COEXISTENCE OF CLUSTER AND MEAN-FIELD DYNAMICS
AND DUALITY OF MANY-NUCLEON WAVE FUNCTION

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an important link between these different states. It is the duality of wave functions
of ground states and some excited states which have both clustering and mean-field-type
characters. The duality is clearly seen in observed large monopole transitions between
cluster states and ground state. The existence of cluster states in addition to mean-field-
type states can be said to be an inevitable consequence of the duality of the ground state.
We demonstrate this fact by the AMD reproduction of tremendously many observables
up to $^{44}$Ti. The duality of the ground state can also be seen in nuclear reactions. We
also discuss that the duality of the nuclear wave function is important for understanding
cluster-gas states and the liquid-gas phase transition, by utilizing the results of AMD
calculation of nuclear caloric curves.

Key words: Coexistence of cluster and mean-field dynamics, duality of many-
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1. INTRODUCTION

When we regard nuclear clustering as being the physics of dynamical assem-
bling and disassembling of nucleons, clustering is a basic nuclear dynamics and ap-
ppears abundantly in many problems of nuclear structure and reactions. The nuclear
clustering is largely because of the saturation property of binding energy and density
of the nucleus which means that nucleons are easy to assemble and disassemble. We
can say that the formation of clusters is a fundamental aspect of nuclear many body
dynamics together with the formation of mean field. The coexistence of mean-field
dynamics and clustering dynamics, which are very different to each other, is a unique
feature of nuclear system.

Although cluster states are very different from mean-field-type states, there is
an important link between these different states. It is the duality of wave functions
of ground states and some excited states. The duality of the wave function means
that the wave function possesses both clustering and mean-field-type characters. A

famous example is the doubly closed shell wave function of \( ^{16}\text{O} \), \((0s)^4(0p)^{12}\). This wave function is known to be equivalent to \(^{12}\text{C} + \alpha\) cluster model wave function \([1]\). The purpose of this paper is to explain and discuss the basic roles of this duality of nuclear wave function for the coexistence of cluster and mean-field dynamics.

We first discuss that the duality is clearly seen in observed large monopole transitions between cluster states and ground state. Then we discuss that the existence of cluster states in addition to mean-field-type states can be said to be an inevitable consequence of the duality of the nuclear ground state. We demonstrate this fact by the AMD reproduction of tremendously many observables up to \(^{44}\text{Ti}\). We also show that the duality of the ground state can also be seen in nuclear reactions. We furthermore discuss that the duality of the nuclear wave function is important for understanding cluster-gas states and the liquid-gas phase transition, on the basis of the results of AMD calculation of nuclear caloric curves.

2. STRONG MONOPOLE TRANSITIONS BETWEEN CLUSTER STATES AND THE GROUND STATE IN SPITE OF LARGE DIFFERENCE OF THEIR STRUCTURES

Cluster states have very different structure from the ground state with mean-field-type structure. For example, the Hoyle state (the \(0^+_2\) state) of \(^{12}\text{C}\) is a \(3\alpha\) gas-like state and its density is about \(1/3\) of the ground state. Also, the 6th \(0^+\) state of \(^{16}\text{O}\) is predicted to be a \(4\alpha\) gas-like state and its density is calculated to be more dilute \([2]\). However, the observed strengths of the monopole \((E0)\) transitions between cluster states and the ground state are large and comparable with the single-nucleon strength in its order of magnitude, in spite of large difference of structures of initial and final states of the transition \([3]\).

