SOME REMARKS ON THE BOSON MASS SPECTRUM
IN A 3-3-1 GAUGE MODEL

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The boson mass spectrum of a 3-3-1 gauge model with right-handed neutrinos is investigated by tuning a unique free parameter within the exact algebraical approach of solving gauge models with high symmetries. The resulting masses and the possible breaking scale of the model are discussed assuming certain phenomenological constrains. A critical point can occur for a particular value of the free parameter, so that the masses in the neutral boson sector and the charged boson sector respectively, become degenerate.

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1. INTRODUCTION

In this paper we analyze the boson mass spectrum in a particular 3-3-1 gauge model and emphasize the fact that a critical point could well occur at a not very high breaking scale. All the Particle Data [1] suggest that the neutral boson $Z$ of whatever extension of the Standard Model (SM) has to be heavier than the neutral boson $Z$ of the SM in order to keep consistency with low energy phenomenology. A very suitable manner to investigate the whole mass spectrum is supplied by the exact algebraical approach for solving gauge theories with high symmetries proposed a decade ago by Cotăescu [2] and developed by one of the authors in a recent series of papers [3] – [7] on the 3-3-1 model with right-handed neutrinos. For some interesting details of the rich phenomenology in such models, the reader is referred also to Refs. [8] – [39] where the “orthodox method” of treating gauge models is involved.

The paper is organized as follows. Section 2 briefly reviews the main results of the theoretical method employed to solve the particular 3-3-1 model with right-handed neutrinos, so that the boson mass spectrum – depending on the unique free parameter $a$ – is obtained. Section 3 deals with the restrictions imposed on the resulting masses from a phenomenological viewpoint and the circumstances under
which the critical point can occur. That is, for $a \geq 0.6$ the neutral bosons of the theory $Z, Z'$ and $X$ gain the same mass! Moreover, at the same time the charged bosons $W^\pm$ and $Y^\pm$ become degenerate. Last section is devoted to sketching our conclusions.

2. BOSON MASS SPECTRUM

The anomaly-free particle content of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$ gauge model with right-handed neutrinos, under consideration here [3]–[39], reads:

Lepton families

$$f_{\alpha L} = \left( \begin{array}{c} \nu^\alpha_L \\ e^\alpha_L \\ e^\alpha_R \end{array} \right) \sim (1,3,-1/3) \quad e_{\alpha R} \sim (1,1,-1) \quad (1)$$

Quark families

$$Q_{\alpha L} = \left( \begin{array}{c} D \_L \\ -d_i \\ u_i \\ \end{array} \right) \sim (3,3',0) \quad Q_{\alpha L} = \left( \begin{array}{c} t \\ b \_L \end{array} \right) \sim (3,3,-1/3) \quad (2)$$

$$b_R, d_R \sim (3,1,-1/3) \quad t_R, u_R \sim (3,1,2/3) \quad (3)$$

$$T_R \sim (3,1,+2/3) \quad D_R \sim (3,1,-1/3) \quad (4)$$

with $i = 1, 2$. The numbers in parenthesis denote – in a self-explanatory notation – the representations and the characters of each fermionic triplet with respect to the gauge group of the theory.

With these representations the particular 3-3-1 model under consideration here is anomaly-free, as one can easily check out by using little algebra. Note that, although all the anomalies cancel by an interplay between families, each family still remains anomalous by itself. This could be a hint to the generation number issue which in turn must be a multiple of 3. Assuming the QCD condition that the number of families must be upper bounded by 5 in order to have quark confinement, one is led to the conclusion that the number of generations is exactly 3.

These representations can be achieved starting with the general method [2] of exactly solving gauge models with high symmetries by just choosing an appropriate set of parameters (for certain details of dealing with the algebraical procedure and the special parameterization involved here, the reader is referred to Ref. [7]). They are:
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\[ e, \theta, \nu_0 = 0, \quad \nu_i = 0, \quad \nu_2 = 1 \quad (5) \]

being imposed by experimental arguments \((e, \theta, \nu)\) [1] or by internal reasons of the general method \((\nu_i)\) [2].

Along with the above parameters, one must add some new ones – as they determine the Higgs sector of the model – grouped in a parameter matrix which reads:

\[ \eta^2 = (1 - \eta_0^2) \text{Diag} \left[ 1 - a, \frac{1}{2}(a + b), \frac{1}{2}(a - b) \right] \quad (6) \]

where, initially, \(a\) and \(b\) are arbitrary non-vanishing real parameters that ensure the condition \(\text{Tr} (\eta^2) = 1 - \eta_0^2\). At the same time, \(\eta_0, a \in [0, 1]\). All the details of the Higgs sector and its involvement in the spontaneous symmetry breakdown (SSB) of the model are explained in Ref. [2]. We do not insist over it, since we are here interested only in the final results – mass spectrum and currents – allowed by the method. They were already obtained in Ref. [7].

