± 8. \times 10^{-6}</td>
<td>f_{π}</td>
<td>130.802 ± 1.335</td>
</tr>
<tr>
<td>V_{ub}</td>
<td>(1.8293 ± 0.6693) \times 10^{-1}</td>
<td>f_{K}</td>
<td>155.685 ± 2.639</td>
</tr>
<tr>
<td>V_{cd}</td>
<td>0.226285 ± 7. \times 10^{-6}</td>
<td>f_{K}/f_{π}</td>
<td>1.119 ± 0.032</td>
</tr>
<tr>
<td>V_{cb}</td>
<td>(38.175 ± 0.03) \times 10^{-3}</td>
<td>f_{D^{0}}</td>
<td>209.6 ± 6.2</td>
</tr>
<tr>
<td>V_{td}</td>
<td>(8.828 ± 0.146) \times 10^{-3}</td>
<td>f_{D^{0}}</td>
<td>278.8 ± 17.8</td>
</tr>
<tr>
<td>V_{cb}</td>
<td>(37.1849 ± 0.0263) \times 10^{-3}</td>
<td>f_{K^{0}}^{s}(0)</td>
<td>956.1 ± 8.3</td>
</tr>
<tr>
<td>V_{ub}</td>
<td>0.999269 ± 3. \times 10^{-7}</td>
<td>f_{K^{0}}^{u}(0)</td>
<td>243.2 ± 20.8</td>
</tr>
<tr>
<td>ΔV_{ub}</td>
<td>(2.399 ± 0.108)%</td>
<td>\mathcal{F}(1)</td>
<td>938.9 ± 78.5</td>
</tr>
<tr>
<td>λ</td>
<td>−1.2694 ± 0.0046</td>
<td>G(1)</td>
<td>979.1 ± 187.4</td>
</tr>
</tbody>
</table>

Result on the parameter $\lambda = g_A/g_V$ was obtained by using the neutron lifetime from [6–14], and the neutron $\beta$-decay asymmetry, $A_0$, [53–56].

From fit we also obtained an experimental value on $δ_c$, which represents the combined radiative and short-range physics corrections, see [15].

The obtained central values for the decay constants and form factors are those normally expected, all the numbers are given in MeV, and the errors are at 1σ level. The results obtained for $f_{π}$ and $f_{K}$ are compatible with those given by PDG2006, [2], although in particular we got a lower value for $f_{K}$. We remark that our value for $f_{K}^{u}(0)$ is compatible to the Leutwyler-Ross theoretical value, [57], and the error is the same!.

More details about our fit, as well as on all physical parameters that can be computed by using our approach, will be given elsewhere.

As one final conclusion we can say that by taking properly into account the unitarity constraints we found one tool which allows the determination of CKM matrix elements, and of various decay constants and form factors directly from experimental data.

Acknowledgements. We acknowledge a partial support from Program CORINT, CNMP contract no 3/2004.
REFERENCES