MONTE CARLO METHOD FOR RADIOLOGICAL X-RAY EXAMINATIONS

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The organ doses and the effective dose of patients exposed at an X-ray beam, having photon energies between 10–150 keV, are analyzed by using a new method named IradMed. Three major radiological procedures, mammography, radiography and Computer Tomography are considered. The Rayleigh scattering, the Compton scattering and the photoelectric effect were taken into account for patient dose calculations. The Compton scattering is modeled by using “classic”, Kahn, Wielopolski and EGS procedures. The best results for angular distribution was obtained when the EGS-based algorithm was used. It can be noted that patient doses determined by using Kahn or “classic” procedures are in good agreement with data from literature when similar computing methods are used. From our data, for patient dose calculations, the EGS procedure is recommended.

Key words: Patient doses, Monte Carlo, Radiological X-ray examinations.

1. INTRODUCTION

The patient dose has often been described by the entrance skin dose in the center of the X-ray beam, primarily because of the simplicity of the measurements. In some cases this definition can be sufficient, for instance in quality control measurements routine for establishing the radiological equipment stability when the same exposure conditions are used at every measurements set. If exposure conditions are changed, then the entrance surface dose is no longer sufficient for the evaluation or comparison of patient doses. In such cases, the patient dose must be related to a quantity which reflects more directly the radiation detriment induced by X-ray [1]. The radiation-induced data cannot be measured directly in patients and are difficult and time consuming to obtain information by experimental measurements using physical phantoms. The computation of the organ doses and the effective dose is most often made using the Monte Carlo calculation method. The accuracy of the calculations depends
on the anatomical model used to describe actual patients and on the
characterization of the radiation field applied in X-ray examination. In our
approach, the MIRD-5 mathematical phantom developed by Oak Ridge National
Laboratory is used for simulating the radiographic examinations and the
computation of the interest doses are done for several phantom types [2]. All
organ doses calculated by our method, IraMed, are given as function of the
patient entrance air kerma (free in air, without backscatter) at the point where the
central axis of the X-ray beam enters the patient. If the entrance dose was
measured directly on patient, in real conditions, then this value must be divided
by the backscatter factor (BSF) which has a typical value of 1.3 (the range for
BSF is 1.1–1.5) [1].

2. DESCRIPTION OF THE METHOD

2.1. MATHEMATICAL PHANTOM USED
FOR RADIOLOGICAL X-RAY EXAMINATIONS

The MIRD-5 phantom, considered in the present analysis as mathemathical
phantom for radiographic and CT examinations, takes into account three main
tissue types with different densities and compositions: lungs, skeleton and soft
tissue (having the density of about 1 g/cm³, such as the muscle tissue) [2]. The
description of the hermaphrodite phantom is presented for several standard
patient ages: new born, 1 year, 5 years, 10 years, 15 years and adult (over
30 years). Each phantom consists of three major sections: an elliptical cylinder
representing the trunk and arms, two truncated circular cones representing the
legs and feet and a circular cylinder on which sets an elliptical cylinder capped
by half an ellipsoid representing the neck and head. Attached to the legs section
there is a small region with a planar front surface to contain the testes. Attached
to the trunk are portions of two ellipsoids representing the female breasts. All
organ descriptions are relative to a reference coordinate system, set at the base of
the trunk. The z-axis is directed upward toward the head, the x-axis is directed to
the phantom’s left and the y-axis is directed toward the posterior side of the
phantom [2].

Based on the tissue composition of the phantom, it was calculated the mass
attenuation coefficients and the mass energy absorption coefficients, for each
X-ray photon energies, in order to construct an adequate database for further
linear interpolations used in Monte Carlo simulation’s routine. An additional
software, named XCOM, created by M. J. Berger and J. H. Hubbell was used [3].