We must mention here that these parameters will determine, after the SSB – which takes place (like in the SM) up to the universal residual \(U(1)_\text{em}\) – a plausible non-degenerate boson mass. The exact expressions of the boson masses are given by the Eqs. (53) - (55) in Ref. [2], namely

\[ M_j^2 = \frac{1}{2} g \, \langle \phi \rangle \sqrt{\left( \eta^{(i)} \right)^2 + \left( \eta^{(j)} \right)^2} \quad (7) \]

for the non-diagonal gauge bosons which usually are charged but – as one can easily observe in the 3-3-1 model under consideration here – one of them comes out neutral, and

\[ \left( M^2 \right)_j = \langle \phi \rangle^2 \text{Tr} \left( B_i B_j \right) \quad (8) \]

with

\[ B_i = g \left[ D_i + \nu \left( D \nu \right) - \frac{1}{\cos \theta} \right] \eta \quad (9) \]

for the diagonal bosons of the model. The angle \(\theta\) is the rotation angle around the versor \(\nu\) orthogonal to the electromagnetic direction in the parameter space [2]. The versor condition holds \(\nu, \nu' = 1\).

Since the electro-weak sector of the model is described now by the chiral gauge group \(SU(3)_L \otimes U(1)_Y\), the two diagonal generators \(D_1\) and \(D_2\) (Ds - stands for the Hermitian diagonal generators of the Cartan subalgebra) in the fundamental
representation of $SU(3)_c$ are: $D_1=T_3$ and $D_2=T_8$ – connected to the Gell-Mann matrices in the manner $T_a = \lambda_a / 2$ – and $D_0=I$ for the chiral new hypercharge.

In our 3-3-1 model, the relation between $\theta$ in the general method [2] and the Weinberg angle $\theta_w$ from SM was established [3, 7] and it is

$$\sin \theta = \frac{2}{\sqrt{3}} \sin \theta_w$$ (10)

**Boson mass spectrum**

By using Eq. (7) one can express the masses of the non-diagonal bosons. They are (according to the parameter order in the $\eta^2$ matrix):

$$m^2_W = m^2 a$$ (11)

$$m^2_Y = m^2 \left[ 1 - \frac{1}{2} (a + b) \right]$$ (12)

$$m^2_X = m^2 \left[ 1 - \frac{1}{2} (a - b) \right]$$ (13)

Throughout this paper we consider the following notation:

$$m^2 = g^2 \langle \phi \rangle^2 \left( 1 - \frac{1}{4} \right)$$

Evidently, $W$ is the “old” charged boson of the SM which links positions 2–3 in the fermion triplet, namely the left-handed neutrino to its charged lepton partner and, respectively, the “up” left-handed quarks to “down” left-handed quarks. The neutral $Y$ boson couples the left-handed neutrino to the right-handed one, and the “classical” up (down) quarks to the “exotic” up (down) quarks, that is positions 1–2 in fermion triplet are involved. The remaining $X$ boson is responsible for the charged weak current between positions 1–3 in each triplet.

The “pure” neutral bosons (diagonal ones) get their mass eigenstates by diagonalizing the resulting matrix:

$$M^2 = m^2 \begin{pmatrix}
1 - \frac{1}{2} a + \frac{1}{2} b & -\frac{1}{\sqrt{3 - 4 s^2}} \left( 1 - \frac{3}{2} a - \frac{1}{2} b \right) \\
-\frac{1}{\sqrt{3 - 4 s^2}} \left( 1 - \frac{3}{2} a - \frac{1}{2} b \right) & -\frac{1}{3 - 4 s^2} \left( 1 + \frac{3}{2} a - \frac{3}{2} b \right)
\end{pmatrix}$$ (14)

after combining Eqs. (8), (9) and (6), where the notation $\sin \theta_w = s$ has been made for simplicity.
One of the two diagonal bosons has to be identical to the neutral boson $Z$ from SM. Therefore, the latter should be an eigenvector of this mass matrix corresponding to the eigenvalue $m_Z^2 = m^2_w / \cos^2 \theta_w$ firmly established in the SM [1]. The eigenvalue problem reads:

$$M^2 \left| Z > = \frac{m^2_a}{1 - s^2} |Z > \right.$$  \hspace{1cm} (15)

That is, one computes $\text{Det} \left| M^2 - m^2 a / (1 - s^2) \right| = 0$ which leads to the constraint upon the parameters: $b = a \tan^2 \theta_w$. Consequently, the parameter matrix (6) becomes:

$$\eta^2 = (1 - \eta^2) \text{diag} \left[ 1 - a, \frac{a}{2 \cos^2 \theta_w}, \frac{a}{2} (1 - \tan^2 \theta_w) \right] \hspace{1cm} (16)$$

Under these circumstances, the boson mass spectrum yields:

$$m^2_W = m^2 a \hspace{1cm} (17)$$

$$m^2_t = m^2 \left( 1 - \frac{a}{2 \cos^2 \theta_w} \right) \hspace{1cm} (18)$$

$$m^2_\chi = m^2 \left[ 1 - \frac{a}{2} (1 - \tan^2 \theta_w) \right] \hspace{1cm} (19)$$

$$m^2_\gamma = \frac{m^2 a}{\cos^2 \theta_w} \hspace{1cm} (20)$$

$$m^2_Z = m^2 \left[ 1 + \frac{1}{5 - 4 \sin^2 \theta_w} - a \left( 1 + \frac{\tan^2 \theta_w}{3 - 4 \sin^2 \theta_w} \right) \right] \hspace{1cm} (21)$$

since $\text{Tr} \left( M^2 \right) = m^2_Z + m^2_Z$ holds.

