ON THE MEASUREMENTS REGARDING RANDOM OBSERVABLES

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Both classical and respectively quantum observables can be modeled as somewhat similar examples of random variables. In such a model the associated measurements preserve the values spectrum of an observable but change the corresponding probabilistic weights (probability density or respectively the wave function). Such a model ensures theoretical estimations for predicted errors specific to the mean values as well as to the fluctuations of both types of observables.

1. INTRODUCTION

According to the modern physics terminology the term observable imply the following two features: (i) it is a quantity which characterize quantitatively an intrinsic property of a physical system and (ii) it can be evaluated by extrinsic experimental devices through adequate measurement process. For classical (non-quantum) systems, the two features are treated theoretically as clearly separate subjects. In the case of quantum systems the theoretical descriptions of the mentioned features are often regarded as fundamentally inseparable things (see the references [1–4] and quoted bibliographies).

In this paper we try to develop (at least partially) a suggestion which reconsiders and removes the actually predominant opinions regarding the differences between the approaches of classical and quantum measurements. Our suggestion is builded on the idea that, by means of a few minimal settings, both classical and quantum observables can be described mathematically as random variables. Within the announced suggestion the theoretical description of measurements offer concomitantly evaluations for measuring changes of mean (expected) values as well as of fluctuations (deviations from the mean) which characterize the observables.

2. THE CASE OF CLASSICAL OBSERVABLES

Mathematically, the observables from classical (non-quantum) physics are variables of both deterministic and random types. The observables of random
type are specific in probabilistic-thermodynamics (known as phenomenological theory of fluctuations) and in statistical mechanics. For a given system in a well specified state, a random observable is characterized not by an unique value but by a spectrum (number) of values associated with corresponding probabilities.

In order to simplify our further discussions we will choose a generic situation. We consider a system in a given state to be characterized by a lot of two independent random variables $X$ and $Y$ which are endowed with the spectra $\Omega_X$ and $\Omega_Y$ respectively by the joint distribution of probability $w(x, y)$. For the same system and state we refer to a set of two derived random quantities $\mathcal{A}$ and $\mathcal{B}$.

In order to compress the discussions we will denote all the observables under attention with the symbols $Z_\alpha$ where

$$\alpha = \{1, 2, 3, 4\}, \quad Z_\alpha \in \{Z_1, Z_2, Z_3, Z_4\} \equiv \{\mathcal{A}, \mathcal{B}, \mathcal{X}, \mathcal{Y}\}$$

(1)

So we can say that the main characteristics of the considered system are given by the random observables $Z_\alpha$, by their spectra of values $\Omega_{Z_\alpha}$ and by the joint distribution of probability $w(x, y)$ [5, 6].

By using the above presented settings our suggestion regarding the description of measurements for classical random observables can be modelled as follows. Any measurement, independently of its technical details, can be assumed that it does not change the spectra $\Omega_{Z_\alpha}$ of the previously presented random variables $Z_\alpha$. But, if one takes into account the inherent imperfections of the experimental devices, it is credible and rightful the idea that the same measurement must be described as a process which change the probability distribution $w(x, y)$. Some simplified versions of the respective idea were discussed in few of our previous works [7–12].

The above mentioned change of $w(x, y)$ during a measuring process can schematized as follows. In such a process the input (in) information regarding the intrinsic properties of the measured system is converted in output (out) information incorporated within the data received on a device recorder. That is why a measurement appears as an information transmission process.

Then, in terms of the above explanations, a measurement regarding the considered system can be modelled theoretically through a transformation of the form

$$w_{\text{in}}(x, y) \rightarrow w_{\text{out}}(x, y)$$

(2)

The concrete analytical expression of this transformation requires justifications by taking into account some of the most general characteristics regarding the measuring devices. Among such characteristics of first interest are the following properties ($P$):
**P1:** A good measuring device is stationary in time, i.e. its performances have a sufficiently long standing viability.

**P2:** The same device is forced to guarantee a linear superposition of the input signals in giving acceptable output records.

