In this work we report an investigation of the behavior of experimental two-neutron separation energies \( S_{2n} \) along the isotopic and isotonic chains. The study refers to the even-even nuclei between Ni and Hg where a rich collective phenomenology is present. The emphasis is on finding non-monotonic behaviors which can be correlated with structure phenomena as shell and subshell closures, phase/shape transition, intruder configurations, etc. In order to enhance the sensitivity of our search, differential variation of the \( S_{2n} \) has been investigated along with the absolute values.

Key words: nuclear structure, two neutron separation energies, nuclear shells, phase/shape transitions.

1. INTRODUCTION

Mass of the atomic nucleus is a basic observable embedding subtle information both on the average nuclear field and nucleon-nucleon correlations. Starting from the nuclear masses can be computed several quantities highly relevant for understanding various features of nuclear structure. In particular for the present work the quantity of interest is the two-neutron separation energies \( S_{2n} \) from the even-even medium mass nuclei.

It is well known that the energy involved in removal of two fermions from a strongly correlated system of identical fermions must be a good indicator for the system stability. Moreover, if the pairing is a dominant component in the binary fermion-fermion interaction, then the energy of separating two fermions will have much higher values for systems with even number of particles than for those with odd number.
Both aforementioned general characteristics can be clearly observed in nuclear systems from the systematic representation of the experimental values of the $S_{2n}$ along the isotopic chains [1]. Gross nuclear structure features like major shell closures (2, 8, 20, 28, 50, 82, 126) are clearly seen from these representations [2]. Moreover, by looking with a proper “eye-glass” at these graphs it appears that a richer phenomenology can be revealed (e.g. subshell closures, phase/shape transition, intruder configurations, etc.). It is the goal of the present study to perform a systematic analysis of all available experimental $S_{2n}$ values in the even-even medium mass nuclei between Ni ($Z=28$) and Hg ($Z=80$), with the main emphasis on revealing non-monotonic evolutions. We expect a rich phenomenology mainly related with the nuclear collectivity and sub-shell closures. The “eye-glass” employed is the differential variations of the $S_{2n}$ along the isotopic and isotonic chains symbolized in the present paper by $dS_{2n}$ and $dS_{2n}^Z$, respectively.

2. THE OBSERVABLE $S_{2n}$ AND ITS DIFFERENTIAL VARIATION ALONG THE ISOTOPIC CHAINS ($dS_{2n}$)

Two neutron separation energy, i.e. the energy required to remove 2 neutrons from a nucleus with $Z$ protons and $N$ neutrons, $S_{2n}(Z, N)$, can be computed from the ground state nuclear masses $M(Z, N)$ and $M(Z, N–2)$ and the neutron mass $m_n$ with the relation:

$$S_{2n}(Z, N) = -M(Z, N) + M(Z, N–2) + 2m_n$$  \hspace{1cm} (1)

We used for the present study the last review of the nuclear masses reported in ref. [2], included in reference databases (e.g. ref [3]) and, where available, very recent data (see below). In this review, $S_{2n}$ values for the even-even nuclei are given with uncertainties up to 0.76 MeV. The percentage of nuclei that have an uncertainty of determination over 0.3 MeV is only 0.01% and this is for $S_{2n}$ values larger than 4 MeV; 10% of the nuclei have an uncertainty of determination between 0.1 and 0.3 MeV, 34% between 0.02 and 0.1 MeV, and the rest of the uncertainties are below 0.02 MeV. In all figures of the present paper the uncertainties of the experimental data which are smaller than the size of the symbols are not represented.

Recently, very precise mass measurements were performed [4–7]. In Table 1 we compare the new $S_{2n}$ values we calculated from these masses (New $S_{2n}$) where available, with the previous (Old $S_{2n}$) data from ref. [2].
### Table 1
Comparison of the new $S_{2n}$ values with the previous data. In the column “Ref.” are indicated the references from where are taken the masses (Mass Excess) used for calculation of the “New $S_{2n}$”. The “Old $S_{2n}$” are taken from Ref. [2].

