ANISOTROPIC COSMOLOGICAL MODEL WITH NEGATIVE CONSTANT DECELERATION PARAMETER AND TIME-DECAYING Λ TERM

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Einstein's field equations are considered for a locally rotationally symmetric Bianchi type II space-time with perfect fluid and a time- decaying cosmological term $\Lambda(t)$. Exact solutions of the field equations for stiff matter are obtained by applying a special law of variation for Hubble's parameter. An anisotropic cosmological model is presented with a negative constant deceleration parameter which corresponds to the accelerated phase of the present universe. The physical and kinematical behaviors of Bianchi type II cosmological models are discussed. We have also studied the consistency of the model with observational parameters through kinematic tests. It has been shown that the model is compatible with the recent observations.

Key words: Cosmology, Bianchi type II, negative constant deceleration parameter, time-decaying cosmological term.

1. INTRODUCTION

General relativistic cosmological models provide a framework for investigation of the evolution of the universe. Present cosmology is based on the Friedmann-Robertson-Walker (FWR) model. In this model, the universe is completely homogeneous and isotropic which is in agreement with the observational data about the large scale structure of the universe. However, there is no reason to believe in a regular expansion for a description of the early stages of the universe. There are theoretical arguments [1,2] and recent experimental data of the cosmic microwave background radiation which support the existence of an anisotropic phase that approaches an isotropic one [3]. This stimulates search for exact anisotropic solution of Einstein's field equations as cosmologically acceptable physical models for universe at least in its early stage of evolution.

There has been considerable interest in the study of spatially homogeneous and anisotropic cosmological models of Bianchi type I-IX [4]. Bianchi type II space-time plays a fundamental role in constructing models with richer structure both geometrically and physically for describing the early stages of evolution of the uni-

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verse. Asseo and Sol [5] emphasized the cosmological importance of Bianchi type II models. Lorentz [6] has presented exact solutions for rotationally symmetric (LRS) Bianchi type II space-time with stiff-matter and an electromagnetic field. Assuming a perfect fluid distribution of matter and using a generating technique, Hajj-Boutros [7] generated exact solutions to Einstein's field equations for LRS Bianchi type II space-time. Hajj-Boutros [8] has also constructed LRS Binchi type II perfect fluid cosmological model with an equation of state that is a function of cosmic time. Shanthi and Rao [9] studied Bianchi type II model in Barber's self-creation theory of gravitation. Venkateswarlu and Reddy [10] have obtained cosmological solutions Bianchi type II stiff fluid models with electromagnetic field. Coley and Wainright [11] have studied models of this type in two-fluid cosmologies. Singh and Kumar [12] have obtained Bianchi type II inflationary model with constant deceleration parameter in general relativity.

It is believed that the early universe evolved through some phase transitions, thereby yielding a vacuum energy density which at present is at least 118 orders of magnitudes smaller than in the Planck time [13]. Such a discrepancy between theoretical expectations and empirical observations constitutes a fundamental problem in the interface uniting astrophysics, particle physics and cosmology is the cosmological constant problem. The recent observational evidence for an accelerated state of the present universe, obtained from distant SNe Ia (Perlmutter *et al.* [14]; Riess *et al.* [15] gave strong support to search for alternative cosmologies. Thus, the state of affairs has stimulated the interest in more general models containing an extra component describing dark energy, and simultaneously accounting for the present accelerated stage of the universe. Some of the recent discussions on the cosmological constant problem and consequences on cosmology with a time-varying cosmological constant Λ have been investigated by Ratra and Peebles [16], Dolgov [17,18,19], Sahni and Starobinsky [20], *etc.*. A variable Λ term or a decaying vacuum energy density is also an ingredient accounting for the accelerated phase of the present universe. Linde [21] has suggested that Λ is a function of temperature and related to the spontaneous symmetry breaking process. Therefore, it could be a function of time in a spatially homogeneous expanding universe. Several ansatz have been proposed and well studied so far in which the Λ term decays with time (see Pradhan *et al.* [22] and references cited therein). One of the ansatz of special interest *viz.* $\Lambda \propto \mathrm{a}^{-2}$, where a is the scale factor of FRW metric, has been discussed by Chen and Wu [23] which was further modified by Abdel-Rahman [24,25], Carvalho *et.al.* [26], Waga [27], Silveira and Waga [28], Vishwakarma [29], *etc.*. Overdin and Cooperstock [30] proposed a model with a cosmological term of the form $\Lambda = \beta(\frac{\ddot{a}}{a})$ where a is the scale factor of the universe and β is a positive constant. Arbab and Cosmo [31,32] have investigated cosmic acceleration with positive Λ and also analyzed the implication of the model based on this ansatz of Λ. This law provides reasonable solution to the cosmological puzzles presently known. One of the motivations for introducing Λ term is to reconcile the age parameter and density parameter of the universe with recent observational data. Cunha *et al.* [33] have discussed the classical cosmological tests for a large class of FRW type models driven by a decaying vacuum energy density. Recently, Pradhan *et al.* [22] have shown that spatially homogeneous Bianchi type II cosmological model with stiff matter and time decaying Λ term is compatible with the recent observations on accelerated universe.

