NON EXISTENCE OF BIANCHI TYPE – V STRING COSMOLOGICAL MODEL WITH BULK VISCOUS FLUID IN GENERAL RELATIVITY

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Received June 8, 2007

Bianchi type – V string cosmological model with bulk viscous fluid is considered in general relativity. It is interesting to note that cosmic strings with bulk viscous fluid do not exist in Bianchi type – V cosmology. Hence the vacuum model is constructed.

Key words: Bianchi type – I space time, cosmic strings, bulk viscous fluid.

1. INTRODUCTION

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe have a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogeneous and anisotropic cosmologies.

The origin of our universe is one of the greatest cosmological mysteries even today. The exact physical situation at early stage of the formation of our universe is still unknown. The concept of string theory was developed to describe events of the early stage of the evolution of the universe. The general relativistic treatment of strings was initiated by Stachel (1980) and Letelier (1983). The gravitational effects of cosmic strings have been extensively discussed by Vilenkin (1981), Gott (1985), Letelier (1983) in general relativity. The string cosmological models with magnetic field are investigated by Chakraborty (1991), Tikekar & Patel (1992). Bhattacharjee and Baruha (2001) studied the problems of cosmic strings taking Bianchi type cosmologies with a self – interacting scalar field. Cosmic strings are important in the early stages of evolution of the universe before the particle creation. The present day observations do not rule out the possible existence of large scale networks of strings in the early universe. Reddy (2003) shown that, the cosmic strings which have received considerable attention in cosmology do not exist in the frame work of Rosen’s (1973) bimetric theory of gravitation.

The role of viscosity in cosmology has been investigated by Weinbergs (1971), Nightingale (1973), Heller and Klimek (1975). Viscosity is important in cosmology for a number of reasons. Misner [1967, 1968] has studied the effect of viscosity on the evolution of cosmological models. Collins and Stewart (1971) have studied the effect of viscosity on the formation of galaxies.


Also several aspects of viscous fluid cosmological models in early universe have been extensively investigated by many authors. Krori and Mukherjee (2000), Singh and Beesham (2000), Bali and Sharma (1995), Bali and Upadhyaya (2003) have discussed locally rotationally symmetric Bianchi type-I string cosmological models with bulk viscosity. Bianchi type-V cosmological models have been studied by other researcher, Reddy (2006), Roy and Prasad (1994), Farnsworth (1967) in different context. Recently Pradhan et al. (2006) studied tilted Bianchi type – I cosmological models filled with disordered radiation in presence of a bulk viscous fluid & head flow also Raj Bali & A. Pradhan (2007) have studied Bianchi type – III string cosmological models with Time dependent Bulk Viscosity.

The purpose of the present work is to obtain vacuum Bianchi type – V string cosmological model with bulk viscous fluid. Our paper is organized as follows.

In section 2, we derive the field equations and their solution in Bianchi type – V String cosmological model with bulk viscous fluid. The last section contains some conclusions.

2. FIELD EQUATIONS OF STRING COSMOLOGICAL MODEL WITH BULK VISCOS FLUID

We consider the Bianchi type-V space-time in the general form

\[ ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2t} (dy^2 + dz^2) , \]  

where \( A \) and \( B \) are functions of \( t \) only.

The energy momentum tensor for a cloud of string dust with a bulk viscous fluid as given by Letelier (1979), Bali and Upadhaya (2003) is

\[ T_{ij} = \rho v_i v^j - \lambda x_i x^j - \zeta v_i (g^j_i + v_j v^j) \]  

where \( v_i \) and \( x_i \) satisfy the condition

\[ v_i v^i = -x_i x^i = -1, \quad v^i x_i = 0. \]
Here $\xi$ is coefficient of bulk viscosity, $\theta = v^j_\ j$ is scalar expansion, $\rho$ is the proper energy density of cloud of strings with particles attached to them. $\lambda$ is the string tension density, $v^j$ is the four velocity of the particles and $x^j$ is a unit spacelike vector representing the direction of strings. If the particle density of the configuration is denoted by $p_p$ then we have
\[ \rho = \rho_p + \lambda \] (4)

The Einstein’s field equations with gravitational units $8 \pi G = C = 1$ read as
\[ R^i_\ j - \frac{1}{2} R g^i_\ j = -T^i_\ j \] (5)
where $R^i_\ j$ is the Ricci tensor, $R = g^{ij} R_{ij}$ is the Ricci scalar.

In comoving co-ordinate system, we have
\[ v^i = (0,0,0,1) \quad \text{and} \quad x^i = \left( \frac{1}{A}, 0, 0, 0 \right). \] (6)

The field equations (5) with (2) subsequently lead to the following system of equations:
\[ 2 B^4_\ A + \frac{B^2_\ A}{B^2} = \frac{1}{A^2} (\lambda + \xi \theta) \] (7)
\[ \frac{A^4_\ A}{A} + \frac{B^2_\ A}{B} + \frac{A^2 B^2_\ A}{AB} = \frac{1}{A^2} \xi \theta \] (8)
\[ 2 \frac{A^2 B^2_\ A}{AB} + \frac{B^2_\ A}{B^2} - \frac{3}{A^2} = \rho \] (9)
\[ 2 \frac{B^2_\ A}{B} - 2 \frac{A^2_\ A}{A} = 0 \] (10)

where the index 4 at the symbols $A$ and $B$ denotes ordinary differentiation with respect to $t$. The physical variables namely the scalar expansion $\theta$ have the following expressions for the above metric (1):
\[ \theta = \frac{A^4_\ A}{A} + 2 \frac{B^2_\ A}{B} \] (11)

