SATURATION OF OPTIMAL RESONANCE LIMITS IN PION-NUCLEUS SCATTERING

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In this paper the experimental evidence on the saturation of PMD-SQS optimal resonance limits in pion-nucleus scattering in the \( \Lambda(1236) \)-resonance region, are presented. The scaling and saturation of the nonextensive entropic optimal resonance limits, for the nonextensive angular momentum entropy, are experimentally evidenced with high accuracy.


In this paper we continue to report the results of our investigations [1, 2] for to get from the available data the experimental evidences for the optimal resonances especially in pion-nucleus scattering in the \( \Lambda(1236) \)-region. So, in the papers [1, 2] by using the Principle of Minimum Distance in Space of Quantum States (PMD-SQS) [3], we obtained a new description of the pion-nucleus scattering in the \( \Lambda(3,3) \)-resonance region, in terms of optimal resonances previously suggested [3–6]. Hence, using the principle of minimum distance in space of quantum states (see Refs. [2]) with the uni-directional constraints

$$\frac{d\sigma}{d\Omega}(E, 1) = \text{fixed} \quad \text{we obtained:} \quad f_N(E, x) = f(E, 1) \frac{K_{L_n}(x, 1)}{K_{L_n}(1, 1)}, \quad 2K(1, l) =$$

$$= \sum_{l=0}^{L_n} (2l + 1)P_l(x)P_l(1) = \hat{P}_{L_n+1}(x) + \hat{P}_{L_n}(x),$$

where the optimal angular momentum is given by:

$$2K(1, 1) = \sum_{l=0}^{L_n} (2l + 1) = (L_n + 1)^2 = \frac{4\pi}{\sigma_{el}} \frac{d\sigma}{d\Omega}(E, 1).$$

(see Ref. [2] for details)

Then, all the essential characteristic features of the pion-nucleus in the optimal resonance limit are derived. All the optimal resonance predictions are found in a good agreement with the available experimental data. Hence, we proved that the “dual diffractive resonances” discovered by us in 1981 and published in the paper [7] are actually genuine optimal resonances. Some of the

Fig. 1 – The saturation of the axiomatic optimal bound $\sigma_{\text{opt}}^2(E) \leq 4\pi \lambda^2 (L_{\text{opt}} + 1)^2 \sigma_{\text{opt}}(E)$ is experimentally evidentiated with high accuracy (see Ref. [2] for details).

Fig. 2 – The saturation of the axiomatic optimal resonance (OR) limits: $\Gamma_{\Delta} \leq \Gamma_{\text{opt}} \leq \Gamma_{\Delta} A^{1/3}$ is experimentally evidentiated with high accuracy. The optimal resonance width are obtained by the best fit to the data with the optimal resonance predictions given in Eq. (6) of Ref. [2].
important results which illustrate this conclusion are presented in Figs. 1–3 [see details in our paper [2].

(i) The pion-nucleus total cross sections, in the energy region corresponding to \( \Delta(1236) \) resonance in the elementary pion-nucleon interaction, are well described by optimal resonance predictions and obey the axiomatic bound 
\[
\sigma_J^2(E) \leq 4 \pi A^2 (L_+ + 1)^2 \sigma_{el}(E)
\]
and (see Fig. 1);

(ii) The total widths of optimal resonances, obtained by fit of the total cross sections in the \( \Delta(1236) \) energy region, are consistent with the optimal resonances predictions \( \Gamma^\Lambda = \Gamma_\Lambda A^{1/3} \) for \( \Gamma_\Lambda = (120 \pm 5) \) MeV (see Fig. 2);

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**Fig. 3** – The saturation of the extensive entropic optimal resonance limits: 
\[
S_J(l) \leq \ln \left( \frac{4\pi}{\sigma_{el}} \frac{d\sigma}{d\Omega}(E, l) \right)
\]
are experimentally evidenced with high accuracy. The angular momentum entropy 
\[
S_J(l) = \sum_{l=0}^{L_{\text{max}}} (2l+1) \rho_l \ln \rho_l
\]
are calculated by using the available pion-nucleus phase shifts analysis [8]. The shadow region around the optimal extensive entropy \( S_J^l(1) \) (solid curve) is calculated assuming an experimental error of order \( \Delta \rho_l = 1 \) for the optimal angular momentum [For the angular momentum nonextensive entropies 
\[
S_J(q) = \left[ 1 - \sum_{l=0}^{L_{\text{max}}} (2l+1) \rho_l^q \right] / (q - 1)
\]
in the pion-nucleus scattering for non-extensivities \( q = 0.75 \) and 1.50 see Ref. [83]].

\[
S_J(1) = \sum_{l=0}^{L_{\text{max}}} (2l+1) \rho_l \ln \rho_l
\]
(iii) The available experimental data on $\sigma_{el}$, $\sigma_{in}$, are also in good agreement with the predictions of the PMD-SQS-resonance mechanism (see Fig. 1) but more experimental data are required; 
(iv) The saturations of the axiomatic entropic bounds in the optimal resonance energy region $E = E_{opt}$ are verified experimentally with high accuracy (see Fig. 3).
(v) The scaling of the angular momentum entropies in the delta resonance region is also presented in Fig. 3 as function on the scaling variable:
\[ n^2 = (L_o + 1)^2 = \frac{4\pi}{\sigma_{el}} \frac{d\sigma}{d\Omega}(E, 1). \]

We note, that an important PMD-SQS prediction [4, 1] is a general scaling function of the angular distributions:
\[ f(E, x) = f(\tau) = \frac{d\sigma}{dt}(E, x) / \frac{d\sigma}{dt}(E, 0) \]
where the scaling variable is given by:
\[ \tau = (2L_o + 1) \sin \theta / 2. \]

Finally, we note that, a detailed quantitative analysis of the experimental data on the angular distributions is also necessary since the general diffractive behaviour is also experimentally verified with high accuracy especially for the number of maxima and minima as function of optimal angular momenta.

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