The \(E0\) single-nucleon strength is roughly given by \((3/5)R^2\) with \(R\) standing for the nuclear radius. It is obtained by calculating \(\langle u_f | r^2 | u_i \rangle\) by using uniform-density approximation for \(u_f(r)\) and \(u_i(r)\), \(u(r) \approx (3/R^2)^{1/2}\) for \(0 \leq r \leq R\). If we adopt \(R \approx 3\) fm for \(R\) for light nuclei like \(^{12}\text{C}\) and \(^{16}\text{O}\), we have \((3/5)R^2 \approx 5.4\) fm\(^2\). The observed \(E0\) value between the Hoyle state and the ground state in \(^{12}\text{C}\) is \(5.4 \pm 0.2\) fm\(^2\), and that between the first-excited \(0^+\) state and the ground state in \(^{16}\text{O}\) is \(3.55 \pm 0.21\) fm\(^2\). The first-excited \(0^+\) state in \(^{16}\text{O}\) is a well-known cluster state which has the structure of \(^{12}\text{C}(0^+_1) + \alpha(S)\) with \(\alpha(S)\) standing for an \(\alpha\) cluster moving around \(^{12}\text{C}(0^+_1)\) with \(S\)-wave \((L = 0)\). In Ref. \([2]\), it is reported that the calculated \(E0\) transition strength between the ground state and the 6th-excited \(0^+\) state with dominant \(4\alpha\) gas-like structure in \(^{16}\text{O}\) is \(1.0\) fm\(^2\).

It looks very contradictory that we have, on one hand, the large difference of structures between cluster states and the ground state with mean-field-type structure, but, on the other hand, we have the large magnitude of the monopole \((E0)\) transitions between cluster states and the ground state. The reason why it looks contradictory
comes from our way of understanding of cluster states as being described by superpositions of many particle-hole excited configurations which are very much complicated compared with the ground state. Therefore in order to resolve this seeming contradiction we have to abandon above-described way of understanding of cluster states in the language of mean-field theory. In the next section we show that this seeming contradiction is resolved very easily when we notice that the ground-state wave function possesses the dual nature which has long been known as the Bayman-Bohr theorem [1]. This theorem says that the ground-state wave function described by an SU(3) shell-model wave function can be equivalently rewritten by a cluster-model wave function. It means that the ground-state wave function described by an SU(3) shell-model wave function possesses the dual nature of mean-field character and clustering character.

3. DUALITY OF MEAN-FIELD-TYPE CHARACTER AND CLUSTERING CHARACTER POSSESSED BY THE GROUND STATE WAVE FUNCTION AND RESOLUTION OF THE E0 TRANSITION PUZZLE

3.1. 12C CASE

We first explain in the case of 12C the dual character of the ground-state wave function and then, by using this ground state character, give the resolution of the puzzle of the strong monopole transition between the ground and Hoyle states. We know that the main component of the ground-state wave function of 12C is given by the SU(3) shell-model wave function, \(|(0s)^4(0p)^8, (\lambda, \mu) = (04)J = 0\rangle\). According to the Bayman-Bohr theorem, this shell-model wave function can be equivalently rewritten by a 3\(\alpha\) cluster-model wave function as follows,

\[
\begin{align*}
| (0s)^4(0p)^8, (04)J = 0 \rangle &= N_\nu A \{ R_{4,0}(\xi_1, (8/3)\nu) R_{4,0}(\xi_2, 2\nu) \phi(\alpha_1) \phi(\alpha_2) \phi(\alpha_3) \} g(X_G, 12\nu), \\
g(r, \gamma) &= (2\gamma/\pi)^{3/4} \exp(-\gamma r^2), \\
\phi(\alpha_j) g(X_j, 4\nu) &= |(0s)^4\rangle, \quad \text{for nucleons} = 4(j - 1) + 1 \sim 4(j - 1) + 4.
\end{align*}
\]