Due to mammography specific geometry, the corresponding phantom is
considered to be an adjustable right cylinder with a standard 4.20 cm thickness
and a 7.00 cm radius having a soft tissue composition [4, 5].
2.2. X-RAY SPECTRUM AND THE EXAMINATION GEOMETRY

The X-ray spectra are estimated either by using a pre-build database based on the XCOMP5R outputs or by using a more accurate calculation based on the SRS78 database [6]. These data are in good agreement with Birch-Marshall theory [7, 8]. The total equivalent filtration of the X-ray tube, the anode angle and the console set voltage were used as input data. The Monte-Carlo simulation was performed for each X-ray spectrum energies. The analysis were made in steps of 0.5 keV for the preset voltages less than 40 kV and in steps of 1 keV for higher voltages.

In mammography case, the center of the coordinate system of the phantom is chosen on the symmetry axis at the top of the cylinder. The z-axis is directed downward in the initial photon direction (see Fig. 1).

The cylinder, used as mathematical phantom for mammographic examinations, is seen from the tube focus by a $\theta_{\text{max}}$ angle of

$$tg\theta_{\text{max}} = R / \text{FSD}$$

where $R$ is the cylinder radius and FSD is the focus-skin distance. The polar angle, $\theta$, is computed based on a random number generator at each initial photon history. The azimuth angle, $\phi$, is sampled from normal distribution, in $[0, 2\pi]$. 

![Fig. 1 – Geometry used in mammographic examinations.](image-url)
range. The directional cosines of the photon are evaluated considering the initial photon direction to be normal at the entrance surface ($\mu_x = \mu_y = 0, \mu_z = 1$) and then the direction is determined by the polar and azimuth angle [9, 10]:

$$u_x = \sin \theta \cos \phi, \quad u_y = \sin \theta \sin \phi, \quad u_z = \cos \theta$$  \hspace{1cm} (2)

In radiography case, the incident photon direction relative to phantom depends on the examination projection type: AP-anterior posterior, PA-posterior anterior, LLat-left lateral or RLat-right lateral. Using the mathematical description of phantom, it is computed the $z$-coordinate of the X-ray field center and the X-ray field dimensions at entrance surface. The next step is the computation of the $z$-coordinate and one of $x$ and $y$ coordinates of the incident photon at entrance surface, by using a random number generator. The remaining coordinate ($x$ or $y$) is assessed assuming that the photon hit the phantom, thus by solving the corresponding equations for the specific region.

It is considered two angles which define different kinds of projection types. The first angle is the projection angle having for RLat the value of $0^\circ$, for AP the value of $90^\circ$, for LLat the value of $180^\circ$ and for PA the value of $270^\circ$. This angle is given by the direction of the central axis of the beam relative to the horizontal axis which crosses the median plane of the phantom from the phantom’s right to the phantom’s left. The second angle is the skull-caudal one and it is defined by the central axis of the beam relative to the vertical axis of the phantom. The skull-caudal angle has a value of $90^\circ$ for all the four projection types taken into account. Let $\xi$ be the skull-caudal angle and let $\lambda$ be the projection angle. It can be shown that the values for directional cosines of the incident photons having directions normal at the entrance surface (if the photon direction is parallel with the beam central axis) are:

$$u_{x0} = \sin \xi \cos \lambda, \quad u_{y0} = \sin \xi \sin \lambda, \quad u_{z0} = \cos \xi$$  \hspace{1cm} (3)

