NONLINEAR EVOLUTION OF DRIFT TURBULENCE: INVERSE CASCADE, ZONAL FLOWS, INTERMITTENCY

M. VLAD, F. SPINEANU

National Institute of Laser, Plasma and Radiation Physics
P. O. Box MG-36, Magurele, Bucharest, Romania,
E-mail: madi@ifin.nipne.ro

Received January 24, 2011

Test modes on turbulent magnetized plasmas are studied taking into account the ion trapping that characterizes the $E \times B$ drift in the background turbulence. We show that trapping provides the physical mechanism for the formation of large scale potential structures (inverse cascade) observed in drift turbulence. Trapping combined with the motion of the potential with the effective diamagnetic velocity determines ion flows in opposite directions, which reduce the growth rate and eventually damps the drift modes leading to intermittent evolution of turbulence.

1. INTRODUCTION

Turbulence evolution in magnetically confined plasmas is a complex problem that is not yet understood besides the huge amount of work on this topic (see [1] and the references there in). Most studies that go beyond the quasilinear stage are based on numerical simulations. They show the generation of large scale structures and of zonal flows that lead to turbulence damping.

The aim of this paper is to study the nonlinear evolution of the drift turbulence on the basis of test modes on turbulent plasmas. A Lagrangian approach is developed using the results we have obtained in the last decade on the statistics of test particle trajectories. Semi-analytical statistical methods (the decorrelation trajectory method [2] and the nested subensemble approach [3]) have been developed for the study of test particle stochastic advection. These are the first methods that describe trajectory trapping or eddying that is specific to the $E \times B$ drift in turbulent plasmas. It was shown that trapping determines memory effects, quasi-coherent behavior and non-Gaussian distribution [3].

We study linear modes on turbulent plasma with the statistical characteristics of the potential considered known. Analytical expressions are derived, which approximate the growth rates and the frequencies of the test modes as functions of the characteristics of the background turbulence. These are the first analytical results.


that give an image on the evolution of turbulence in the strongly nonlinear regime, which is in agreement with experiments and numerical simulations. A new understanding of important aspects of the physics of drift type turbulence in the non-linear phase is obtained. The main role in the processes of inverse cascade, zonal flows and intermittency is shown to be played by ion trapping.

2. TEST MODES ON TURBULENT PLASMA

We consider the drift instability in slab geometry with constant magnetic field. We start from the basic gyrokinetic equations for the distribution of electrons and ions.

The solution for the potential in the zero Larmor radius limit is
\[ \phi(x, z, t) = \phi_0(x - V_\ast t, z), \]
where \( \phi_0(x, z) \) is the initial condition and \( V_\ast = T \partial_x [\ln(n_0)]/eB \) is the diamagnetic velocity produced by the gradient of the average density \( n_0(x) \). This shows that the potential is not changed but displaced with the diamagnetic velocity. The special case \( \phi_0(x, z) = A \exp(ikx + ikz) \) leads to the drift waves.

The finite Larmor radius effects combined with the non-adiabatic response of the electrons destabilizes the drift waves. The frequency \( \omega \) and the growth rate \( \gamma \) of the modes are
\[ \omega = k_y V_\ast \Gamma_0, \quad V_\ast \Gamma_0 = V_\ast \frac{\Gamma_0}{2 - \Gamma_0}, \]
\[ \gamma = \sqrt{\pi} \frac{k^2}{2 - \Gamma_0}, \]
where \( \Gamma_0 = \exp(-b)I_0(b) \), \( b = k^2_\perp \rho_L^2/2 \) and \( \rho_L \) is the ion Larmor radius. The wave number components are \( k_i, i = x, y, z \) and \( k_\perp = \sqrt{k_x^2 + k_y^2} \). These are the characteristics of the linear (universal) drift instability on quiescent plasmas.