In this paper we consider Einstein's field equations for anisotropic LRS Binchi type II space in the presence of stiff matter with a time-decaying cosmological term of the form $\Lambda = \beta(\frac{\ddot{a}}{a})$. An exact solutions is obtained by using a special law of variation for Hubble's parameter proposed by Berman [34]. The solution represents an anisotropic Bianchi type II stiff fluid cosmological model with negative constant deceleration parameter which corresponds to the accelerated state of the present universe. Analytic expressions for the proper distance, luminosity distance, and look-back time in the framework of Bianchi type II cosmological models are derived and their meaning are discussed in detail.

2. THE METRIC AND FIELD EQUATIONS

We consider the locally rotationally symmetric (LRC) metric for anisotropic Bianchi type II in the form

$$ds^2 = dt^2 - (Sdx + SZdy)^2 - (Rdy)^2 - (Rdz)^2 \tag{1}$$

where R and S are functions of cosmic time t.

For the metric (1), the spatial average factor a is given by

$$a = \left(R^2 S\right)^{1/3}. \tag{2}$$

The volume scale factor V will also be useful, and is given by

$$V = a^3. \tag{3}$$

We also define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) \tag{4}$$

where $H_1 = H_2 = \dot{R}/R$, $H_3 = \dot{S}/S$ are the directional Hubble's factors in the directions of x, y and z respectively. Here an overdot denotes derivative with respect to t. From Eqs. (2)–(4), the average Hubble's parameter may be generalized in anisotropic cosmological model as

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{2\dot{R}}{R} + \frac{\dot{S}}{S}\right). \tag{5}$$

For a perfect fluid, the energy-momentum tensor is

$$T_{ij} = (\rho + p)\, u_i u_j - p g_{ij} \tag{6}$$

where ρ and p are, respectively, the energy and pressure of the cosmic fluid, and u_i is the fluid four-velocity vector such that $u^i u_i = 1$.

For energy-momentum tensor in (6) and the LRS Bianchi type II space-time (1), Einstein's field equations

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -8\pi T_{ij} \tag{7}$$

in comoving coordinates lead to the following set of three independent and non-linear differential equations

$$\frac{2\dot{R}}{R}\frac{\dot{S}}{S} + \frac{\dot{R}^2}{R^2} - \frac{S^2}{4R^4} = 8\pi\rho + \Lambda \tag{8}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{3S^2}{4R^4} = -8\pi p + \Lambda \tag{9}$$

$$\frac{\ddot{S}}{S} + \frac{\ddot{R}}{R} + \frac{\dot{R}}{R}\frac{\dot{S}}{S} + \frac{S^2}{4R^4} = -8\pi p + \Lambda \tag{10}$$

where $\Lambda(t)$ is the cosmological term. For complete determinacy of the system of equations (8)-(10), we consider the barotropic equation of state

$$p = \gamma\rho, \qquad 0 \leq \gamma \leq 1. \tag{11}$$

The continuity equation $T^{j}_{i;j} = 0$ leads to the equation

$$\dot{\rho} + (\rho + p)\left(\frac{2\dot{R}}{R} + \frac{\dot{S}}{S}\right) = -\frac{\dot{\Lambda}}{8\pi}. \tag{12}$$

