SYSTEMATICS OF HINDRANCE FACTORS IN ALPHA DECAY OF EVEN-EVEN TRANS-LEAD NUCLEI

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The experimental values of the hindrance factors in the alpha decay of even-even trans-lead nuclei, corresponding to the states $2^+_1$, $4^+_1$, $6^+_1$, $0^+_2$, and $1^+_1$ are examined as a function of the mass number and collectivity indicators $N_pN_n$ and $P$. Their evolution is discussed in connection with the main trends of the collectivity in this region.

Key words: Even-even actinides, alpha decay hindrance factors, experimental systematics.

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1. INTRODUCTION

Alpha decay is one of the oldest observed nuclear structure phenomena, but it is still important as an experimental tool for investigating unstable nuclei, especially in the heavy and superheavy regions. The calculation of the absolute values of the alpha decay rates is a problem that is still not fully solved. Experimentalists often need tools to evaluate expected alpha decay rates for nuclei that are not studied yet. Empirical systematics, such as the Geiger-Nuttall law [1], or its generalization by Viola and Seaborg [2] are very useful in this respect. Another aspect is that the most studied alpha decay processes are those implying the unhindered transitions (with $L = 0$), that are mostly measured for the ground state to ground state transition of even-even nuclei. By contrast, the population of excited states in the daughter nuclei (the fine structure of $\alpha$-decay), especially for non-zero spin states is much less studied both experimentally and theoretically.

In the present work we review the up-to-date experimental data on the $\alpha$-decay fine structure for the even-even nuclei above lead. We propose a systematic of the hindrance factors known for excited states based on the $N_pN_n$ scheme [3], and highlight a striking behavior of the hindrance factors for the lowest members of the ground state band of the rotational nuclei from this region.

2. THE ANALYZED EXPERIMENTAL DATA

The considered alpha-decay data were those of the even-even parent nuclei with $Z \geq 84$ for which at least the branch for the $2^+_1$ state was measured besides that of the...
ground state. In addition to the $2^+_1$ state, other states considered in our analysis were the $4^+_1$ and $6^+_1$ states that were measured in most of the rotational nuclei, and also the $0^+_2$ and $1^-_1$ states (less numerous data). For all these states we have examined the measured hindrance factors. Systematics of these quantities were published before by Ellis and Schmorak [4] (for $A \geq 229$), and by Akovali [5], but we have also checked these experimental data with the most recent ENSDF evaluations [6].

The hindrance factors $HF$ in the alpha decay have the following definition:

$$HF = \frac{T_{1/2}(\alpha)_{\text{exp.}}}{T_{1/2}(\alpha)_{\text{theor.}}}$$

where $T_{1/2}(\alpha)_{\text{exp.}}$ is the partial half-life for the excited state having a given $\alpha$-decay branch ($T_{1/2}(\alpha)_{\text{exp.}} = T_{1/2}/\alpha$-branch). Because they comprise a theoretically calculated quantity, they are model dependent. In the convention adopted by the ENSDF (therefore, for all $HF$'s considered by us) the theoretical half-life $T_{1/2}(\alpha)_{\text{theor.}}$ is obtained from the spin-independent equations of Preston [7]. Moreover, for the even-even nuclei, the $HF$ for the ground state to ground state ($0^+ \rightarrow 0^+$) transition is taken 1 by definition, therefore all other hindrance factors are relative to this value. By this procedure, the hindrance factors are basically an $\alpha$-decay half-life corrected for the barrier penetration, leaving only the dependence on the structure of the initial and final states. This makes the $\alpha$-decay a strong spectroscopic tool for investigating the properties of the nuclear states.

In total, we have found experimental $HF$ values for 62 $2^+_1$ states, 29 $4^+_1$ states, 17 $6^+_1$ states, 11 $0^+_2$ states, and 15 $1^-_1$ states, respectively, in daughter nuclei with $Z \geq 82$. Hindrance factors are also known for other excited states in this set of considered nuclei, but in smaller numbers.