Here \(\nu\) is the single-nucleon oscillator parameter (\(\nu = m\omega/2\hbar, m=\text{nucleon mass}\)), \(X_G\) is the total center-of-mass coordinate and \(\xi_k\) are Jacobi coordinates of 3 \(\alpha\) clusters, \(\xi_1 = X_1 - (X_2 + X_3)/2, \xi_2 = X_2 - X_3, \) with \(X_j\) standing for the center-of-mass coordinate of \(j\)-th \(\alpha\) cluster. \(R_{N,\ell}(\xi, \mu)\) stands for the harmonic oscillator function with the oscillator parameter \(\mu\), where \(\ell\) is the angular momentum and \(N\) is the number of oscillator quanta \(N = 2n + \ell\) with \(n\) denoting the number of nodes. This equality relation due to the Bayman-Bohr theorem just means that the ground-state wave function of 12C whose dominant component is \(|(0s)^4(0p)^8, (\lambda, \mu) = (04)J = 0\rangle\) has the dual character of mean-field-type character and clustering-type character.
On the other hand, the Hoyle state of $^{12}$C is a $3\alpha$ cluster state and has long been known to be well described by the $3\alpha$ RGM (resonating group method) wave function [4, 6] and $3\alpha$ GCM (generator coordinate method) wave function [5, 6]. These $3\alpha$ RGM and $3\alpha$ GCM wave functions were shown [8] to be almost 100% equivalent to $3\alpha$ THSR wave functions [7], which implies the $3\alpha$ Bose-condensation-like character of the Hoyle state. The $3\alpha$ THSR wave function which is orthogonalized to $|{(0s)^4(0p)^8}, (04)J = 0\rangle$ has the following form,

$$
\Phi_{3\alpha \text{THSR}} = N_\beta A \left[ P_A \exp \left[ -\beta \sum_{j=1}^{3} (X_j - X_G)^2 \right] \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3) \right]
$$

(4)

$$
= N_\beta A \left[ P_A \exp \left[ -\beta((8/3)\xi_1^2 + 2\xi_2^2) \right] \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3) \right],
$$

(5)

where $N_\beta$ is the normalization constant, and $P_A$ stands for the projection operator onto the functional space of $\xi_1$ and $\xi_2$ whose elements have the number of total oscillator quanta larger than 8. Therefore the $E0$ transition between the ground state and the Hoyle state is essentially the $E0$ transition between the $3\alpha$ relative-motion wave functions of the ground state and the Hoyle state;

$$
R_{4,0}(\xi_1, (8/3)\nu)R_{4,0}(\xi_2, 2\nu) \overset{E0 \text{ transition}}{\leftrightarrow} P_A \exp \left[ -\beta((8/3)\xi_1^2 + 2\xi_2^2) \right]
$$

(6)

Actually we can show [9] that in the $E0$ transition operator,

$$
O(E0, ^{12}C) = (1/2) \sum_{i=1}^{12} (r_i - X_G)^2
$$

$$
= O(E0, \alpha_1) + O(E0, \alpha_2) + O(E0, \alpha_3) + (1/2)((8/3)\xi_1^2 + 2\xi_2^2),
$$

(7)

only the relative-motion part, $(1/2)((8/3)\xi_1^2 + 2\xi_2^2)$, contributes to the $E0$ transition. Here $O(E0, \alpha_j)$ is the $E0$ transition operator of the $j$-th $\alpha$ cluster, $(1/2) \sum_{i=k_j+1}^{k_j+4} (r_i - X_j)^2$ with $k_j = 4(j - 1)$.

By using the cluster-model representation of $|{(0s)^4(0p)^8}, (04)J = 0\rangle$, the analytic formula of the $E0$ transition matrix element between $|{(0s)^4(0p)^8}, (04)J = 0\rangle$ and $\Phi_{3\alpha \text{THSR}}$ is calculated to be [9]

$$
M(E0, 0^+_1 \leftrightarrow 0^+_1) = \langle (0s)^4(0p)^8, (04)J = 0 | O(E0, ^{12}C) | \Phi_{3\alpha \text{THSR}} \rangle \times g(X_G, 12\nu) = \sqrt{7/6} \sqrt{\left| \frac{F_4}{F_5} \right|} \sigma_5 \langle R_{40}(r, \nu) | r^2 | R_{60}(r, \nu) \rangle,
$$

(8)

$$
\langle F_n \rangle = \langle Q_n | A \{ Q_n \} \rangle, \quad Q_n = F_n(\xi_1, \xi_2)\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3),
$$