Einstein's field equations (8)-(10) are a coupled system of highly non-linear differential equations and we seek physically realistic solutions to the field equations for applications in cosmology and astrophysics. In order to obtain exact solution, we normally assume a form for the matter content or relation between the metric functions. The solutions to the field equations may also be generated by applying the law of variation for Hubble's parameter, initially proposed by Berman [34] for FRW models, which yields a constant value of the deceleration parameter. Berman and Gomide [35], Johri and Desikan [36], Singh and Desikan [37], Pradhan and Vishwakarma [38], Reddy *et al.* [39], Adhav *et al.* [40], Singh and Baghel [41] and others have studied cosmological models with constant deceleration parameter. With the help of this special law of variation for Hubble's parameter, we obtain exact solutions of the field equations which represent an anisotropic cosmological model with negative constant deceleration parameter.

3. SOLUTION OF FIELD EQUATIONS

We consider the constant deceleration parameter q defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = constant \tag{13}$$

where a is the average scale factor defined in (2). For an accelerating model of the universe, we take the constant as negative.

The solution of (13) is given by

$$a = (c_1 t + c_2)^{\frac{1}{1+q}} \tag{14}$$

where c_1 and c_2 are integration constants. Eq. (14) implies that the condition of expansion is $1 + q > 0$.

In order to obtain physically viable solutions of Eqs. (8)–(10), we consider the case when the space-time is filled with stiff-matter $(\gamma = 1)$. In the case of stiff-matter, Eqs. (8) and (10) give

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{3\dot{R}}{R}\frac{\dot{S}}{S} + \frac{\dot{R}^2}{R^2} = 2\Lambda. \tag{15}$$

Subtracting Eq.(10) from Eq. (9), we obtain the condition for isotropy of pressure as

$$\frac{\ddot{R}}{R} - \frac{\ddot{S}}{S} + \frac{\dot{R}^2}{R^2} - \frac{\dot{R}}{R}\frac{\dot{S}}{S} = \frac{S^2}{R^4}. \tag{16}$$

Eq.(16), on integration, yields

$$R^2\dot{S} + R\dot{R}S = \int 2\Lambda(R^2 S)dt + h \tag{17}$$

where h is an integration constant.

We now use the phenomenological decay law for $\Lambda(t)$ of the from

$$\Lambda = \beta\left(\frac{\ddot{a}}{a}\right) \tag{18}$$

in (17). By the use of (2) and (14) in (17) and integrating, we obtain

$$R^2\dot{S} + R\dot{R}S = \frac{-2\beta q c_1}{(1+q)(2-q)}(c_1 t + c_2)^{\frac{2-q}{1+q}} + h \tag{19}$$

provided $q \neq -1$. Dividing (19) by $R^2 S$ and integrating, we obtain

$$\frac{\dot{R}}{R} + \frac{\dot{S}}{S} = \frac{-2\beta q c_1}{(1+q)(2-q)}(c_1 t + c_2)^{-1} + h\,(c_1 t + c_2)^{\frac{-3}{1+q}}. \tag{20}$$

Eq. (20), on integration, gives

$$RS = m\,(c_1 t + c_2)^{\frac{-2\beta q}{(1+q)(2-q)}} \exp\left[\frac{h(1+q)}{c_1(q-2)}(c_1 t + c_2)^{\frac{2-q}{1+q}}\right] \tag{21}$$

where m is a constant of integration. From (2) and (14), we also have

$$R^2 S = (c_1 t + c_2)^{\frac{3}{1+q}} . \tag{22}$$

From (21) and (22), we obtain solutions for the scale factors $R(t)$ and $S(t)$ as given by

$$R(t) = \frac{1}{m} (c_1 t + c_2)^{\frac{6-3q+2\beta q}{(1+q)(2-q)}} \exp\left[\frac{h(1+q)}{c_1(2-q)} (c_1 t + c_2)^{\frac{q-2}{1+q}}\right] \tag{23}$$

$$S(t) = m^2 (c_1 t + c_2)^{\frac{3q-4\beta q-6}{(1+q)(2-q)}} \exp\left[\frac{-2h(1+q)}{c_1(2-q)} (c_1 t + c_2)^{\frac{q-2}{1+q}}\right]. \tag{24}$$

By the use of (16), (23) and (24) in (8), we obtain the expression for energy density as given by

$$8\pi\rho = \frac{A}{(c_1 t + c_2)^2} + \frac{B}{(c_1 t + c_2)^{\frac{4+q}{1+q}}} - \frac{3h^2}{(c_1 t + c_2)^{\frac{6}{1+q}}} \tag{25}$$

where A and B are constants expressible in terms of β and q which are not needed in further discussion.