It can be shown that the new directional cosines are [9, 10]:

$$u_x = \sin \theta \cos \phi, \quad u_y = \sin \theta \sin \phi, \quad u_z = \text{SIGN}(u_{z0}) \cos \theta \quad \text{if } u_{z0} > 0.99$$  \hspace{1cm} (4)

and in general case:

$$u_x = \frac{\sin \theta}{\sqrt{1 - u_{z0}^2}} [u_{x0} u_{z0} \cos \phi - u_{y0} \sin \phi] + u_{x0} \cos \theta$$

$$u_y = \frac{\sin \theta}{\sqrt{1 - u_{z0}^2}} [u_{x0} u_{z0} \cos \phi + u_{x0} \sin \phi] + u_{y0} \cos \theta$$  \hspace{1cm} (4')

$$u_z = -\sin \theta \cos \phi \sqrt{1 - u_{z0}^2} + u_{z0} \cos \theta$$
In CT case, it can be considered that each individual slice scan is composed of four radiographic RLat, AP, LLat and PA projections having the same weight in computation of the dose. The sum of entrance exposures for each projection type is regarded as the CT entrance dose. Usually, for this input data it is considered the CT specific physical quantity named CTDI [5, 11]. All mathematical considerations are therefore identical to those involved in the corresponding radiographic projection types.

2.3. INTERACTION SAMPLING

For radiodiagnostic energy range it is taken into account three photon interactions with matter: the photoelectric effect, the incoherent Compton scattering and the coherent Rayleigh scattering. The interaction probability for photoelectric effect, \( p_f \), is computed based on the mass attenuation coefficients:

\[
p_f = \frac{\mu_f}{\mu_t}
\]  

(5)

where \( \mu_t \) is the total mass-attenuation coefficient at a specific photon energy and \( \mu_f \) is the photoelectric mass-attenuation coefficient.

If \( r_1 \leq p_f \), where \( r_1 \) is a random number, then the photon is considered to be absorbed by photoelectric effect. Otherwise, it is computed the coherent Rayleigh scattering probability, \( p_{coh} \):

\[
p_{coh} = \frac{\mu_{coh}}{\mu_t - \mu_f}
\]  

(6)

where \( \mu_{coh} \) is the coherent (Rayleigh) mass-attenuation coefficient.

If \( r_2 \leq p_{coh} \), where \( r_2 \) is a random number, then the photon will suffer a coherent Rayleigh scattering, otherwise an incoherent (Compton) scattering will occur.

By photoelectric effect, all photon energy is considered to be locally absorbed and deposited in the organ, in other words, it is assessed that the absorbed dose is equal with the kerma in organs (the kerma approximation). The boundary effects would have a little effect in the determination of average dose to the larger organs. The one exception for the organs under study would be the active bone marrow, where a small increase in dose due to the size of the marrow cavities is expected to appear from increased photoelectron emission by surrounding bone. The evaluation of energy deposited in active bone marrow is separately treated and depends only by energy deposition in different skeleton regions [12].

By coherent Rayleigh scattering the photons suffer elastic processes without energy deposition. Differential Thomson cross section is defined by:
\[ d\sigma_r(\theta) / d\Omega = r_e^2 [1 + \cos^2 \theta] / 2 \]  

(7)

where \( \theta \) is the scattering angle and \( r_e \) is the classical electron radius.

Let to be \( u = 1 - \cos \theta \) having the maximum value of 2. The variable \( u \) is sampled by \( 2r \), where \( r \) is a random number in \([0, 1]\) interval. Then, the scattering angle \( \theta \) is given by:

\[ \theta = a \cos(1 - u) \]  

(8)

and the weighting variable \( w \) is computed by:

\[ w = [1 + \cos^2 \theta] \]  

(9)

If,

\[ w > 2r \]  

(10)

then, the value of \( \theta \) is accepted, otherwise, the procedure is repeated.

The condition (10) makes the scattering (polar) angle, \( \theta \), to follow the Rayleigh distribution where the scattering at small angles is most often to occur. The azimuth angle, \( \varphi \), follows the normal distribution and is defined in \([0, 2\pi]\) range.