<table>
<thead>
<tr>
<th>Element</th>
<th>Z</th>
<th>N</th>
<th>New $S_{2n}$ [MeV]</th>
<th>Ref.</th>
<th>Old $S_{2n}$ [MeV]</th>
<th>Comparison (new-old $S_{2n}$) [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni</td>
<td>28</td>
<td>42</td>
<td>11.892 ± 0.004</td>
<td>[2],[6]</td>
<td>11.829 ± 0.345</td>
<td>0.063 ± 0.345</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>44</td>
<td>11.155 ± 0.039</td>
<td>[6]</td>
<td>10.933 ± 0.557</td>
<td>0.222 ± 0.558</td>
</tr>
<tr>
<td>Sr</td>
<td>38</td>
<td>58</td>
<td>10.229 ± 0.012</td>
<td>[2],[5]</td>
<td>10.241 ± 0.028</td>
<td>-0.012 ± 0.030</td>
</tr>
<tr>
<td></td>
<td>38</td>
<td>60</td>
<td>9.648 ± 0.014</td>
<td>[5]</td>
<td>9.849 ± 0.038</td>
<td>-0.201 ± 0.040</td>
</tr>
<tr>
<td></td>
<td>38</td>
<td>62</td>
<td>9.540 ± 0.014</td>
<td>[5]</td>
<td>9.716 ± 0.130</td>
<td>-0.176 ± 0.130</td>
</tr>
<tr>
<td></td>
<td>38</td>
<td>64</td>
<td>9.395 ± 0.111</td>
<td>[2],[5]</td>
<td>9.001 ± 0.169</td>
<td>0.394 ± 0.202</td>
</tr>
<tr>
<td>Zr</td>
<td>40</td>
<td>46</td>
<td>22.683 ± 0.007</td>
<td>[4]</td>
<td>21.961 ± 0.032</td>
<td>-0.152 ± 0.032</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>48</td>
<td>21.809 ± 0.007</td>
<td>[4]</td>
<td>21.961 ± 0.032</td>
<td>-0.152 ± 0.032</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>58</td>
<td>11.991 ± 0.011</td>
<td>[2],[5]</td>
<td>11.987 ± 0.020</td>
<td>0.004 ± 0.022</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>60</td>
<td>11.128 ± 0.014</td>
<td>[5]</td>
<td>11.460 ± 0.041</td>
<td>-0.232 ± 0.043</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>62</td>
<td>11.357 ± 0.014</td>
<td>[5]</td>
<td>11.281 ± 0.061</td>
<td>0.076 ± 0.062</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>64</td>
<td>10.281 ± 0.014</td>
<td>[5]</td>
<td>10.281 ± 0.014</td>
<td>–</td>
</tr>
<tr>
<td>Mo</td>
<td>42</td>
<td>54</td>
<td>16.522 ± 0.006</td>
<td>[2],[4]</td>
<td>16.523 ± 0.001</td>
<td>-0.001 ± 0.006</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>56</td>
<td>15.465 ± 0.009</td>
<td>[4]</td>
<td>15.464 ± 0.001</td>
<td>0.001 ± 0.009</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>58</td>
<td>14.216 ± 0.009</td>
<td>[2],[4]</td>
<td>14.215 ± 0.006</td>
<td>0.001 ± 0.010</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>60</td>
<td>13.529 ± 0.012</td>
<td>[2],[5]</td>
<td>13.516 ± 0.020</td>
<td>0.013 ± 0.023</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>62</td>
<td>12.927 ± 0.014</td>
<td>[5]</td>
<td>12.914 ± 0.058</td>
<td>0.013 ± 0.059</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>64</td>
<td>11.928 ± 0.014</td>
<td>[5]</td>
<td>12.069 ± 0.057</td>
<td>-0.141 ± 0.058</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>66</td>
<td>10.764 ± 0.014</td>
<td>[5]</td>
<td>10.764 ± 0.014</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>68</td>
<td>9.930 ± 0.026</td>
<td>[5]</td>
<td>9.930 ± 0.026</td>
<td>–</td>
</tr>
<tr>
<td>Ru</td>
<td>44</td>
<td>62</td>
<td>14.365 ± 0.010</td>
<td>[2],[7]</td>
<td>14.376 ± 0.007</td>
<td>-0.011 ± 0.012</td>
</tr>
<tr>
<td></td>
<td>44</td>
<td>64</td>
<td>13.487 ± 0.014</td>
<td>[7]</td>
<td>13.493 ± 0.116</td>
<td>-0.006 ± 0.116</td>
</tr>
<tr>
<td></td>
<td>44</td>
<td>66</td>
<td>12.555 ± 0.014</td>
<td>[7]</td>
<td>12.452 ± 0.128</td>
<td>0.103 ± 0.128</td>
</tr>
<tr>
<td></td>
<td>44</td>
<td>68</td>
<td>11.700 ± 0.016</td>
<td>[7]</td>
<td>11.644 ± 0.091</td>
<td>0.056 ± 0.092</td>
</tr>
<tr>
<td></td>
<td>44</td>
<td>70</td>
<td>10.732 ± 0.014</td>
<td>[7]</td>
<td>10.732 ± 0.014</td>
<td>–</td>
</tr>
<tr>
<td>Pd</td>
<td>46</td>
<td>66</td>
<td>14.121 ± 0.014</td>
<td>[2],[7]</td>
<td>14.130 ± 0.017</td>
<td>-0.009 ± 0.022</td>
</tr>
<tr>
<td></td>
<td>46</td>
<td>68</td>
<td>13.313 ± 0.011</td>
<td>[7]</td>
<td>13.303 ± 0.029</td>
<td>0.010 ± 0.031</td>
</tr>
<tr>
<td></td>
<td>46</td>
<td>70</td>
<td>12.484 ± 0.011</td>
<td>[7]</td>
<td>12.607 ± 0.060</td>
<td>-0.123 ± 0.061</td>
</tr>
<tr>
<td></td>
<td>46</td>
<td>72</td>
<td>11.703 ± 0.011</td>
<td>[7]</td>
<td>11.648 ± 0.217</td>
<td>0.055 ± 0.217</td>
</tr>
<tr>
<td></td>
<td>46</td>
<td>74</td>
<td>11.062 ± 0.013</td>
<td>[7]</td>
<td>10.826 ± 0.244</td>
<td>0.236 ± 0.244</td>
</tr>
</tbody>
</table>
In some cases the differences are larger than the uncertainties and, as we will see below, it is very important to have very precise mass measurements in order to interpret correctly the evolution of $S_{2n}$. We used, where available, new $S_{2n}$ values from Table 1 instead of those from ref. [2].