3. SYSTEMATICS OF THE HINDRANCE FACTORS

In this section we examine the evolution of the experimental hindrance factors from the set defined above, as a function of different structure parameters. Fig. 1 shows the $HF$ values for the first excited state (the $2^+_1$ state). The upper panel displays the evolution with the mass number $A$ of the daughter nucleus. It shows a rather smooth evolution, at least above $A = 220$, but this representation has the disadvantage that it does not distinguish between isobars.

We have represented the same $HF$ values against two other quantities, known to describe very well the development and evolution of collectivity over wide nuclear regions. The first of these is the product $N_pN_n$, between the numbers of active protons ($N_p$) and neutrons ($N_n$), as counted with respect to the nearest magic number. This product, roughly proportional to the strength of the neutron-proton interaction, was shown to give very compact trajectories for the evolution of many structure indicators, such as the ratio $R(4/2) = E(4^+_1)/E(2^+_1)$, the electromagnetic transition...
probability \( B(E2; 2^+_1 \rightarrow 0^+_1) \), etc. \([3, 8]\). It can be seen (the middle panel of Fig. 1) that the representation after \( N_pN_n \) is useful, because at least for the collective nuclei \((N_pN_n \) larger than about 80) all values follow a rather compact trajectory (in this case, an exponential increase). A similar representation is shown in the bottom panel, against the quantity \( P = N_pN_n/(N_p + N_n) \), which roughly represents the ratio between the strengths of the neutron-proton and pairing interactions, and is also a very useful parameter, giving rather compact trajectories for the evolution of various structure indicators over different mass regions \([9]\). One sees that the \( P \)-representation of the \( HF \)-values also leads to a nice exponential trajectory for \( P \)-values above 3–4 that characterize the collective nuclei \([9]\). One should emphasize that in Fig. 1 the number of active nucleons (particles or holes) were counted for the daughter nuclei, with respect to the shell gap numbers 82 and 114 for protons, and 126 and 184 for neutrons, respectively.

In Fig. 2 we compare the two representations \((N_pN_n \) and \( P \)\) for the hindrance factor of the \( 2^+_1 \) state, in both situations, when we count the number of active particles either in the daughter nuclei, or in the parent ones. It appears that counting the number of active particles in the daughter nuclei provides slightly smoother trajectories, therefore in the following we will adopt this choice.

Fig. 3 shows the systematic of the \( HF(4^+_1) \) values. Again, the \( N_pN_n \) or the \( P \)-representations lead to relatively smooth trajectories, emphasizing that this way of representing the data may be valuable for predicting expected values for other nuclei. It is worth to mention that the behavior for the \( 4^+_1 \) state is completely different from the one of the \( 2^+_1 \) state (Fig. 1).

In the following, we adopt these two schemes (representations) also for other states.

4. DISCUSSION

Figure 4 shows a comparison of the evolution of the known \( HF \)-values for the \( 2^+_1, 4^+_1 \) and \( 6^+_1 \) states from the ground state band, in the \( N_pN_n \)-scheme. Fig. 5 shows the same data, but represented in the \( P \)-scheme. In both cases, the three states show very different behaviors. As remarked above, the \( 2^+ \) state shows a practically exponential increase for the deformed nuclei (large \( N_pN_n \), or \( P \) values). Over the same interval of \( N_pN_n \) or \( P \)-values, the \( 4^+ \) state shows a marked maximum (around \( N_pN_n \approx 250 \), corresponding to \( P \approx 7.5 \)). Between \( P \approx 4 \) and the maximum at \( P \approx 7.5 \), \( HF(4^+_1) \) increases by almost two order of magnitudes, very different from the increase by a factor of about 3 of the \( 2^+ \) state. The \( 6^+ \) state shows yet a different behavior, in the \( P \) interval from about 4 to 7 being out of phase with the \( 4^+ \) state: it decreases by more than one order of magnitude, with a possible increase for \( P > 7 \). These different behaviors of the three states are also clear in the representations as a
function of the mass number shown in the systematic survey of Ref. [4].