(9)
\[ F_n(\xi_1, \xi_2) = \frac{1}{4\pi} \sum_{n_1+n_2=n} \sqrt{\frac{(2n_1+1)!!(2n_2+1)!!}{2n_1!!2n_2!!}} \times R_{2n_1,0}(\xi_1, (8/3)\nu) \times R_{2n_2,0}(\xi_2, 2\nu), \quad (10) \]

\[ \Phi^{3n_{THSR}} = \sum_{n=5}^{\infty} \sigma_n(e_n A\{Q_n\}), \quad ||e_n A\{Q_n\}|| = 1. \quad (11) \]

The quantity \( \langle F_n \rangle \) represents the effect of the antisymmetrization but in the above analytical formula, it appears in the form of ratio, \( \langle F_4 \rangle / \langle F_5 \rangle \), whose magnitude is close to unity. Thus the effect of antisymmetrization has only little influence on the \( M(E0) \) value. The quantity \( \sigma_5 \) is the amplitude of the 2\( h\omega \)-jump component contained in \( \Phi^{3n_{THSR}} \) and its magnitude is around 0.25. The \( E0 \) matrix element of the relative motion, \( \langle R_{40}(r, \nu)|r^2|R_{60}(r, \nu) \rangle \), is larger than the corresponding \( E0 \) matrix elements of the single-nucleon motion, \( \langle R_{00}(r, \nu)|r^2|R_{20}(r, \nu) \rangle \) and \( \langle R_{11}(r, \nu)|r^2|R_{31}(r, \nu) \rangle \), by about 50%. Thus the order of magnitude of \( M(E0, 0^+_2 \leftrightarrow 0^+_1) \) is the same as the single-nucleon strength.

The numerical value of \( M(E0, 0^+_2 \leftrightarrow 0^+_1) \) calculated with the above formula with suitable parameter values for the wave functions is 1.3 fm\(^2\). This value is surely of the same order of magnitude as the observed value, 5.4 \( \pm \) 0.2 fm\(^2\). In [9] it is reported that the ground-state correlation due to the clustering degree of freedom described by the Bayman-Bohr theorem makes the magnitude of the calculated \( E0 \) matrix element much closer to the observed value.

### 3.2. \( ^{16}O \) CASE

In order to confirm that the arguments in the \( ^{12}C \) case given in the previous subsection are of general character, we here explain, in the case of \( ^{16}O \), the dual character of the ground state and show that the resolution of the puzzle of strong monopole transitions between the ground state and cluster states is obtained by using this dual character of the ground state.

The wave function of the ground state of \( ^{16}O \) has, as its dominant component, \( |(0s)^4(0p)^{12}, J = 0 \rangle = (1/\sqrt{16!}) \det |(0s)^4(0p)^{12} | \) which is the doubly-closed-shell wave function of 0\( s \) and 0\( p \) shells. According to the Bayman-Bohr theorem, this shell-model wave function can be equivalently rewritten by \( ^{12}C + \alpha \) cluster-model wave functions and also by \( 4\alpha \) cluster-model wave functions as follows,

\[ |(0s)^4(0p)^{12}, J = 0 \rangle = (1/\sqrt{16!}) \det |(0s)^4(0p)^{12} | = C_L A \{ R_{4,L}(rC_{-\alpha}, 3\nu)|Y_L(rC_{-\alpha})\phi_L(^{12}C)|J = 0 \phi(\alpha) \} g(X_G, 16\nu), \quad (12) \]

\[ D_{(L_1, L_2, L_0)} A \{ R_{12,J = 0}(\xi_1, \xi_2, \xi_3)\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \} g(X_G, 16\nu), \quad (13) \]
\begin{equation}
R_{L_1,L_2,L_0}^{12,J=0}(\xi_1,\xi_2,\xi_3) = ([R_{4,L_1}(\xi_1,(8/3)\nu)R_{4,L_2}(\xi_2,2\nu)]L_0 R_{4,L_0}(\xi_3,3\nu)]_{J=0}.
\end{equation}

Here \( X_G \) is the total center-of-mass coordinate of \(^{16}\)O and \( \xi_k \) are Jacobi coordinates of \( 4\alpha \) clusters, \( \xi_1 = X_1 - (X_2 + X_3)/2, \xi_2 = X_2 - X_3, \xi_3 = X_4 - (X_1 + X_2 + X_3)/3 \) with \( X_j \) standing for the center-of-mass coordinate of \( j \)-th \( \alpha \) cluster. Here \( L \) is arbitrary among \( L = 0 \sim 4 \), and also any of \( L_1, L_2, \) and \( L_0 \) is arbitrary among \( 0 \sim 4 \).

