THE PAIR NEUTRINO ENERGY LOSS FOR NUCLEI $^{56}Fe$ AT THE LATE STAGES OF STELLAR EVOLUTION

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Based on the Weinberg-salam theory, the pair neutrino energy loss rates for nuclei $^{56}Fe$ are canvassed for the wide range of density and temperature. The pair neutrino energy loss rates are sensitive to temperature. The results of ours ($QLJ$) is compared with those of Beaudet, Petrosian, and Salpeter’s ($QBPS$) and show that the pair neutrino energy loss rates of $QBPS$ are always larger than $QLJ$. The $QBPS$ will deviate 12.57%, 12.86%, 14.99%, 19.80% times higher than $QLJ$ correspond to the temperature of $T_9=0.385, 1.0, 5.0, 10$.

Key words: neutrino energy loss; stellar evolution.

I. INTRODUCTION

According to stellar evolution theory, when the temperature go up to $5 \times 10^7 K$ or the more and the density is no more than $10^7 g/cm^3$, the matter has been nodegenerate nature in the cores of massive stars ($M > (60 \sim 100) M_\odot$). At this time, the average energy of the hot photons has already exceeded the quiescent energy of the electron. It is known that the radiation of pair neutrino would be dominated over the other electromagnetic radiations. During this evolution period, a good many neutrinos which can escape unhindered in circumstances where photons are trapped, are produced and carried away large quantity of messages and energy from star. On the other hand, the gravitation of the massive star will be exceeded the pressure of the electron gas out and away. It may be lead to the gravitation collapse of the massive star. Some researches show that the pair neutrino energy loss is large enough to make the core of the massive star cool greatly and the pressure of the electron gas would be went down quickly. So the pair neutrino energy loss (hereafter PNEL) plays a key role and is the major factor for unstable gravitation collapse.

Some authors investigated extensive results of their calculation of the PNEL, such as Weinberg; [1] Salam [2] and Dicus. [3] Basing on the Feynmann-Gell-Mann theory, Pinave [4] and Beaudet, Petrosian, and Salpeter (hereafter BPS) [5] remarked the PNEL. The PNEL rates were also investigated by Naoki Itoh et al. [6–8] based on the Weinberg-salam theory. S. Esposito et al. [9] had investigated the PNEL at wide density-temperature region at the late stages of stellar evolutions. Based on the Weinberg-salam theory, the PNEL for nuclei $^{56}_{Fe}$ will be investigated at the Late Stages of Stellar Evolution. We reconsider the PNEL rates, according to the method of Itoh’s, for the wide range of the density and temperature. The results, we obtained, will be compared with BPS’s which will be reinvestigated according to the method of BPS’s. The present paper is organized as follows. In the next section, the calculation of the PNEL rates is formulated. In section III some numerical results on the PNEL rates will be presented. Some conclusions are given in section IV.

II. THE PNEL RATES

Based on the Weinberg-salam theory, The PNEL rates per unit volume per unit time due to the pair neutrino process is written as [1, 2, 8]

$$Q_{i\nu} = \frac{1}{2} \left[ \left( C_{i\nu}^2 + C_{i\nu}^\prime + C_{i\nu}^\prime\right) + n \left( C_{i\nu}^2 + C_{i\nu}^\prime + C_{i\nu}^\prime\right) \right] Q_{\text{pair}}$$

$$+ \frac{1}{2} \left[ \left( C_{i\nu}^2 - C_{i\nu}^\prime + C_{i\nu}^\prime\right) + n \left( C_{i\nu}^2 - C_{i\nu}^\prime + C_{i\nu}^\prime\right) \right] Q_{\text{pair}}$$

(1)

where $C_{i\nu} = \frac{1}{2} + 2\sin^2 \theta_w$; $C_{i\nu}^\prime = -1 - C_{i\nu}$; $C_{i\nu}^\prime = 1 - C_{i\nu}$ and $\sin^2 \theta_w = 0.2319 \pm 0.0005$.