We studied $S_{2n}$ as a function of the neutron number $N$ on each isotopic chain of even-even nuclei, ranging from Ni ($Z=28$) to Hg ($Z=70$). We have also studied the differential variation of the two-neutron separation energy $dS_{2n}(Z, N)$ defined as

$$dS_{2n}(Z, N) = \frac{S_{2n}(Z, N + 2) - S_{2n}(Z, N)}{2}$$

(2)
as a function of the neutron number $N$, in a correlated manner with $S_{2n}$.

3. BASIC SINGLE PARTICLE AND COLLECTIVE NUCLEAR PHENOMENOLOGY

Despite the complexity of the nuclear interaction some simple phenomenological facts emerged from the bulk of the properties of the low-lying states in the even-even atomic nuclei. These features can be arranged in two groups: (i) single-particle properties [8] and (ii) collective properties [9]. Although these properties are clearly seen in the nuclear excited states, the ground state and in particular $S_{2n}$ is sensitive also to this collective/single-particle inter-play [1]. Furthermore, some basic ingredients of the single-particle and collective features needed in our discussion will be mentioned.

Following various observations of the nuclear proprieties Maria-Goepert Mayer and Jenssen [10] made an analogy between nuclear excitations and atomic excitations where the forces are clearly of central type. Based on this analogy they introduced the “shell model of the atomic nucleus”, an enterprise which evolves from the simple “single particle” version to the complex multi-particle with residual interaction developments.

The basic idea of this model is that nuclear excitations are generated by a single-nucleon dynamics in a central potential. The influence of other nucleons can be “gathered” in a residual interaction.

Even in its simplest single-particle version, by considering a Woods-Saxon central potential well and a spin-orbit interaction this model generates by diagonalisation of a one-nucleon Hamiltonian, the energy levels illustrated in Figure 1 and explain the experimental magic numbers at the nucleon numbers 2, 8, 20, 28, 50, 82, 126. These are the major shell closures predicted by the spherical shell-model and we expect to observe a stronger stability in the nuclei which have the number of protons or neutrons equal with one of them. In section 4 we show that these major shell closures are strongly seen also in the $S_{2n}$ behavior.
At the other extreme of the nuclear dynamics there are collective excitations. At the low excitation energy these excitations involve coherent dynamics of all valence nucleons, i.e. of nucleons outside the highest closed shell. Therefore, these type of excitations occur in nuclei which are not doubly magic and are located lower in energy, when the nuclei have the nucleon numbers close to the middle of the major shells.