We consider a turbulent plasma with given statistical characteristics of the stochastic potential. Linear test modes on this turbulent state are studied. The growth rates and the frequencies of the test modes are determined as functions of the statistical characteristics of the background turbulence with potential \( \phi(x, t) \). A wave type perturbation of the potential \( \delta \phi(x, z, t) = \phi_{k, \omega} \exp(ik \cdot x + ikz - i\omega t) \) is introduced. It is small (\( \delta \phi \ll \phi \)) and thus it has a negligible influence on particle trajectories. The solutions for the perturbations of electron and ion densities are obtained using the method of characteristics as integrals along particle trajectories in the background potential of the source terms determined by the density gradient. The characteristics \( x^\alpha(\tau), z^\alpha(\tau) \) are the trajectories obtained by integrating the equation of motion...
Nonlinear evolution of drift turbulence: inverse cascade, zonal flows, intermittency

backwards in time with the condition at time \( t \), \( x^\alpha(t) = x \), \( z^\alpha(t) = z \)

\[
\frac{d x^\alpha}{d\tau} = -\nabla \phi(x^\alpha, z^\alpha, \tau) \times e_z, \quad \frac{d z^\alpha}{d\tau} = v_z^\alpha.
\]

The background turbulence produces two modifications in the response. One consists in the stochastic \( E \times B \) drift that appears in the characteristics (4) and the other is the fluctuation of the diamagnetic velocity due to the fluctuations of the density \( \delta n \) in the background turbulence. Both effects are important for ions while the response of the electrons is approximately the same as in quiescent plasma. The perturbed distribution function is averaged over the stochastic trajectories. The dispersion relation (quasi-neutrality condition) of a mode with frequency \( \omega \) and wave number \( k \) is obtained

\[
2 + i \sqrt{\frac{\pi}{\omega - k_y V_s}} = i\Pi \Gamma_0 [\omega + V_s (k_y + ik_j k_j R_{ij})].
\]

The background turbulence appears in this equation in the average propagator

\[
\Pi = \int_{-\infty}^{-\infty} d\tau \exp (-ik \cdot x^\alpha(\tau)) \exp (i\omega(t - \tau))
\]

and in the tensor \( R_{ij} \), which is the integral of a Lagrangian correlation

\[
R_{ji}(\tau,t) \equiv \int_{\tau}^{t} d\theta' \int_{-\infty}^{\tau-\theta'} d\theta \left\langle v_j \left( x^i(\theta'), z, \theta' \right) \partial_2 v_i \left( x^i(\theta), z, \theta \right) \right\rangle,
\]

where \( v_j \) is the \( E \times B \) drift velocity component. The average propagator (6) contains the effects of the stochastic trajectories and the tensor \( R_{ij} \) yields from the fluctuations of the diamagnetic velocity.

3. DRIFT TURBULENCE EVOLUTION

The growth rate and the frequency of the drift modes give an image of the turbulence evolution starting from a weak initial perturbation with very broad wave number spectrum. We show that a sequence of processes appear at different stages as transitory effects and that the drift turbulence has an oscillatory (intermittent) evolution.

The effective diamagnetic velocity (2) is a function of \( k \) due to finite Larmor radius of the ions. This means that the potential does not translate as in the solution (1) corresponding to \( \rho_L = 0 \), but it also changes due to \( k \)-dependence of the growth rate and of the effective diamagnetic velocity. Two characteristic times are associated with this variation: the potential drift time \( \tau_s = \lambda_c / V_d \) and the correlation time \( \tau_c \). \( \lambda_c \) is the correlation length of the background potential and \( V_d \) is the average of \( V_{d,eff} \) over wave-numbers. \( \tau_s \) is the characteristic time for the potential motion while \( \tau_c \)
accounts for the modification of the shape of the potential. The latter is essentially related to the growth rates of the modes and also depends on the width of the turbulence spectrum. The ordering of the characteristic times for the drift turbulence is

\[ \tau_e^{\parallel} \ll \tau_s \ll \tau_c \ll \tau_i^{\parallel}, \]

(8)

where \( \tau_e^{\parallel}, \tau_i^{\parallel} \) are the parallel decorrelation times for electrons and ions (\( \tau_e^{\parallel} = \lambda^{\parallel}/v_{th}^{e} \) with \( \lambda^{\parallel} \) the parallel correlation length, and \( v_{th}^{e,i} \) the thermal velocity).