The cosmological term $\Lambda(t)$ is given by

$$\Lambda(t) = \frac{-\beta q c_1^2}{(1+q)^2} (c_1 t + c_2)^{-2} \tag{26}$$

since $q < 0$, $\Lambda(t)$ is always positive. The directional Hubble's factors H_1, H_2, and H_3 are given by

$$\begin{aligned} H_1 = H_2 &= \frac{(6-3q+2\beta q)c_1}{(1+q)(2-q)(c_1 t + c_2)} - \frac{h}{(c_1 t + c_2)^{\frac{3}{1+q}}}, \\ H_3 &= \frac{(3q-4\beta q-6)c_1}{(1+q)(2-q)(c_1 t + c_2)} + \frac{2h}{(c_1 t + c_2)^{\frac{3}{1+q}}}. \end{aligned} \tag{27}$$

The average Hubble's parameter H is given by

$$H = \frac{c_1}{(1+q)(c_1 t + c_2)}. \tag{28}$$

The expansion scalar θ has the value

$$\theta = \frac{2\dot{R}}{R} + \frac{\dot{S}}{S} = \frac{3c_1}{(1+q)(c_1 t + c_2)}. \tag{29}$$

The shear scalar σ is given by

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{R}}{R} - \frac{\dot{S}}{S}\right) = \frac{(4+2\beta q-2q)c_1}{(1+q)(2-q)} (c_1 t + c_2)^{-1} - h (c_1 t + c_2)^{\frac{-3}{1+q}} . \tag{30}$$

4. SOME PHYSICAL PROPERTIES

The metric (1) with $R(t)$ and $S(t)$, given by (23) and (24), represents an exact stiff fluid LRS Bianchi type II cosmological model with negative constant deceleration parameter and time-decaying positive cosmological term $\Lambda(t)$. We observe that this is an accelerating model of the universe.

The spatial volume is zero at $t = t_0$ where $t_0 = -\frac{c_2}{c_1}$. At this epoch the energy density is infinite. One of the scale factors $R(t)$ vanishes while the other one $S(t)$ diverges at $t = t_0$. Therefore the model has a cigar-type singularity. The scalars of expansion and shear are infinite at $t = t_0$. These show that the universe starts evolving with zero-volume at t_0 and expands with time t. The model is well behaved for $t < \infty$. The expansion scalar θ tends to zero as $t \to \infty$ which shows that the universe is expanding with increase of the time and the rate of expansion decreases with increase of time. The shear scalar σ is non-zero for $t > t_0$ and tends to zero as $t \to \infty$. We also find that $\frac{\sigma^2}{\theta}$ does not tend to zero as $t \to \infty$, which indicates that the model does not approach isotropy for large time. As $t \to \infty$, the energy density also tends to zero. Thus the present model gives essentially an empty space for large time.

The solutions for the scale factors have combination of power-law term and the exponential term in the product form. Initially the exponential term is more significant and it is possible to have inflationary scenario during the evolution of the universe. The interesting feature of the solution is that it is possible to exist from exponential inflationary scenario if $\beta = 0$ and $h = 0$. After some inflation time the power-law expansion begins to dominate the dynamics, the universe will continue to expand with power-law expansion.

5. KINEMATIC TESTS

The expression for $a(t)$ derived in Eq.(14) may be useful to extend the kinematic tests for any arbitrary redshifts. We now study the consistency of our model with the observational parameters through kinematic tests.