By incoherent Compton scattering, the photons suffer an energy loss by interaction with atomic electrons or with free electrons, and deposit energy in tissue. The photon energy and the energy of the recoil are given by [4, 10, 13]:

\[ E = \frac{E_0}{1 + \left( \frac{E}{E_0} \right) \left( 1 - \cos \theta \right)} \]  

(11)

where \( m_0 c^2 \) is the rest electron mass in energy unit (511 keV) and \( \theta \) is the scattering angle,

\[ T = E_0 - E \]  

(12)

It can be shown that the polar angle must follows the Klein-Nishina distribution given by the following differential equation [4, 13]:

\[ \frac{d\sigma_{KN}(E, \theta)}{d\Omega} = \frac{r_e^2}{2} \frac{E}{E_0} \left[ 1 + \left( \frac{E}{E_0} \right)^2 - \frac{E}{E_0} \sin^2 \theta \right] \]  

(13)

One way to sample the polar angle from (13) is to consider the major \( \theta \) dependency (weight) as being given by:

\[ w = \left[ 1 + \left( \frac{E}{E_0} \right)^2 - \frac{E}{E_0} \sin^2 \theta \right] \]  

(14)

which has a maximum value of 2 [4].
Let \( u = 1 - \cos \theta \), having a maximum value of 2. From (11) it can be obtained:

\[
   u = \frac{m_0 c^2}{E_0} \left[ \left( 1 + 2 \frac{E_0}{m_0 c^2} \right)^r - 1 \right]
\]

(15)

where \( r \) is a new random number defined in \([0, 1]\) range.

The scattering (polar) angle, \( \theta \), is then computed applying the equation (8) and it is tested if \( w \) from (14) satisfies the condition (10) in the same way as it was discussed for Rayleigh scattering. These assessments make the polar angle to follow the Klein-Nishina distribution where the scattering at small angles is most often to occur [4].

The azimuth angle, \( \phi \), follows the normal distribution and has a value within \([0, 2\pi]\) range. Other similar methods which avoid the solving of Klein-Nishina equation are the Kahn method and, recently, the Wielopolski-Arinc method [10]. A more accurate algorithm for polar angle sampling is used in EGSnrc Monte Carlo routines [14, 15].

IradMed presents the possibility of selection for above mentioned algorithms: EGS, “classic”, Kahn and Wielopolski. A preliminary study for polar angle sampling using these algorithms has been made by choosing a 15 degrees step in the 0–180 degrees range. For each interval, 0–15, 15–30, …, 165–180 degrees, the angle probability of occurrence (in \%) was calculated for several incident photon energies: 0.1 MeV, 0.7 MeV, 1.5 MeV and 2.6 MeV. The best probability distribution over the entire interval range was obtained for EGS

Fig. 2 – The Compton scattering (polar) angle distribution in 15 degrees steps covering 0–180 degrees range by using different algorithms.
algorithm at each photon energy [10]. This is followed by the Kahn and the “classic” ones and the worst angle distribution was obtained using the Wielopolski algorithm [10]. For instance, the 0.7 MeV angle distribution is presented in Fig. 2.

2.4. PATHLENGTH SAMPLING

The beam attenuation in a medium of thickness $x$ is given by:

$$I = I_0 \exp(-\mu x)$$

where $I$ is the beam intensity and $\mu$ is the linear total attenuation coefficient of the medium for incident radiation energy. It can be shown that the equation (16) is the statistical law for the photon probability to traverse a distance $x$ in medium without interaction. Let $r_x$ be this probability ($r_x$ can be generated by random numbers in $[0, 1]$ range). It can be shown that:

$$x = -\ln(r_x) / \mu$$

At each interaction site, IradMed computes in what organ this interaction takes place relative to the main coordinate system. Also, the corresponding attenuation coefficient and the mass-absorption coefficients are computed. The updated coordinates as function of the old ones are calculated taking into account the directional cosines [9]:

$$\mu_x = s \hat{i}, \quad \mu_y = s \hat{j}, \quad \mu_z = s \hat{k}$$

where $\hat{i}, \hat{j}, \hat{k}$ refer to unit vectors for $x, y$ and $z$ axis and $s$ is the photon vector relative to actual coordinate system. If $d$ is the distance to the new interaction site (computed by relation 17) then the updated coordinates as function of the old ones are given by:

