In this paper, a study on the multifractal characteristics of intermittency dynamics in a discharge plasma is presented. In order to characterize the singularity spectra, we propose two relevant parameters and investigate the influence of the applied voltage on their magnitude. With increasing of the disorder in the experimental discharge current time series, an evolution of the fluctuations towards Brownian type motion is clearly observed.

Key words: Plasma intermittency, multifractal analysis, singularity spectrum.

1. INTRODUCTION

Multifractal method of analysis represents a modern time series technique of investigation, suitable for the description of statistical characteristics of intermittency. The multifractal scaling analysis has been used largely in the study of turbulence of fluids and, more recently in plasma turbulence [1, 2].

In this paper we present a singularity spectrum analysis on intermittency observed in the dynamics of a discharge plasma [3, 4]. We observe a significant change of the spectrum parameters versus the discharge voltage. The multiscale nature of the time series is associated with the presence of spatial and temporal nonlinear structures responsible for this kind of instabilities.

2. EXPERIMENTAL RESULTS AND MULTIFRACTAL ANALYSIS

The experimental signals used in our fractal analysis are the current oscillations collected by an additional positive electrode immersed in a low temperature discharge plasma [3]. The structure of the oscillations changes when
the externally applied voltage on the additional electrode is modified. This is illustrated in Fig. 1a and Fig. 1c for two values of the voltage in the range of the considered experiment.

The multifractal character of the time series is demonstrated from the plot of the singularity spectrum. It is a curve representing the Hausdorff dimension \( f(\alpha) \) as function of the singularity parameter \( \alpha \) presenting a characteristic maximum. We may interpret \( f(\alpha) \) as the fractal dimension of the set of points with the scaling index in the range \( \alpha, \alpha + d\alpha \). The maximum value of \( f(\alpha) \) corresponds to the (average) box counting dimension of the object (the time series).

The multifractal spectrum is related to the set of generalized dimensions \( D_q \) quantified by the order \( q \) which can have positive and negative values as well as zero [5]. In Fig. 1b and Fig. 1d the computed spectra for the following values of the order of the generalized dimensions \( q = -4, -3, -2, -1, 0, +1, +2, +3, +4 \) are shown.

We define the following parameters for the characterization of the multifractal spectrum: \( \Delta f(\alpha) \) and \( \delta \alpha \). Their significance is presented in Fig. 2:
\[ \Delta f(\alpha) = |f_{-\Delta}(\alpha) - f_{+\Delta}(\alpha)|, \quad \delta \alpha = \Delta \alpha / \alpha_{\text{max}} \]  

(1)

where the subscript on \( f(\alpha) \) denotes the corresponding values of \( q \).

In the computation of the singularity spectra the algorithm based on the wavelet transform [6] was used.

The changes induced in the singularity spectrum by variation of the voltage of the additional electrode are observed as alterations of the parameters defined by equation (1), as shown in Fig. 3.

Figure 3a shows that the dependence of \( \Delta f(\alpha) \) on the applied voltage is well fitted as a decreasing exponential having as asymptote the value corresponding to Brownian motion represented by the broken line. The equation of the fitting curve is written on the graph.

![Fig. 2 – Singularity spectrum for two voltages (marked in the figure) and the definition of the measures considered.](image)

![Fig. 3 – Dependences on the voltage of \( \Delta f(\alpha) \) (a) and \( \delta \alpha \) (b). On each graph, the broken line represents the values corresponding to the Brownian noise.](image)
Unlike this, Fig. 3b shows that the dependence of $\delta\alpha$ on the applied voltage is best fitted by a straight line whose final point is above the broken line corresponding to the Brownian noise. This indicates that for higher values of the voltage, the resulting current fluctuations are well approximated as a Brownian motion.

In a previous paper [7] we considered that the changing of the additional electrode potential has two effects: first, it can induce changes in the excitable dynamics of a highly nonlinear oscillator modeling the anodic double layer and second, it can enhance the noise level of the discharge. The present results demonstrate that for high enough biasing potential, the dynamics evolves to correlated fluctuations that can be modeled as Brownian motion.

Although theoretically the Brownian noise is known to be monofractal, which means that the singularity spectrum consists in a single point, the computer generated Brownian noise, irrespective of the used algorithm is never strictly monofractal. One of the factors to contribute to the apparent multifractality is the limited length of any analyzed time series [8]. In order to compute the defined parameters for Brownian noise, we averaged the results obtained from 10 computed time series having the same length as our experimental time series.

The multifractal characteristics of intermittency dynamics in a discharge plasma is studied. We propose two relevant parameters and investigate the influence of the applied driving voltage on their magnitude. The choice of these parameters is justified by the fact that they quantitatively describe the departure of a process from the ideal Brownian noise. We observe that with increasing of the disorder in the experimental discharge current time series, an evolution of the fluctuations towards Brownian type motion is taking place.

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REFERENCES