We obtained a mass spectrum depending on a single free parameter $a$ to be tuned. One can observe that, although the fermion representations and even the order in the parameter matrix $\eta^2$ are not the same with those chosen in Ref. [3], the resulting mass spectrum exhibits the same structure. That means there are equivalent ways to choose the parameters in the general method in order to reach the same particle content of the model and the same physics. In addition, by inspecting the above mass spectrum one recovers the decoupling theorem.
3. PHENOMENOLOGICAL CONSEQUENCES

The method presented above relies heavily on the role played by the parameter $a$. In fact, it determines the breaking scale and the structure of the mass spectrum in the model. For instance, from Eq.(17) it is obviously that assuming $m(W) = 80.4$ GeV, the smaller the parameter, the greater the mass scale $m$ (and consequently the breaking scale of the model).

When inspecting the boson mass spectrum - Eqs. (17) - (21) - one can enforce certain conditions on the parameter $a$ as to obtain realistic values, in accordance with the available experimental data. Furthermore, the neutrino phenomenology was investigated [4] and, because a very high breaking scale $\langle \phi \rangle$ was required, the method suggested a natural see-saw mechanism [5], embedded in order to keep consistency with the tiny observed masses in the neutrino sector.

However, a special and unexplored yet opportunity is offered by our method. It was for the first time mentioned by Cotăescu in a communication [40] on the well known Pisano-Pleitez-Frampton 3-3-1 model [41, 42] and consequently developed in a regular paper devoted to exactly solving this model [43].

As long as the exact masses of the new bosons have not been experimentally determined to date, one is entitled to ask if there is no screening between them or – more precisely – if the new neutral boson does not “cover” the old one. Are their masses degenerate? And if so, what kind of consequences has such a hypothesis? What kind of hidden symmetry can unfold?

From Eqs. (20) and (21) it results that the free parameter has to be

$$a = \frac{2 \cos^2 \theta_w}{3 - 2 \sin^2 \theta_w}$$

in order to achieve $m_Z = m_{\nu}$. That is $a = 0.6$ if we consider $\sin^2 \theta_w$ [1].

Furthermore, what are the values gained by the masses of the remaining bosons? Embedding (22) in (17), (18) and (19) respectively, one obtains the amazing results:

$$m_{\nu}^2 = m_{\nu}^2 = m_{\nu}^2 = m^2 \frac{2 \cos^2 \theta_w}{3 - 2 \sin^2 \theta_w}$$

and simultaneously

$$m_Z^2 = m_{\nu}^2 = m_{\nu}^2 = m^2 \frac{2}{3 - 2 \sin^2 \theta_w}.$$ 

These are the well-known values predicted by SM, namely 91.2 GeV for the neutral bosons, and 80.4 GeV for the charged ones. Assuming that in the SM
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\[ m_W = \frac{g}{2} \langle \phi \rangle_{SM} \]  \hspace{1cm} (25)

holds, one can estimate the required breaking scale \( \langle \phi \rangle \) of the 3-3-1 model under consideration here, by comparing it to (17). That is \( \langle \phi \rangle \geq \langle \phi \rangle_{SM} / \sqrt{a} \). This leads to \( \langle \phi \rangle \geq 320 \text{ GeV} \). However, we present below a plot with the mass spectrum depending on the parameter \( a \). The most plausible region of interest lies around the \( m \geq 1 \text{ TeV} \), corresponding to \( a \geq 0.0065 \).

Fig. 1 – The masses of the bosons X, Y, Z'.

4. CONCLUDING REMARKS

We have proven in this brief report that the exact algebraical approach for solving gauge models with high symmetries offers - when it is applied to a 3-3-1 model with right-handed neutrinos - a plausible way to investigate the boson mass spectrum. It allows one to make predictions regarding the resulting masses of the “new” bosons, while keeping the “old” boson ones at their established values from SM.
The critical point of the model can be seen as a point where the free parameter of the model enforces the breakdown of the gauge group of the model up to the SM's one. This can occur at a not very high breaking scale $\langle \phi \rangle \gtrsim 320 \text{ GeV}$. Since this result exactly holds at tree level, one is entitled to ask if the radiative corrections do not significantly alter it. Of course, a detailed analysis has to be done in a future work [44] regarding the way the oblique parameters S, T, U (computed already for this particular 3-3-1 model in Ref. [13]) can influence these results.

Therefore, we consider that the strange coincidence that simultaneously occurs - namely, $m_{W} = m_{Z}$ and $m_{\gamma} = m_{\gamma}'$ - for a particular value of the free parameter seems more than a simple “fit”. It suggests a possible deeper identity between the “same charge” bosons. This hypothesis must not be ruled out a priori, since more accurate results regarding the decays of the “new” bosons and high-energy scatterings involving their couplings to fermions - experimental details which can reveal some new restrictions on the parameter $a$ - have to be more exactly investigated at LHC.

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