From equation (10), we get $A = \mu B$ (12)
Without loss of generality, we can take $\mu = 1$. We get, $A = B$. (13)

With the help of equation (1), the set of equations (7)–(9), reduces to
\[ 2 \frac{B^4_\ A}{B} + \left( \frac{B^2_\ A}{B} \right)^2 - \frac{1}{B^2} = (\lambda + \xi \theta) \] (14)
\[ 2 \frac{B^4_\ A}{B} + \left( \frac{B^2_\ A}{B} \right)^2 - \frac{1}{B^2} = \xi \theta \] (15)
Solving equation (15), (16), we get

$$\frac{6B_{44}}{B} = (3\xi\theta - \rho)$$  \hspace{1cm} (17)

The field equations (14)–(16) are highly non-linear three differential equations in five unknown parameters $B$, $\rho$, $\lambda$, $\theta$, $\xi$. One additional constraint relating these parameters is required to obtain a determinate solution. Therefore, we assume a relation between the coefficient of bulk viscosity $(\xi)$, scalar expansion $(\theta)$ and density $(\rho)$ given by

$$3\xi\theta = \rho$$  \hspace{1cm} (18)

Using above equation (18) into equation (17), we have $B_{44} = 0$  \hspace{1cm} (19)

Integrating, we obtain $A = B = C_1 t + C_2$  \hspace{1cm} (20)

where $C_1$ and $C_2$ are constants of integration.

Comparing equations (14) and (15), we get, $\lambda = 0$.  \hspace{1cm} (21)

$$\lambda = -\frac{3(C_1^2 - 1)}{(C_1 t + C_2)^2}$$  \hspace{1cm} (22)

when $C_1 = 1$ then we get $\rho = 0$ and $\xi = 0$.

$\therefore \lambda = 0$ & $\rho = 0$ which immediately shows that cosmic strings do not occur in Bianchi type – V space time with bulk viscosity.

Hence Bianchi type – V string cosmological model with bulk viscous fluid in general relativity can be expressed as

$$ds^2 = -dt^2 + \left(C_1 t + C_2\right)^2 dx^2 + \left(C_1 t + C_2\right)^2 e^{2\xi} (dy^2 + dz^2)$$

Which can be reduced (after a proper choice of coordinates and constants) in the form

$$ds^2 = -dT^2 + T^2 dX^2 + T^2 e^{2\xi} (dY^2 + dZ^2).$$  \hspace{1cm} (23)

3. STRING COSMOLOGICAL MODEL WITHOUT BULK VISCOUS FLUID

In this section we obtain a string cosmological model without bulk viscous fluid. The field equation (5) with equation (2) (only without bulk viscous fluid at string), lead to the following

$$2\frac{B_{44}}{B} + \frac{B}{B^2} - \frac{1}{A^2} = \lambda.$$  \hspace{1cm} (24)
By the same arguments as in section 2, the equation (24)–(27) reduces to
\[
\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_{4}B_{4}}{AB} - \frac{1}{A^2} = 0
\]
(25)
\[
2\frac{A_{4}B_{4}}{AB} + \frac{B_{4}^2}{B^2} - \frac{3}{A^2} = \rho
\]
(26)
\[
\frac{2B_{4}}{B} - \frac{2A_{4}}{A} = 0
\]
(27)

To obtain a determinate solution of the above equations (28)–(30) which are highly non-linear field equations, we assume a relation \( \rho = \lambda \), i.e., proper energy density for a cloud string is equal to the string tension density. (31)

From equations (28) and (29), we obtain \( \lambda = 0 \), hence \( \rho = 0 \).

From equations (28), (30) and (31), we obtain
\[
B_{44} = 0
\]
(32)
Integrating it, we obtain
\[
A = B = C_3 t + C_4
\]
(33)
where \( C_3 \) and \( C_4 \) are constants of integration.

Comparing equation (28) and (29), we obtain
\[
\lambda = 0
\]
\[
\therefore \lambda = \rho = 0
\]
(34)
which immediately means that Bianchi type – V string cosmological model do not exist without bulk viscous fluid in general relativity. Hence in this case also we obtain the vacuum model given by equation (23).

4. PHYSICAL AND KINEMATICAL PROPERTIES

Spatial Volume \( V = \sqrt{-g} = (C_1 t + C_2)^3 e^{\psi} \), where \( C_1 \) and \( C_2 \) are constants.

Expansion Scalar \( \theta = \sqrt{g} \) \( = 3B_4 / B = 3C_1 / (C_1 t + C_2) \)

Shear Scalar \( \sigma^2 = \frac{7}{2} \left( \frac{C_1}{C_1 t + C_2} \right)^2 \)
Deceleration Parameter $q = 0$. Therefore universe is accelerating due to introduction of bulk viscosity. As such present day observations suggest that our universe is accelerating.

**CONCLUSION**

It may be noted, here, that Krori *et al.* (1994) have shown that in the context of general relativity cosmic strings do not occur in Bianchi type – V cosmology. Our finding is similar to above. It is also observed that bulk viscosity plays no role in the evolution of the universe.

**REFERENCES**