The linear and the nonlinear regimes are determined by the position of the time of flight (or eddying time) \( \tau_{fl} = \lambda_c/V \) in this ordering. \( \lambda_c \) is the amplitude of the \( E \times B \) drift. More specifically, the quasilinear regime is characterized by \( \tau_s \ll \tau_{fl} \) (or \( V \ll V_d \)) while the nonlinear effects appear when \( \tau_{fl} \gtrsim \tau_s \) (\( V \gtrsim V_d \)).

3.1. TRAJECTORY DIFFUSION AND DAMPING OF SMALL k MODES

The statistics of trajectories is Gaussian for small amplitudes of the stochastic velocity \( V \ll V_d \) and the diffusion of trajectories is isotropic. The solution of the dispersion relation (5) shows that \( \omega \) and \( V^{eff}_{\ast} \) are not changed [Eqs. (2)] and that the growth rate becomes:

\[ \gamma = \frac{\sqrt{\pi}}{|k_z| v_T c} \frac{k_0^2 (V^\ast - V^{eff}_{\ast})}{2 - \Gamma_0} - k_i^2 D_{i}^{2} \frac{2}{2 - \Gamma_0}. \]

(9)

This is the well known result of Dupree [4] which shows that a stabilizing contribution is produced by the ion diffusion in the background turbulence, which leads to the damping of the large \( k \) modes.

3.2. TRAJECTORY STRUCTURES AND LARGE SCALE CORRELATIONS

The increase of the turbulence amplitude \( V \) above \( V_d \) determines ion trapping or eddying. As we have shown, this strongly influences the statistics of trajectories. The distribution of the trajectories is not more Gaussian due to trapped trajectories that form quasi-coherent structures. The fraction of trapped particles \( n_{tr} \) increases with the increase of \( V \). The nonlinear effects that appear for weak trapping with \( n_{tr} \ll 1 \) are mainly produced by the modification of the probability of ion displacements, \( P(x,t) \). We have shown that it has a pronounced peaked shape. It can be modeled by \( P(x,y,t) = n_{tr}G(x;S) + n_{fr}G(x;S'), \) where \( G(x;S) \) is the 2-dimensional Gaussian distribution with dispersion \( S = (S_x, S_y) \). The first term has a small constant dispersion that is the size \( \bar{S} \) of the trajectory structures and describes the trapped trajectories. The free trajectories are described by the second term, which has diffusive dispersion \( S_{i}' = S_i + 2D_i t, \) \( i = x, y \). This distribution modifies the average propagator by a factor \( F \) that is determined by the average size of the trapped
Nonlinear evolution of drift turbulence: inverse cascade, zonal flows, intermittency

The solution of the dispersion relation (5) leads only to the modification of the effective diamagnetic velocity

$$V_{\text{eff}}^* = V_* \frac{\Gamma_0 \mathcal{F}}{2 - \Gamma_0 \mathcal{F}},$$

while the growth rate equation is not modified. The decrease of the effective diamagnetic velocity produces the displacement of the position of the maximum of $\gamma$ toward small $k$. The maximum of $\gamma$ moves to smaller $k$ values of the order of $1/S_i$ and the size of the unstable $k$ range decreases. The maximum growth rate decreases.

Thus, ion trapping determines the increase of the correlation length of the potential and the decrease of the average frequency (proportional with $k_2$). In this nonlinear stage, turbulence has large scale potential cells, its evolution becomes slower and leads to ordered states (narrower spectra with maximum at smaller $k$).