The $\theta_w$ is the Weinberg angle and the $n$ is the number of the neutrino flavors other than the electron neutrino, whose masses can be neglected compared with $K_n T$. According to Ref [8], the pair neutrino energy loss rates are expressed in units of $\text{ergs cm}^{-3} \text{s}^{-1}$ as

$$Q_{i\nu} = \frac{1}{2} \left[ \left( C_{i\nu}^2 + C_{i\nu}^\prime + C_{i\nu}^\prime\right) + n \left( C_{i\nu}^2 + nC_{i\nu}^\prime \right) \right]$$

$$\times \left[ 1 + \frac{\left( C_{i\nu}^2 - C_{i\nu}^\prime + C_{i\nu}^\prime\right) + n \left( C_{i\nu}^2 - C_{i\nu}^\prime + C_{i\nu}^\prime\right) q_{\text{pair}}}{\left( C_{i\nu}^2 + C_{i\nu}^\prime + C_{i\nu}^\prime\right) + n \left( C_{i\nu}^2 + C_{i\nu}^\prime + C_{i\nu}^\prime\right)} \right] \times g(\lambda) e^{-\lambda} f_{\text{pair}}$$

(2)
The pair neutrino energy loss for nuclei $^{56}$Fe

$$q_{\text{pair}} = \left(10.7480 \lambda^2 + 0.3967 \lambda^{0.5} + 1.0050 \right)^{-1.0} \times \left[1 + \left(\frac{\rho}{\mu_e}\right)\left(7.692e + 07 \lambda^3 + 9.715e + 06 \lambda^{0.5}\right)\right]^{-0.3} \tag{3}$$

$$f_{\text{pair}} = \frac{a_0 + a_1 \xi + a_2 \xi^2}{\xi^3 + b_1 \lambda^{-1} + b_2 \lambda^{-2} + b_3 \lambda^{-3}} e^{-\xi} \tag{4}$$

$$g(\lambda) = 1 - 13.04 \lambda^2 + 133.5 \lambda^4 + 1534 \lambda^6 + 918.6 \lambda^8 \tag{5}$$

$$\xi = \left(\frac{\rho / \mu_e}{10^9 \text{g/cm}^3}\right) \lambda^{-1} \tag{6}$$

$$\lambda = \left(\frac{T}{5.9302 \times 10^9 K}\right) \tag{7}$$

where $\rho/\mu_e$ is the density in units of g/cm$^3$ and $T$ is the temperature in units of K. We use the natural unit in which $\hbar = c = 1$ in this article unless specified explicitly. The constant $a_0, a_1, a_2, b_1, b_2, b_3, c$ will be found in Table 1.

Table 1

<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 10^{10}$K</td>
<td>6.002e+19</td>
<td>2.084e+20</td>
<td>1.872e+21</td>
<td>9.383e-1</td>
<td>-4.141e-1</td>
<td>5.829e-2</td>
<td>5.5924</td>
</tr>
<tr>
<td>$\geq 10^{10}$K</td>
<td>6.002e+19</td>
<td>2.084e+20</td>
<td>1.872e+21</td>
<td>1.2383</td>
<td>-0.8141</td>
<td>0.0000</td>
<td>4.9924</td>
</tr>
</tbody>
</table>

Basing on the Feynmann-Gell-Mann theory, BPS calculated the NEL rates for the pair neutrino process. They fitted their results to [5]

$$Q_{\text{BPS}} = \frac{g^2}{18 \pi^2} \left(\frac{m c^2}{\hbar}\right)^3 \left(\frac{m c^2}{\hbar}\right) \times \left[\frac{G_0}{1\over 2} \left(7G_{1/2}^* + 5G_{3/2}^*\right) + G_0' \left(7G_{1/2}^* + 5G_{3/2}^*\right)\right] + G_i \left(8G_{1/2}^* - 2G_{3/2}^*\right) + G_i' \left(8G_{1/2}^* - 2G_{3/2}^*\right) \tag{8}$$

$$G_{\nu}^b (\lambda, \nu) = \lambda^{3 + 2n} \int_{\lambda^{-1}}^{\infty} \frac{x^{2n+1} (x^2 - \lambda^{-2})^{1/2}}{1 + e^{x \nu}} dx \tag{9}$$
\[ g = G m^7 = (3.002 \pm 0.006) \times 10^{-12} \] (10)

In order to compare the results of \( Q_{LJ} \) with those of \( Q_{BPS} \) for the nuclei \(^{56}\text{Fe}\) at the different temperatures, the factors \( C \) are defined as follows

\[ C = \frac{Q_{BPS} - Q_{LJ}}{Q_{LJ}} \] (11)