Fig. 6 shows the $P$-systematics of the $HF$ for the second excited $0^+$ state ($0^+_2$) and the $1^-_1$ state. The number of such states with measured $HF$ is smaller in this case therefore it seems premature to speculate about the occurrence of certain structures.

The very different behaviors of the $HF$ values of the $2^+$, $4^+$, $6^+$ states from the ground state band is so conspicuous that it calls for a simple explanation, or some correlation with the evolution of other structure indicators of these nuclei. Efforts in this direction were recently made in Refs. [10, 11], where alpha-decay widths for the $2^+$ and $4^+$ states of nuclei in this mass region were calculated within the stationary coupled channels approach, describing the collective states with the rigid rotor model. A good description for the decay widths of the $2^+$ state was obtained [10], while for the $4^+$ state a good agreement was obtained only for the $Z = 90$ isotopes but the calculations fail by about 1.5 orders of magnitude in the region of the peak observed around $Z = 94$ (corresponding to the peak in Figs. 4 and 5). In Ref. [11], interesting correlations were observed between the alpha-decay widths and the deformation parameters predicted by the calculations of [12]. The decay widths for the $2^+$ states were found rather well correlated with the quadrupole deformation parameter $\beta_2$. A certain correlation is also observed between the decay widths of the $4^+$ state and the hexadecapole deformation parameter $\beta_4$ predicted in [12], but not as good as in the $2^+$ state case, and the large discrepancy between experiment and theory in the region of the peak at $Z \approx 94$ still remained.

In the following we discuss the properties of the nuclei from this region by examining some of their low-excitation energy observables. Figure 7 shows such quantities, either experimentally measured or derived from the energies of the ground state band. One of these is the moment of inertia (MoI). Fig. 7(a) shows the experimental MoI as derived from the energy of the $2^+_1$ state with the rotor model formula $[J = \hbar^2 I(I + 1)/2E(2^+_1)]$, normalized to the MoI calculated for a rigid ellipsoid having a $\beta_2$ deformation equal to either the experimental value (as extracted from the tables of [13]) or, when the experimental value was not known, as predicted by [12]. One can see that roughly after the $P = 6$ value the moment of inertia stabilizes at a maximum value which is about half the rigid-body value, similar to the situation from the deformed rare-earth nuclei (fig. 11.2 of [14]). More details about how the rotational properties of these nuclei change with the number of nucleons can be inferred from the graphs (b) and (c) of Fig. 7, which show the two Harris parameters $J_0$ and $J_1$ resulted from a fit of the g.s.b. energies with the VMI formula [15]:

$$E = \frac{1}{2} \omega^2 (J_0 + \frac{3}{2} \omega^2 J_1)$$

Graph 7(b) shows that $J_0$ is very close to the MoI $J$ displayed in Fig. 7(a). The $J_1$ parameter is related to the rigidity of the nucleus, relatively small $J_1$ values associ-
ated with large $J_0$ values indicating an increased rigidity (a behavior closer to that of rigid rotor). The variation of $J_1$ shown in graph 7(c) shows that the nuclei with $P$ values above 3 show a rather monotonous decrease of $J_1$, indicating a rigidity that increases with the increase of the $P$-number. A detailed understanding of the behavior of $J_1$ is more difficult to achieve (see, for example, the discussion in [16]), but, as a partial conclusion, from graphs (a), (b), and (c) of Fig. 7 we can say that above $P \approx 6$, where $J_0$ stabilizes around half the rigid-body value, the nuclei of our set have the largest rigidity (or, they behave almost like a rigid rotor, in the sense that their intrinsic structure changes little with the excitation energy or rotation). The rest of three graphs in Fig. 7 show the evolution of different other structure indicators and strengthen this conclusion. The energy ratio $R(4/2) = E(4_1^+/2_1^+) / E(2_1^+/2_1^+)$ (graph (d)) evolves smoothly with $P$, and the transition towards good rotor ($R(4/2) > 3.27$) takes place around $P = 5$ as stated in [9]. The experimental quadrupole deformation $\beta_2$ (graph (e)) shows a smooth increase with $P$, reaching about 0.25 around $P = 5$. It is seen that $\beta_2$ has a very steep increase with $R(4/2)$ after $R(4/2) \gtrsim 3.27$ (graph (f)).