$$x = x_0 + \mu_x d, \quad y = y_0 + \mu_y d, \quad z = z_0 + \mu_z d$$

3. COMPARISONS WITH OTHER DATA AND CONCLUSIONS

For mammographic examinations, the average glandular dose (AGD) according to IAEA standard was computed [5]. As input data it was considered: the standard breast radius of 7 cm, the standard breast thickness of 5 cm, focus–skin distance of 55 cm and the specific tube parameters such as the voltage of 28 kV, total filtration 0.5 mm Al, pre-computed HVL of 0.32 mm Al, anode angle of 17 degrees and free-in air dose without backscatter at entrance surface of 7.52 mGy. The number of histories for each photon energies in the spectrum
was 2000 which is enough for a good statistic (<1% error). The dose comparison for this specific mammography is presented in Table 1.

### Table 1

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Breast</td>
<td>1.323</td>
<td>1.467</td>
<td>1.399</td>
<td>1.40</td>
</tr>
</tbody>
</table>

For radiographic comparison, it was chosen the lungs X-ray examination, PA projection code, with the following input data: voltage of 125 kV, total filtration of 2.5 mm Al, anode angle of 17 degrees, focus-skin distance of 160 cm and it was chosen a standardized value for free in air entrance dose of 1 Gy. The comparison with literature data [1] is presented in Table 2.

### Table 2

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Lungs</td>
<td>448</td>
<td>691</td>
<td>638</td>
<td>716</td>
<td>719</td>
<td>680</td>
<td>–</td>
<td>1046</td>
</tr>
<tr>
<td>Marrow</td>
<td>84</td>
<td>138</td>
<td>119</td>
<td>242</td>
<td>235</td>
<td>150</td>
<td>207</td>
<td>361</td>
</tr>
<tr>
<td>Uterus</td>
<td>2.6</td>
<td>2.5</td>
<td>2.8</td>
<td>4.3</td>
<td>4.6</td>
<td>&lt; 10</td>
<td>6.8</td>
<td>1.6</td>
</tr>
<tr>
<td>Thyroid</td>
<td>16</td>
<td>84</td>
<td>61</td>
<td>97</td>
<td>92</td>
<td>90</td>
<td>93</td>
<td>228</td>
</tr>
</tbody>
</table>

For effective dose comparison, the same procedure was considered with few exceptions: voltage of 70 kV and a free in air entrance dose without backscatter of 5.00 mGy. The effective dose comparison for this specific radiography is presented in Table 3.

### Table 3

<table>
<thead>
<tr>
<th>Organ</th>
<th>IradMed EGS</th>
<th>IradMed “classic”</th>
<th>IradMed Kahn</th>
<th>PCXMC [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole body</td>
<td>0.71</td>
<td>0.91</td>
<td>0.85</td>
<td>0.98</td>
</tr>
</tbody>
</table>

For CT comparison it was used a specific PC program named CTDose, developed by Niels Baadegaard from National Institute of Radiation Hygiene, Denmark [16]. This software does not perform a real-time Monte Carlo simulation but it uses pre-computed Monte Carlo coefficients for different kind of CT scanners. It was chosen the thoracic CT routine having the following presets: voltage of 100 kV, total filtration of 2.5 mm Al, anode angle of 17 degrees,
focus-symmetry axis of 70 cm, normalized CTDI of 3.0 mGy/mAs, tube load of 10 mAs, slice thickness of 10 mm, start z coordinate of 76.6 cm and stop z coordinate of 36.6 cm. The scanner type for CTDose estimation was CTPicker PQ2000 with default presets for total filtration and focus-symmetric axis distance. The organ doses and the effective doses for this specific CT examination is presented in Table 4.