The theoretical collective shapes [11] form a triangle in which every nucleus finds its place as represented in Figure 2. The benchmarks shapes are: spherical harmonic vibrator, axially symmetric rotor and deformed γ-soft [11]. Each of these geometrical forms has its correspondent algebraic limit in the Interacting Boson Model (IBM): U(5), SU(3) and O(6), respectively. In the evolution of spherical to deformed shape, there are two critical point phase/shape transitions [13, 14]: E(5) – between spherical vibrator and deformed γ-soft and X(5) – between spherical vibrator and axially symmetric rotor. Although the majority of the nuclei find their place inside the triangle, there are some nuclei on the borders where the two shapes coexist (at the critical phase transition point). This can be seen in section 4, where we show that dS2n discloses such a behavior.
4. RESULTS AND INTERPRETATION

The evolution of $S_{2n}$, as a function of neutron number, shows the well-known regularities [2]: for any fixed number of protons, $S_{2n}$ decreases smoothly as the number of neutron increases. This decrease has sharp discontinuities at neutron spherical closures magic numbers, i.e. the energy necessary to remove 2 neutrons from a nucleus $(Z, N_{\text{magic}}+2)$ is much smaller than that to remove 2 neutrons from the nucleus $(Z, N_{\text{magic}})$, and much smaller than is expected from the regular smooth trend. The curves for various isotopic chains are roughly parallel to each-other. From these general characteristics of the $S_{2n}$ curves we expect that the derivative, $dS_{2n}$, is negative, practically constant as a function of $N$ for various $Z$ values, with sharp drops (very negative values) at neutron magic numbers. Deviation from this general trend will disclose additional nuclear structure features.

In order to present them clearly we will show $S_{2n}$ and $dS_{2n}$ separately for various ranges of $N$ and $Z$.

Figure 3 presents the variation of $S_{2n}$ and $dS_{2n}$ respectively, for the isotopic chains of Ni $(Z=28)$, Zn $(Z=30)$, Ge $(Z=32)$, Se $(Z=34)$ and Kr $(Z=36)$.

We notice large discontinuities in the monotonic evolution of $S_{2n}$ shown in Figure 3a, which are reinforced in Figure 3b by the evolution of the derivative. These discontinuities reflect the major shell closures at $N=28$ and $N=50$ predicted by the shell model. In addition, Figure 3b discloses some interesting structure features which are not seen (or it is very hard to be seen) in the $S_{2n}$ evolution. The
non linear behavior, especially in Ni isotopes at N=40 and 42, indicates a local change in nuclear structure, likely to be correlated with the filling of the g9/2 orbital.

Fig. 3. $S_{2n}$ and $dS_{2n}$ for the isotopic chains of Ni, Zn, Ge, Se and Kr.
Figure 4 presents the variation of \( S_{2n} \) and \( dS_{2n} \) for the isotopic chains of Sr (Z=38), Zr (Z=40) and Mo (Z=42). We notice again the major shell closure at \( N=50 \). In addition, \( dS_{2n} \) shows clearly a drop at \( N=56 \). This is the subshell closure predicted by the shell model at \( d{5/2} \), which, as the figure shows, is strong for the Sr and Zr isotopic chains and attenuates for Mo. Another non-linear behavior can be observed at \( N=60 \) where \( dS_{2n} \) curves, especially for Sr and Zr are up-picked. Based
on similarities with the behavior at N=90 (see figure 7), where is known to be present an X(5) critical phase/shape transition, we mark it in the same way.

Figure 5 presents the variation of $S_{2n}$ and $dS_{2n}$, for the isotopic chains of Ru (Z=44), Pd (Z=46), Cd (Z=48), Sn (Z=50) and Te (Z=52). As predicted by the shell model, in this neutron number range the major shell closure is for N=82 and this can be seen clearly in both parts of the figure. For Cd isotopes with N=72–78, Sn with N=52–60, and especially Te with N=56–64, $dS_{2n}$ varies from one point to another in the form of saw tooth,
showing that there are changes in nuclear structure due to the detailed filling of the single-particle orbitals in these nuclei close to the magic proton number 50. Beside this, $dS_{2n}$ varies very little with the neutron number. We notice that the region around $N=56$ ($^{102}$Pd) which is known [15, 16], from the excited states properties, to be an example of E(5) critical point phase/shape transition, does not show from the ground state masses embedded in $S_{2n}$ any non-monotonic behavior.