However, we could not correlate the behavior of the hindrance factors for the $4_1^+$ and $6_1^+$ states with any of these structure indicators. It remains to understand why this peculiar behavior takes place in the region where the nuclei are closest to rigid rotors.

5. CONCLUSIONS

We have studied the evolution of the known hindrance factors in the alpha-decay of the even-even trans-lead nuclei, for several excited states in the daughter nucleus, namely the $2_1^+, 4_1^+, 6_1^+, 0_2^+, \text{ and } 1_1^+$. The main results of this study are the following.

First, for the $2_1^+, 4_1^+, \text{ and } 6_1^+$ members of the ground state bands, the experimental values of the hindrance factors show rather compact systematics within the $N_pN_n$ and $P$ schemes of Casten [3, 9]. These systematics could be used for predicting fine structure alpha-decay branchings in nuclei where they are not measured. They may represent a useful alternative to the Geiger-Nuttall law approach that was recently shown to be valid for the hindered, fine structure transitions as well [17].

Second, the $2_1^+, 4_1^+, \text{ and } 6_1^+$ states show strikingly different behaviors. In particular, the behavior of the hindrance factors for the $4_1^+$ and $6_1^+$ states in the region of the nuclei showing the most rigid-rotor like behavior is very conspicuous and its understanding represents a big challenge for nuclear structure models.

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REFERENCES

5. Y.A. Akovali, Nuclear Data Sheets 84, 1 (1998).
Fig. 1 – Hindrance factors for the $2^+_1$ excited state as a function of mass number $A$ (top panel), the quantity $N_p N_n$ (middle), and $P = N_p N_n / (N_p + N_n)$ (bottom). See text for the definition of $N_p$ and $N_n$. Here the mass number $A$ and the $N_p$ and $N_n$ values are those of the daughter nuclei (see discussion in text).
Fig. 2 – Hindrance factor for the $2_1^+$ state, represented as a function of the product $N_p N_n$, with $N_p$, $N_n$ calculated (a) for the daughter nuclei; (b): for the parent nuclei.
Fig. 3 – Same as Fig. 1, but for the $4^+_1$ state.
Fig. 4 – Comparison of the hindrance factors of the $2^+_1$, $4^+_1$, and $6^+_1$ states as a function of the $N_p N_n$ quantity.
Fig. 5 – Same as Fig. 3, but as function of the $P$ quantity.
Fig. 6 – Hindrance factors of the $0_2^+$ and $1_1^-$ excited states as a function of $P$. 
Fig. 7 – Evolution of different structure indicators of the considered nuclei (that is, those for which at least $HF(2^+_1)$ has been measured). (a) Ratio of the experimental moment of inertia (calculated form the energy of the $2^+_1$ state) and the calculated one (corresponding to the $\beta_2$ quadrupole deformation (either experimental [13], or from [12]); (b) and (c): Harris parameters $J_0$ and $J_1$ from fits to the g.s.b. energies; $J_0$ is also compared to the moment of inertia extracted from the $2^+_1$ energy; (d) $R(4/2) = E(4^+_2)/E(2^+_1)$ as a function of $P$; the experimental $\beta_2$ quadrupole deformation as a function of: (e) $P$, and (f) $R(4/2)$. 