These properties can be incorporated naturally into the alluded modelling of a measurement if the transformation (2) is written as follows

\[
 w_{\text{out}}(x, y) = \int_{\Omega_X} \int_{\Omega_Y} G(x, y | x', y') \cdot w_{\text{in}}(x', y') \cdot dx' \cdot dy'
\]

Regarded from the physics perspective the term \( G(x, y | x', y') \) incorporates the theoretical description of all the characteristics of the measuring device.

The kernel \( G(x, y | x', y') \) must satisfy the following relations

\[
\int_{\Omega_X} \int_{\Omega_Y} G(x, y | x', y') \cdot dx \cdot dy = 1
\]

\[
\int_{\Omega_X'} \int_{\Omega_Y'} G(x, y | x', y') \cdot dx' \cdot dy' = 1
\]

where, according to the above stipulations, we take \( \Omega_X \equiv \Omega_{X'}; \ \Omega_Y \equiv \Omega_{Y'} \).

Then, for the respective observables, the measuring errors ("uncertainties"), induced by the discussed kind of measurement, can be evaluated theoretically by the following first order indicators:

\[
\text{PEI}\{Z_{\alpha}\} = \langle Z_{\alpha}\rangle_{\text{out}} - \langle Z_{\alpha}\rangle_{\text{in}}
\]

where the symbol \( \text{PEI}\{Q\} \) signifies the predicted error indicator of the quantity \( Q \).

### 3. THE CASE OF QUANTUM OBSERVABLES

We consider a simple microparticle (quantum system) endowed only with orbital motions (characteristics). The state of such a system is described by the wave function \( \Psi(\vec{r}, t) \). But the wave function describes only the presence of the particle in a location in space but not the travel of the same particle through the respective location. Such a travel is described by the probability current \( \vec{j}(\vec{r}, t) \).

The physical properties of the particle are associated with the observables \( A_\alpha \) (\( \alpha = 1, 2, \ldots, n \)), described by the quantum operators \( \hat{A}_\alpha \). The respective operators are generalized random variables.

By reporting our discussions to the above reminded quantum notions now we try to develop a description of quantum measurements by adopting some
viewpoints presented in the previous section about classical measurements. Firstly we note that a measurement of a random observable does not change its spectrum of values. This fact means that a quantum measurement must be regarded as an action that preserves the mathematical expressions of the operators $\hat{A}_q$. On the other hand, by taking into account the imperfections of experimental devices, it is credible and rightful the idea that the same action must be regarded also as a process in which the input (in) information (probabilities) regarding the intrinsic properties of the measured system are converted in output (out) information incorporated within the data received on a recorder device. So a quantum measurement appears also as an information transmission process.

Because, as a rule the devices which measures the quantities $\rho(\vec{r},t)$ and $j(\vec{r},t)$ are technically distinct objects, the mentioned changes must be described through separate transformations. Similarly with the classical situations for the description of the quantum measurements we have to take into account the fact that, mainly, the measuring devices must have the same properties $P1$ and $P2$ mentioned in previous section.

Based on the above noted facts we consider that the mentioned transformations can be taken of the forms

$$\rho_{out}(\vec{r},t) = \int_{\mathbb{R}^3} \Gamma(\vec{r} | \vec{r}') \cdot \rho_{in}(\vec{r}',t) d^3\vec{r}'$$

(6)

$$j_{out,\mu}(\vec{r},t) = \sum_{\nu=1}^{3} \int_{\mathbb{R}^3} \Lambda_{\mu\nu}(\vec{r} | \vec{r}') \cdot j_{in,\nu}(\vec{r}',t) d^3\vec{r}'$$

(7)

Note that by omitting the time $t$ in the expressions of kernels $\Gamma(\vec{r} | \vec{r}')$ and $\Lambda_{\mu\nu}(\vec{r} | \vec{r}')$, in our model, we consider the measurements as instantaneous actions (i.e. we neglect the relativistic effects connected with retarded influences).

In the relations (6) and (7), consonantly with the viewpoint of physics, the kernels $\Gamma(\vec{r} | \vec{r}')$ and $\Lambda_{\mu\nu}(\vec{r} | \vec{r}')$ depict theoretically the actions of the measuring devices.