Table 4

<table>
<thead>
<tr>
<th>Organ</th>
<th>IradMed</th>
<th>IradMed</th>
<th>IradMed</th>
<th>CTDose</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EGS</td>
<td>“classic”</td>
<td>Kahn</td>
<td>[16]</td>
</tr>
<tr>
<td>Breast</td>
<td>11.18</td>
<td>7.13</td>
<td>9.06</td>
<td>6.2</td>
</tr>
<tr>
<td>Active bone marrow</td>
<td>1.19</td>
<td>1.94</td>
<td>1.68</td>
<td>3</td>
</tr>
<tr>
<td>Adrenals</td>
<td>1.78</td>
<td>2.76</td>
<td>2.55</td>
<td>6.6</td>
</tr>
<tr>
<td>Brain</td>
<td>0.44</td>
<td>0.69</td>
<td>0.58</td>
<td>0.3</td>
</tr>
<tr>
<td>Stomach</td>
<td>1.00</td>
<td>1.69</td>
<td>1.49</td>
<td>3.4</td>
</tr>
<tr>
<td>Heart</td>
<td>2.28</td>
<td>3.61</td>
<td>3.28</td>
<td>9.5</td>
</tr>
<tr>
<td>Small intestine</td>
<td>0.12</td>
<td>0.33</td>
<td>0.25</td>
<td>0.19</td>
</tr>
<tr>
<td>Kidneys</td>
<td>0.57</td>
<td>1.04</td>
<td>0.83</td>
<td>1.7</td>
</tr>
<tr>
<td>Liver</td>
<td>1.36</td>
<td>2.16</td>
<td>1.98</td>
<td>4.5</td>
</tr>
<tr>
<td>Lungs</td>
<td>5.98</td>
<td>9.70</td>
<td>8.90</td>
<td>7.6</td>
</tr>
<tr>
<td>Ovaries</td>
<td>0.01</td>
<td>0.05</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>Pancreas</td>
<td>1.03</td>
<td>1.87</td>
<td>1.67</td>
<td>5.4</td>
</tr>
<tr>
<td>Spleen</td>
<td>1.33</td>
<td>2.16</td>
<td>1.93</td>
<td>3.88</td>
</tr>
<tr>
<td>Thymus</td>
<td>3.77</td>
<td>5.42</td>
<td>4.55</td>
<td>10</td>
</tr>
<tr>
<td>Thyroid</td>
<td>5.68</td>
<td>7.18</td>
<td>6.67</td>
<td>13</td>
</tr>
<tr>
<td>Urinary bladder</td>
<td>0.007</td>
<td>0.027</td>
<td>0.018</td>
<td>0.063</td>
</tr>
<tr>
<td>Utherus</td>
<td>0.019</td>
<td>0.061</td>
<td>0.042</td>
<td>0.012</td>
</tr>
<tr>
<td>Remainder</td>
<td>2.46</td>
<td>2.68</td>
<td>2.58</td>
<td>2.4</td>
</tr>
<tr>
<td>Effective dose</td>
<td>2.80</td>
<td>3.82</td>
<td>3.55</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Applying the “classic” or the Kahn algorithm, the value of doses generated by IradMed are, in general, in good agreement with other data and the differences could be explained by using different kind of phantom types. IradMed uses the MIRD-5 phantom (a modified CHIRSTY phantom), PCXMC uses the original CHIRSTY phantom and CTDose uses the ADAM phantom. In addition, the differences for active bone marrow dose (see Table 2) can be explained by using the Rosenstein method which is based on a less accurate database for the involved coefficients [12].

By using the EGS algorithm, which is known to give the best results for Compton sampling, smaller doses are obtained compared with those when other algorithm is followed. A good description was also obtained by using Kahn or
Monte Carlo method for radiological X-ray examinations

“classic” algorithm to sample the incoherent Compton scattering. The values of patient doses calculated with IradMed method are more reliable when the EGS algorithm is selected.

REFERENCES