Fig. 6. $S_{2n}$ and $dS_{2n}$ for the isotopic chains of Xe, Ba, Ce, Nd, Sm, Gd, Dy.
Figure 6 presents the variation of $S_{2n}$ and $dS_{2n}$ for $N<86$ and the isotopic chains of Xe ($Z=54$), Ba ($Z=56$), Ce ($Z=58$), Nd ($Z=60$), Sm ($Z=62$), Gd ($Z=64$) and Dy ($Z=66$). The major shell gap for $N=82$ can be noticed in all chains. Except this major shell gap, $dS_{2n}$ is constant at approximate $-0.4$ MeV. The well known $E(5)$ phase transition point at $^{134}$Ba ($N=78$) [17] it is not noticeable once again in the $dS_{2n}$ plot.

![Graphs showing $S_{2n}$ and $dS_{2n}$ for isotopic chains of Ba, Ce, Nd, Sm, Gd, Dy, Er.](image)

Fig. 7. – $S_{2n}$ and $dS_{2n}$ for the isotopic chains of Ba, Ce, Nd, Sm, Gd, Dy, Er.
Figure 7 presents the variation of the $S_{2n}$ and $dS_{2n}$, for the isotopic chains of Ba ($Z=56$), Ce ($Z=58$), Nd ($Z=60$), Sm ($Z=62$), Gd ($Z=64$), Dy ($Z=66$) and Er ($Z=68$) with $N> 82$. In this case, the almost constant evolution of $dS_{2n}$ of approximate −0.4 MeV is interrupted by a special behavior of the derivative around $N=90$. These nuclei with $N=90$ are known [18–22] to be examples of the critical point X(5) phase/shape transition, so we can associate this type of behavior, i.e. a “bump” in $dS_{2n}$, with this type of nuclear phenomena (see section 3). In addition, $dS_{2n}$ curve for Dy isotopes presents a minimum at $N=98$, behavior similar to a subshell closures.

![Graph of $S_{2n}$ and $dS_{2n}$ for the isotopic chains of Ba, Ce, Nd, Sm, Gd, Dy, Er.]

Fig. 8. – $S_{2n}$ and $dS_{2n}$ for the isotopic chains of Yb, Hf, W, Os, Pt, Hg.
Figure 8 presents the variation of the $S_{2n}$ and $dS_{2n}$, for the isotopic chains of Yb ($Z=70$), Hf ($Z=72$), W ($Z=74$), Os ($Z=76$), Pt ($Z=78$) and Hg ($Z=80$). The evolution of $dS_{2n}$ is almost constant at approx. $0.3$ MeV for all neutron number with 2 notable exceptions: Yb and Hf at $N=104$ and Hf, W, Os at $N=108$ which have lower values, behavior similar with subshell closures.

Fig. 9. – The evolution of $dS_{2n}$ ($Z, N_{\text{magic}}$) with proton number $Z$, for $N_{\text{magic}}=50, 82$. 
Regarding major shell closures, we notice that the variation of $S_{2n}$ and $dS_{2n}$ is not the same for various magic numbers. In Figure 9 we illustrate $dS_{2n}$ at $N_{\text{magic}}=50$ and 82.

We notice that the variation of $S_{2n}$ at major shells as function of proton number is almost smooth, with maximum values at proton magic numbers, i.e. the absolute difference between $S_{2n}(N_{\text{magic}}, Z) - S_{2n}(N_{\text{magic}}+2, Z)$ is the largest for $Z=Z_{\text{magic}}$. This fact, somehow expected, reflects the maximum stability for the $(N, Z)$ system when both $N$ and $Z$ are magic numbers. This is valid also in the cases of the subshell closures at $Z=38, 40$ and $Z=64$.