### 3.3. ION FLOWS AND TURBULENCE DAMPING

The evolution of the potential determines the increase of the fraction of trapped ions. This produces another effect on the test modes. The potential continues to move with the average diamagnetic velocity $V_d$. This determines an average flux of the trapped particles $n_{tr} V_d$. As the $E \times B$ drift has zero divergence, the probability of the Lagrangian velocity is time invariant, i.e. it is the same with the probability of the Eulerian velocity. The average Eulerian velocity is zero and thus the flux of the trapped ions that move with the potential has to be compensated by a flux of the free particles. These particles have an average motion in the opposite direction with a velocity $V_{fr}$ such that $n_{tr} V_d + n_{fr} V_{fr} = 0$. The velocity on structures method that we have recently developed shows that the probability of the displacements splits in two components that move in opposite direction. Thus, opposite ion flows are generated by the moving potential in the presence of trapping. They modify both the effective diamagnetic velocity and the growth rate

$$\gamma = \frac{\sqrt{\pi} \ k_0^2 \ (V_* - V_{\text{eff}}^*(0)) \ (V_{\text{eff}}^* - nV_*)}{2 - \Gamma_0 \mathcal{F}} - k_0^2 D_i \frac{2 - \Gamma_0 \mathcal{F} n_{tr}}{2 - \Gamma_0 \mathcal{F}},$$

where $n = n_{tr}/n_{fr}$. The maximum of the drive term of the drift instability (the first term in Eq. (13), $\gamma_1$) decreases with the increase of $n$. The effective diamagnetic velocity (12) shows that it increases for all values of $k$ and becomes larger than $V_*$ first for small $k$. This determines the damping of these modes. As $n$ increases modes with larger $k$ are damped and, for $n = 1$, $\gamma_1 < 0$ for any $k$. 

Trajectory structures

$$\mathcal{F} \equiv \exp \left( - \frac{1}{2} k_i^2 S_i^2 \right).$$
Thus, trapping combined with potential drift produces ion opposite flows, which lead to turbulence damping.

3.4. GENERATION OF ZONAL FLOW MODES

The fluctuations of the density produced by the background turbulence (7) determine an additional term in the growth rate: \( \approx k_1 k_j R_{ij} V_\ast \). The component \( R_{11} \) is very interesting because it generates modes with \( k_2 = 0 \) and \( \omega = 0 \), if this term is positive. This are static oscillations in the direction of the average density gradient, which are known as zonal flow modes and have been intensely studied in the last decade in connection with internal transport barriers (see [5] and the references therein). We have found that \( R_{11} \) essentially depends on \( n \). It increases when the ion flows become important up to a positive maximum and then it decreases to zero. It is essentially determined by the anisotropy that is generated by the difference in the average velocity of the trapped ions \( V_d \) and the average velocity of the free ions \( V_{fr} = -n V_d \). When \( n = 1 \), the ion flows are symmetrical and \( R_{11} \) vanishes. A clear connection of the zonal flow modes with the ion flows induced by the moving potential appears.

4. CONCLUSIONS

A different physical perspective on the nonlinear evolution of drift turbulence is obtained. The main role is played by the trapping of the ions in the stochastic potential that moves with the effective diamagnetic velocity. The trapped ions determine the evolution of the turbulence toward large wave lengths (the inverse cascade). They also determine a slower increase of the amplitude of the potential fluctuations and the evolution to more ordered states. The influence of the ion flows produced by the moving potential appears later in the evolution of the turbulence. The ion flows determine the damping of the small \( k \) modes, the decay of the growth rate and eventually the damping of the drift modes with any \( k \). The ion flows also determine transitory zonal flow modes (with \( k_y = 0 \) and \( \omega = 0 \)) in connection with the fluctuation of the diamagnetic velocity due to the background turbulence. Thus, in this perspective, there is no causality connection between the damping of the drift turbulence and the zonal flow modes. Both processes are produced by ion trapping in the moving potential, which determines ion flows. The drift turbulence does not saturate but has an intermittent evolution.

Acknowledgements. We acknowledge financial support from the Romanian Ministry of Education and Research under Project LAPLAS2, contract No. PN 09 39.
REFERENCES