III. SOME NUMERICAL RESULTS ON PNEL RATES

Figures 1–4 show that the NEL rates for nucleus \(^{56}\text{Fe}\) vary with density \( \rho/\mu_\text{e} \) at the temperature of \( T_9 = 0.385, 1.0, 5.0, 10 \) respectively (\( T_9 \) is the temperature in units of \( 10^9 \text{K} \)). One can find that the PNEL rates are sensitive to the temperature. The higher the temperature is (such as \( T_9 = 15 \)), the smaller the affection on the NEL is. For example, the PNEL rates \( Q_{LJ} \) of \(^{56}\text{Fe}\) is \( 1.5 \times 10^4 \text{ergs cm}^{-3} \text{ sec}^{-1} \) at the density of \( 10^4 \text{g/cm}^3 \) and \( T_9 = 0.385 \) in Fig. 1, whereas it will increase to \( 6.2 \times 10^2 \text{ergs cm}^{-3} \text{ sec}^{-1} \) at \( T_9 = 5 \) in Fig. 3. This is because that the Boltzmann factor \( e^{-(2/\lambda)} \) is very small at lower temperature (such as \( T_9 = 0.385 \)) but it is very important at medium and high temperature (such as \( T_9 = 5 \)), especially at lower density (such as density of \( 10^4 \text{g/cm}^3 \)).

![Diagram showing PNEL rates of QBPS and Q_LJ versus density for nuclide \(^{56}\text{Fe}\) at the temperature \( T_9 = 0.385 \).](image)

Fig. 1. – The PNEL rates of \( Q_{BPS} \) and \( Q_{LJ} \) versus the density for nuclei \(^{56}\text{Fe}\) at the temperature of \( T_9 = 0.385 \).
On the other hand, the comparison of the results of $Q_{LJ}$ with those of $Q_{BPS}$ for the nucleus $^{56}\text{Fe}$ at the different temperatures will be shown in the four Figs. One can also find that the PNEL rates of $Q_{BPS}$ are always larger than $Q_{LJ}$. The higher the
temperature is (such as $T_g=10$), the larger the difference of the PNEL rates between $Q_{BPS}$ and $Q_{LJ}$ is. It is reason that the pair neutrino process would be degenerate at relativity high temperature due to the strong dependence of the number density of the electron and positrons. It is readily seen that the NEL rates of BPS are higher than ours due not to consider the plasma affection on the electron-positron pairs by BPS.

Fig. 4. – The PNEL rates of $Q_{BPS}$ and $Q_{LJ}$ versus the density for nuclei $^{56}$Fe at the temperature of $T_g=10$.

Fig. 5. – The factor $C$ versus the density for nuclei $^{56}$Fe at different temperature of $T_g=0.385$, 1.0, 5.0, 10.
The numerical results of the factor $C$ versus the density for nuclei $^{56}Fe$ at different temperature will be given in Fig. 5. It has been found that it is readily seen that the maximum difference are 12.57%, 12.86%, 14.99%, 19.80% correspond to the temperature of $T_9=0.385, 1.0, 5.0, 10$.

In summary, one can conclude that the influence of the temperature on the PNEL is very obvious. The higher the temperature is (such as $T_9=10$), the larger the affection on the PNEL is from above five Figs, we can also find that the density has different effect on PNEL rates for nuclei $^{56}Fe$ at the different temperature. The higher the density is the larger influence on PNEL is at the same temperature. Because the electron Fermi energy is so high at high density that the PNEL rates are influenced so much. On the other hand, at the low density and high temperature the electron Fermi energy is so small and electron average energy is high enough. It may lead to a great deal of neutrinos produce which comes from the pair neutrino process.

**IV. CONCLUDING REMARKS**

We calculate the neutrino energy loss rates due to the pair neutrino process using the Weibberg-salam theory. By analysis PNEL rates of the nuclei $^{56}Fe$ at different temperature-density region, we draw the following results that the PNEL rates are sensitive to the temperature. On the other hand, the PNEL rates of $QBPS$ are always larger than $QLJ$. The $QBPS$ for the nuclei $^{26}Fe$ will deviate 12.57%, 12.86%, 14.99%, 19.80% times higher than $QLJ$ correspond to the temperature of $T_9=0.385, 1.0, 5.0, 10$.

As is well known, with escaping of a great many of neutrinos by pair neutrino process, the neutrino energy loss gives one of the main contributions to the cooling of stellar interior in the late stages of star evolution. It is helpful to the collapse and the explosion of the supernova. Therefore the investigations on the neutrino energy loss have been the important questions and the conclusion, we obtained in this study, may have significant help on further research of nuclear astrophysics and neutrino astrophysics.

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