These equality relations due to the Bayman-Bohr theorem just mean that the ground-state wave function of \(^{16}\)O whose dominant component is \((0s)^4(0p)^{12},J = 0)\) has the dual character of mean-field-type character and clustering-type character.

As cluster states, we here discuss three states, namely \( 0^+_2 \) at 6.05 MeV, \( 0^+_0 \) at 12.1 MeV, and \( 0^+_3 \) at 15.1 MeV. As mentioned before, the \( 0^+_2 \) state is known to have \(^{12}\)C(0\(^+_1\)) + \( \alpha \)(\( S \)) structure where the \( \alpha \) cluster moves around the \(^{12}\)C(0\(^+_1\)) cluster with relative \( S \)-wave [6,10,11]. The observed \( E0 \) transition strength between the \( 0^+_2 \) state and the ground state is as large as 3.55 ± 0.21 fm\(^2\). The \( 0^+_3 \) state is regarded as having \(^{12}\)C(2\(^+_2\)) + \( \alpha \)(\( D \)) structure where the \( \alpha \) cluster moves around the \(^{12}\)C(2\(^+_2\)) cluster with relative \( D \)-wave [2,6,11] and the observed \( E0 \) transition strength between this \( 0^+_3 \) state and the ground state is also large as being 4.03 ± 0.09 fm\(^2\). As for the \( 0^+_0 \) state, experimental information is insufficient and no \( M(E0) \) data are available. But the \( 4\alpha \) OCM study of Ref. [2] temporarily assigns the \( 4\alpha \)-condensate-like structure to this state. The \( 4\alpha \) OCM calculation reproduces the \( 0^+_0 \) excitation energy well and also gives, for the \( 0^+_0 \) level width, 140 keV which is close to the observed value 170 keV. This \( 4\alpha \) OCM calculation gives, as the \( E0 \) transition strength between the \( 0^+_0 \) and the ground states, the value \( M(E0)_{cal} = 1.0 \) fm\(^2\) whose order of magnitude is the same as that of the single-nucleon strength.

3.2.1. \( 0^+_2 \) state

Since \((0s)^4(0p)^{12},J = 0)\) is equivalently rewritten by the cluster-model wave function \( C_0A\{R_{4,0}(r)[Y_0(\vec{r})\phi_0(12\text{C})]\}_{J=0} \phi(\alpha)\), and the \( 0^+_2 \) state is described by \( A\{\chi_0(\vec{r})[Y_0(\vec{r})\phi_0(12\text{C})]\}_{J=0} \phi(\alpha)\}, \) the \( E0 \) transition between the ground and \( 0^+_2 \) states is essentially the \( E0 \) transition between the \(^{12}\)C(0\(^+_1\)) - \( \alpha \) relative wave functions of the ground and \( 0^+_2 \) states;

\begin{equation}
R_{4,0}(r_{C-\alpha},3\nu) \xrightarrow{E0 \text{ transition}} \chi_0(r_{C-\alpha})
\end{equation}

Just like the previous \(^{12}\)C case, we can show [9] that in the \( E0 \) transition operator

\begin{equation}
O(E0)^{16}\text{O} = \frac{1}{2} \sum_{i=1}^{16} (r_i - X_G)^2 = O(E0)^{12}\text{C} + O(E0,\alpha) + \frac{1}{2} \left( 12 \times \frac{4}{16} \right) r_{C-\alpha}^2.
\end{equation}

only the relative-motion part, \( \frac{1}{2} (12 \times 4/16) r_{C-\alpha}^2 \), contributes to the \( E0 \) transition.