5.1. PROPER DISTANCE d(z)

Let a source be at $r = r_s$, $t = t_s$ and an observer be at $r = 0$, $t = t_0$. Let a photon be emitted from the source and received by observer. The proper distance between the source and observer is given by

$$d(z) = a_0 \int_a^{a_0} \frac{da}{a\dot{a}} \tag{31}$$

where a_0= present scale factor. Also

$$r_s = \int_{t_s}^{t_0} \frac{dt}{a} = \frac{{a_0}^{-1}{H_0}^{-1}}{q}[1-(1+z)^{-q}] \tag{32}$$

where H_0= Hubble constant at present in $Kms^{-1}Mp_0^{-1}$. Hence

$$d(z) = r_s a_0 = \frac{H_0^{-1}}{q}\left[1-(1+z)^{-q}\right]. \tag{33}$$

For a given redshift z, the expression of scale factor of the universe a is related to a_0 by

$$1+z = \frac{a_0}{a}. \tag{34}$$

Therefore, from Eq.(14), we have

$$1+z = \left[\frac{c_1 t_0 + c_2}{c_1 t + c_2}\right]^{\frac{1}{1+q}}. \tag{35}$$

For small z, Eq.(33) reduces to

$$H_0 d(z) = z - \frac{(1+q)}{2}z^2 + \ldots \tag{36}$$

From Eq.(33) it can be observed that the distance $d(z)$ is maximum at $z \to \infty$ and hence

$$d(z \to \infty) = \frac{H_0^{-1}}{q}. \tag{37}$$

5.2. LUMINOSITY DISTANCE-REDSHIFT d(z)

The luminosity distance of a light source is defined as

$$d_L^2 = \frac{L}{4\pi l} \tag{38}$$

where L is detected energy flux and l is an apparent luminosity. Eq.(38) takes the from

$$d_L = a_0 r_s(z)(1+z). \tag{39}$$

Using Eq.(32) into Eq.(39), we get

$$H_0 d_L = \frac{(1+z)}{q}\left[1-(1+z)^{-q}\right]. \tag{40}$$

For small z, Eq.(40) gives

$$H_0 d_L = z + \frac{(1-q)}{2}z^2 + \ldots.. \tag{41}$$

For $q=1$ and $q=0$, we have

$$H_0 d_L = z \tag{42}$$

and

$$H_0 d_L = (z+\frac{z^2}{2}) \tag{43}$$

respectively.

5.3. LOOK-BACK TIME-REDSHIFT d(z)

From Eq.(35), we have

$$1+z = \frac{a_0}{a} = \left[\frac{c_1 t_0 + c_2}{c_1 t + c_2}\right]^{\frac{1}{1+q}}. \tag{44}$$

Eq.(44) gives

$$(c_1 t + c_2) = (c_1 t_0 + c_2)(1+z)^{-(1+q)}. \tag{45}$$

This equation can also be expressed as

$$H_0(t_0 - t) = \frac{1}{(1+q)}[1-(1+z)^{-(1+q)}]. \tag{46}$$

For small z, Eq.(46) gives

$$H_0(t_0 - t) = z - \frac{q}{2}z^2 + \dots \tag{47}$$

Taking limit $z \to \infty$ in Eq.(46), the present age of universe (the extrapolated time back to the bang) is

$$t_0 = \frac{H_0^{-1}}{(1+q)}. \tag{48}$$

6. CONCLUSION

The cosmological constant problem is one of the outstanding problems in cosmology. In this paper, we have analyzed Einstein's field equations for LRS Bianchi type II stiff fluid space-time with time-decaying cosmological term of the form $\Lambda = \beta(\frac{\ddot{a}}{a})$. There are recent observational evidences for the accelerated state of the present universe. It is held that the deceleration parameter q was positive in the early phases of the matter dominated era and becomes negative during the later stages of the evolution. Therefore to obtain exact solutions to the field equations, which would correspond to an accelerated universe, we have applied a special law of variation for Hubble's parameter initially proposed by Berman. The expressions for some important cosmological parameters have been obtained and their physical behavior is discussed. The model, obtained, has a barrel singularity at the initial epoch $t = -c_2/c_1$. The corresponding universe expands indefinitely with acceleration while all the physical parameters diverge at this epoch. As t increases the physical parameters decrease and ultimately tend to zero as $t \to \infty$. The universe becomes essentially empty for large time. It is seen that σ^2/θ does not tend to zero as $t \to \infty$ which means that the universe is not isotropic for large time. The proper distance, the luminosity distance-redshift and look-back time for the model have been analyzed through kinematic tests. It has been seen that this LRS Bianchi type II universe filled with stiff fluid is compatible with the recent observations.

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