The kernels $\Gamma(\vec{r} | \vec{r}')$ and $\Lambda_{\mu\nu}(\vec{r} | \vec{r}')$ must satisfy the following conditions:

$$\int_{\mathbb{R}^3} \Gamma(\vec{r} | \vec{r}') \cdot d^3\vec{r}' = \int_{\mathbb{R}^3} \Gamma(\vec{r} | \vec{r}') \cdot d^3\vec{r}' = 1$$

(8)

$$\sum_{\mu=1}^{3} \int_{\mathbb{R}^3} \Lambda_{\mu\nu}(\vec{r} | \vec{r}') \cdot d^3\vec{r}' = \sum_{\nu=1}^{3} \int_{\mathbb{R}^3} \Lambda_{\mu\nu}(\vec{r} | \vec{r}') \cdot d^3\vec{r}' = 1$$

(9)
Associated to the two kinds of situations $\eta = in, out$, for a set $A_\alpha$, $(\alpha = 1, 2, \ldots, n)$, of quantum observables described by the operators $\hat{A}_\alpha$, the following probabilistic parameters can be evaluated

$$< A_\alpha >_\eta = \int_{\mathbb{R}^3} \Psi_\eta^* (\vec{r}, t) \hat{A}_\alpha \Psi_\eta (\vec{r}, t) \cdot d^3 \vec{r}$$

(10)

$$\text{Var}_\eta (A_\alpha) = \sigma^2_\eta (A_\alpha) = \left( < A_\alpha - < A_\alpha >_\eta >_\eta \right)^2$$

(11)

$$\text{Cov}_\eta (A_\alpha, A_\beta) = \left( < A_\alpha - < A_\alpha >_\eta >_\eta < A_\beta - < A_\beta >_\eta >_\eta \right), \quad \alpha \neq \beta$$

(12)

$$\text{Cov}_\eta (A_\alpha^m, A_\beta^n) = \left( < A_\alpha^m - < A_\alpha^m >_\eta >_\eta < A_\beta^n - < A_\beta^n >_\eta >_\eta \right), \quad \left\{ \begin{array}{l} \alpha = \text{or} \neq \beta \\ m + n = s \geq 3 \end{array} \right.$$ 

(13)

Now, within the above promoted model about the description of measurements, let us note what are the indicators of the predicted errors ($\mathcal{PEI}$), specific for quantum measurements. Taking into account the formulas (10)–(13) now we can state that the roles of such indicators can be performed by the following quantities

$$\mathcal{PEI} \left[ < A_\alpha > \right] = < A_\alpha >_{out} - < A_\alpha >_{in}$$

(14)

$$\mathcal{PEI} \left[ \text{Var} (A_\alpha) \right] = \text{Var}_{out} (A_\alpha) - \text{Var}_{in} (A_\alpha)$$

(15)

$$\mathcal{PEI} \left[ \sigma (A_\alpha) \right] = \sigma_{out} (A_\alpha) - \sigma_{in} (A_\alpha)$$

(16)

$$\mathcal{PEI} \left[ \text{Cov} (A_\alpha, A_\beta) \right] = \text{Cov}_{out} (A_\alpha, A_\beta) - \text{Cov}_{in} (A_\alpha, A_\beta), \quad \alpha \neq \beta$$

(17)

$$\mathcal{PEI} \left[ \text{Cov}_\eta (A_\alpha^m, A_\beta^n) \right] = \text{Cov}_{out} (A_\alpha^m, A_\beta^n) - \text{Cov}_{in} (A_\alpha^m, A_\beta^n), \quad \left\{ \begin{array}{l} \alpha = \text{or} \neq \beta \\ m + n = s \geq 3 \end{array} \right.$$ 

(18)

4. CONCLUSIONS

In this paper we have suggested that the measurements regarding the cases of classical respectively quantum observables to be described theoretically in somewhat similar manners. In such a description a measurement preserves the values spectrum of an observable but changes the corresponding probabilistic weights (classical probability density or, respectively, the quantum wave function).
Our modeling of measurements ensures theoretical estimations for predicted error indicators specific to the mean values as well as to the fluctuations of both classical and quantum types of observables.

REFERENCES


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