5. $S_{2n}$ EVOLUTION ALONG THE ISOTONIC CHAINS AND ITS DIFFERENTIAL VARIATION

As was seen in the previous section the behavior of $S_{2n}$ discloses, besides the magic spherical shell closures at $N=50$ and $N=82$, two other structural features: subshells closures and $X(5)$ type behavior. Both features appear at specific neutron numbers but with different intensities for different proton numbers. In order to follow better the $Z$ dependence of these effects we draw the same $S_{2n}$ values as function of $Z$ for different isotonic chains and its differential variation calculated with the relation

$$dS_{2n}^Z(Z, N) \equiv \frac{S_{2n}(Z+2, N) - S_{2n}(Z, N)}{2} \quad (3)$$

Figure 10 presents the variation of the $S_{2n}$ and $dS_{2n}$ for the $N \approx 90$ region. The general trend of $S_{2n}$ as function of $Z$ shows a monotonic increase along almost parallel curves [1]. We expect roughly almost constant $dS_{2n}^Z$ values. Therefore, the observed deviations are fingerprints for local structure features. At $Z=64-68$ the isotonic chains behave as 2 branches: $N=86, 88$ branch as “subshell closure” and $N=90, 92$ branch as $X(5)$ phase/shape transition. At $Z=58$ the $N=92$ chain has a notable non-linear behavior.

Figure 11 presents the variation of the $S_{2n}$ and $dS_{2n}^Z$, for the $N \approx 60$ region. Unexpectedly, at $Z=50$, $dS_{2n}^Z$ for different isotopic chains, does not show a significant deviation from the almost constant behavior. This indicates that removing 2 neutrons is not sensitive to the filling of the major proton shell. It is not the case for $Z=40$ (which is similar to $Z=64$) where the proton subshell closure is depending of the neutron number. It is well known [11] that $Z=64$ is a subshell closure for $N<90$ and disappears for $N\geq 90$. Similarly $Z=40$ is a subshell closure for
N<60 and disappears for N≥60 [11]. If $dS^Z_{2n}$ behavior reflects Z=64 subshell closure, at Z=40 this picture is only partially fulfilled. The evolution for N=58 shows indeed a subshell closure at Z=40 and that for N=62 shows a phase/shape transition behavior; however the behaviour of N=54, 56 and 60 isotopic chains does not support this picture.

Fig. 10. – $S^Z_{2n}$ and $dS^Z_{2n}$ for N=86, N=88, N=90, N=92 isotonic chains.
Figure 11. – $S_{2n}^Z$ and $dS_{2n}^Z$ for $N=54$, $N=56$, $N=58$, $N=60$, $N=62$ isotonic chains.

Figure 12 presents the variation of the $S_{2n}^Z$ and $dS_{2n}^Z$, for the $N \approx 40$ region. The $dS_{2n}^Z$ values for the $N=38-44$ isotonic chains show a constant behavior including at the spherical major shell closure $Z=28$. The fact that $dS_{2n}^Z$ is not
sensitive to the major proton shell closure at $Z=28$ is similar to the behavior at $Z=50$, discussed above.

Fig. 12. – $S_{2n}$ and $dS_{2n}$ for $N=38, N=40, N=42, N=44$ isotonic chain.
6. CONCLUSIONS

It is shown that two neutron separation energies disclose rich nuclear structure information. They indicate very clearly the major shell closures at \( N = N_{\text{magic}} \) reflected by strong discontinuities of \( S_{2n} \) as a function of \( N \). The evolution of nuclear collectivity is reflected as a smooth variation of \( S_{2n} \) as a function of \( N \) or \( Z \). Besides these well known gross properties, the ‘eyed-glasses’ analysis of its derivative show that the information contained in \( S_{2n} \) is more subtle. First of all, it shows exactly where are the neutron subshell closures and its dependence of the proton number; if the major proton spherical shell closures do not influence the two neutron separation energies, the proton subshell closures due to their nature (proton-neutron interaction) are reflected in the behavior of \( S_{2n} \). Another effect which is due to the proton-neutron interaction, the critical point of the transition from spherical to deformed shapes, is reflected in the variation of \( S_{2n} \) but the effect is small and is visible only in the evolution of the derivative of \( S_{2n} \) as function of neutron or proton number. We note that the X(5) critical point is seen in the \( N = 90 \) and \( N = 60 \) regions but the E(5) critical point is not seen neither in \( S_{2n} \) nor in \( dS_{2n} \). Further investigations like the study of \( S_{2p} \) or the correlations of the mass properties with spectroscopic information are necessary.

The two neutron separation energies and their evolution with neutron and proton number constitute a very good starting point in testing various nuclear structure models. The analysis presented in this paper shows the need of very precise data on masses and extension of this type of information to nuclei very far from stability.

REFERENCES
