COHERENT FLOWS IN MAGNETIZED PLASMAS, IN LASER PLASMA AND IN PLANETARY ATMOSPHERE

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There is a fundamental nonlinearity that governs the coherent and quasi-organised flows in magnetized plasma, high power laser-produced plasmas and planetary atmosphere. It consists of the convection of a vector field by its own velocity field. Unfolding the complexity of this self-interaction is only possible in few, non-generic cases. However a new conceptual approach is possible if we provide freedom to the system, extending the description to a field theoretical framework. We write the Lagrangian densities for ideal fluids and for confined plasma and examine various applications.

Key words: Euler fluid, non-Abelian bosonic theory, Chern-Simons, tropical cyclone, crystals of vortices.

1. INTRODUCTION

The fluids and plasma exhibit in two-dimensions a strong tendency to self-organisation in the undriven evolution towards stationary states. As shown by experiments and numerical simulation [1], the 2D fluids and plasmas reach states of high coherency of the flow, generated by concentration of vorticity in few large scale vortical flows. The process is essentially non-dissipative since the energy is almost conserved during this process. The presence of dissipation is however essential since breaking up of streamlines and reconnection into larger structures is only possible in the presence of irreversible resistive-like mechanisms. The vortex merging is the typical process and a large number of studies have been done both experimentally and by numerical simulation. When the initial state is turbulent one may invoke arguments related with the inverse cascade of energy in 2D but this approach has limited relevance when the process has evolved to the point where the number of quasi-coherent structures is large: the high number of irreducible


correlations necessary to describe a statistical ensemble of turbulence with embedded structures invalidates any perturbative considerations. A theoretical model of the asymptotic stationary states of fluids cannot be based on only the conservation laws (density, momentum, energy, etc.) since they allow for a large class of functions that could represent the final flow pattern. A natural approach is to first identify functionals of the fluid's state and apply variational procedures. The two-dimensional geometry of the fluid flow makes this possible. The 2D ideal incompressible fluid (Euler) can be represented as a discrete system of point-like vortices interacting by a long range potential. The 2D atmosphere is equivalent with a discrete system of point-like vortices interacting by a short range potential. These are well-known models that have been used in various applications. A fundamental property of these models consists of the fact that they formulate the dynamics in terms of matter, field and interaction. We then look again to the continuum limit of these models but preserving this structure. In both cases we obtain a classical field theory for the matter field (the density of point-like vortices) the gauge field (the long or short range potential) and interaction. The essential benefit of this approach is that it provides a Lagrangian density, whose integral on 2D space and time is the action functional.

2. THE IDEAL FLUID

For the 2D ideal fluid the Euler equation $d\omega/dt = 0$ is expressed for the discrete system of point-like vortices as $d\mathbf{r}_i^\mu / dt = \mathbf{e}^\nu / \partial \mathbf{r}_i^\mu \sum_{n=1}^{N} \alpha_{n} G(\mathbf{r}_i - \mathbf{r}_n)$, where $\mathbf{r}_i \equiv (x, y)$, $\omega_\alpha$ is the elementary vorticity carried by each point-like vortex and $\mathbf{e}^\nu$ is the antisymmetric tensor. The potential is long range $G(\mathbf{r}, \mathbf{r}') \approx -\pi/\rho \ln \left| \mathbf{r} - \mathbf{r}' \right|/L$ where $L$ is the spatial extension of the flow. The continuum limit leads to a Lagrangian density expressed in terms of two fields: $A_\mu$ is the ("gauge") field representing the potential between vortices, $\phi$ is the complex scalar ("matter") field representing the increase or decrease of local vorticity (equivalently: increase/decrease of density of vortices). The fields $A_\mu$ and $\phi$ are $2 \times 2$ matrices with complex entries that belong to $su(2)$, as required by the chirality arising from the spin-like nature of the point vortices. Looking for the extremum of the action $S = \int d^2 \mathbf{x} dt dL$ we are guided by the observation that the asymptotic states of the fluid are coherent structures, possibly integrable. Since all known integrable structures (including solitons, instantons) are obtained at "self-duality" we look for this property of $S$. In practical terms this means to write $S$ as a sum of squares plus a topological term. Indeed $S$ for the Euler has this property (the topological term is zero) and $S$ can be minimised by taking to zero the square
terms. This leads to two equations (the "self-duality" equations) which, with an algebraic ansatz, lead to the sinh-Poisson equation \([2]\). The latter is known to describe the asymptotic structures reached by Euler fluid at stationarity.

3. THE PLASMA AND THE PLANETARY ATMOSPHERE

For the atmosphere the interaction between point-like vortices is short-range:

\[ G \sim K_0 \left( \left| r_k - r_r \right| / \rho_g \right) \]

where \( K_0 \) is the modified Bessel function and \( \rho_g \) is the Rossby radius. The Lagrangian, similar to the Euler fluid case, consists of the kinetic

\[ L = -\text{tr} \left( \begin{pmatrix} D^\nu \phi \\ D_\mu \phi \end{pmatrix} \right)^2 - \kappa \varepsilon^{\mu \nu \rho} \text{tr} \left( \epsilon_\mu A_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right) - V(\phi, \phi^+) \]  \( (1) \)

term for the \( \phi \) field, the Chern-Simons term of the \( A_\mu \) field and the potential \( V \) for the self-interaction of the scalar field.

As suggested by the Euler model we look for self-duality and write the action \( S \) as a sum of squares and an additional term. However it is not possible to find for the latter a topological meaning, when the fields \( \phi \) and \( A_\mu \) belong to \( su(2) \). This leaves a certain ambiguity in the separation into squares and "residual" energy. Taking to zero the square terms we derive the reduced set of equations \([3]\). Under the same algebraic ansatz we find

\[ \Delta \psi + \frac{1}{2p^2} \sinh \psi \left( \cosh \psi - p \right) = 0 \]  \( (2) \)

where \( p \) is a positive constant and \( \psi \) is the streamfunction of the flow.

The Eq.(2) has to be validated by successful applications to physical systems that are characterized by (1) the nonlinearity \( \left[ \left( -\nabla \psi \times \hat{e}_z \right) \cdot \nabla \right] \nabla^2 \psi \), and (2) the presence of an intrinsic length, like Larmor radius, Rossby radius, etc. Such systems are: the tropical cyclone, the plasma in strong magnetic field (tokamak), the non-neutral plasma, the current density (or, equivalently, the perturbed magnetic field) of the plasma generated by high density laser pulses, etc.
For the tropical cyclone the physical quantities are normalised using the Rossby radius \( \rho_g \) and the Coriolis frequency \( f_0 \). After the phase of cyclogenesis the balance of mechanical and thermal processes is realised around a flow configuration representing a solution of Eq.(2). We have compared the results of integration of the Eq.(2) with the observations, in particular on the tropical cyclones of north-east Pacific, where stationarity can be reached before landfall. The agreement is very good. In the spirit of the main idea of this work, we note that the stationarity coincides with an almost vanishing magnitude of the nonlinearity which drives vortices.

For the quasi-two-dimensional plasma of the tokamak we found the same suppression of the vorticity convection nonlinearity in the states where the plasma reaches the High Confinement. This means that the poloidal rotation profile (or radial electric field) is described by Eq.(2) to a good approximation. The fact that is a “privileged” configuration, which in our terms means extremum of the action functional, means that the High Confinement (H-mode) state is preferred by the plasma, once the threshold of poloidal damping is overcome.

We show in Figures 1 the results of solving the differential equation for parameters appropriate to tropical cyclone (a section through the profile of the azimuthal velocity) [4] and tokamak plasma (radial profile of the density in the H-mode).

4. THE LASER-GENERATED PLASMA AND THE CRYSTALS OF VORTICES

The differential equation for the magnetic field in a plasma generated by a high-power laser pulse has the same nonlinearity as the Euler equation. The associated current sheets are broken into filaments and this can be observed experimentally.
The stationary state would be a set of discrete quasi-coherent structures similar to vortices in the case of vorticity sheets. It is interesting to note that the differential equation for the fluid governed by an elementary intrinsic length Eq.(2) is able to reproduce this structure finding that a sequence of filaments has lower action compared with the sheet of current.

In the case of non-neutral plasma, the experiments have shown formation of crystals of vortices with long time of persistence [5]. The system has the same nature as the planetary atmosphere or confined plasma, therefore the Eq.(2) should be applicable. Indeed we obtain structures consisting of vortices placed in a regular geometry like crystals.

In the physics of the atmosphere it has been identified structures consisting of discrete vortices placed in a symmetric form, like a crystal (of tornadoes) [6]. The same approach has been taken using Eq.(2) and the possibility of these patterns is confirmed.

The particularity of these crystals is the slow time evolution toward a single, centrally placed and circularly symmetric vortex. Indeed we find from Eq.(2) crystals as quasi-solutions, which means flow configurations that obey Eq.(2) with a lower precision. They correspond to a degenerate direction of the function space, along which the action functional has only weak variation. The symmetric vortex belongs to the same line and this explains the slow evolution observed in experiments.

The laser-produced plasma has a magnetic field which is driven toward structured patterns by the convection nonlinearity [7]. This explains why we obtain, from Eq.(2) the filaments which are similar to what is observed in experiments with high power laser beam.

![Fig. 2 – Filamentation of current sheets and crystals of vortices in plasma. The contours of the streamfunction ψ and the velocity vector are represented.](image)

Fig. 2 – Filamentation of current sheets and crystals of vortices in plasma. The contours of the streamfunction ψ and the velocity vector are represented.
In conclusion the field theoretical description is able to determine states of coherent flows in fluids, plasmas, planetary atmosphere. The development is also very promising: a much better understanding can be obtained from the fermionic version of the field theory, where the Thirring-type self-interaction can explain the Lorentz motion of the point-like vortices.

REFERENCES