By using the cluster-model representation of \((0s)^4(0p)^{12},J = 0)\), which is \( C_0A\{R_{4,0}(r)[Y_0(\vec{r})\phi_0(12\text{C})]\}_{J=0} \phi(\alpha)\}, \) the analytic formula of the \( E0 \) transition
matrix element between \(|(0s)^4(0p)^2, J = 0\) and \(A\{\chi_0(r)[Y_0(\vec{r})\phi_0(^{12}\text{C})]_{J=0} \phi(\alpha)\}\) is calculated to be \([9]\)

\[
M(E0, 0^+_2 \leftrightarrow 0^+_1) = \frac{1}{2} \sqrt{\frac{\tau_{0,4}}{\tau_{0,6}}} \eta_6 \langle R_{40}(r, \nu) | r^2 | R_{60}(r, \nu) \rangle,
\]

\[
\tau_{L,N} = \langle \Psi_{L,N} | A\{\Psi_{L,N}\} \rangle,
\]

\[
\Psi_{L,N} = R_{N,L}(r_{C,-\alpha,3\nu})[Y_L(\vec{r}_{C,-\alpha})\phi_L(^{12}\text{C})]_{J=0} \phi(\alpha),
\]

\[
A\{\chi_0(r)[Y_0(\vec{r})\phi_0(^{12}\text{C})]_{J=0} \phi(\alpha)\} = \sum_{N=6}^{\infty} \eta_N(C_N A\{\Psi_{0,N}\}),
\]

\[
||C_N A\{\Psi_{0,N}\}|| = 1.
\]

The quantity \(\tau_{L,N}\) represents the effect of the antisymmetrization and actually is fairly smaller than unity in general for non-large \(N\). However, in the above analytical formula, quantities \(\tau_{0,N}\) appear in the form of ratio, \(\tau_{0,4}/\tau_{0,6}\), and the magnitude of the ratio is close to unity, which implies that the effect of antisymmetrization has only little influence on the \(M(E0)\) value. The quantity \(\eta_6\) is the coefficient of the \(2\hbar c\) jump component contained in \(A\{\chi_0(r)[Y_0(\vec{r})\phi_0(^{12}\text{C})]_{J=0} \phi(\alpha)\}\), and its magnitude is around 0.4. Note that \(\eta_6\) is not percentage quantity, \((\eta_6)^2\). The \(E0\) matrix element of the relative motion, \(\langle R_{40}(r, \nu) | r^2 | R_{60}(r, \nu) \rangle\) is larger than the corresponding \(E0\) matrix elements of the single-nucleon motion, \(\langle R_{00}(r, \nu) | r^2 | R_{20}(r, \nu) \rangle\) and \(\langle R_{11}(r, \nu) | r^2 | R_{31}(r, \nu) \rangle\), by about 50%. We thus see that the order of magnitude of \(M(E0, 0^+_2 \rightarrow 0^+_1)\) is the same as the single-nucleon strength.

The numerical value of \(M(E0, 0^+_2 \rightarrow 0^+_1)\) calculated with the above formula with suitable parameter values for the wave functions is 1.97 fm\(^2\). This value is surely of the same order of magnitude as the observed value, 3.55 \(\pm\) 0.21 fm\(^2\). In Ref. \([9]\), it is reported that the ground-state correlation due to the \(^{12}\text{C} - \alpha\) clustering degree of freedom described by the Bayman-Bohr theorem makes the magnitude of the calculated \(E0\) matrix element much closer to the observed value.

### 3.2.2. \(0^+_3\) state

The explanation of the large observed \(E0\) transition strength between the ground and \(0^+_3\) states on the basis of the dual nature of the ground state wave function is almost the same as that of the \(E0\) transition between the ground and \(0^+_2\) states given above. As mentioned before, the \(0^+_3\) state is a cluster state described by the wave function \(A\{\chi_2(r)[Y_2(\vec{r})\phi_2(^{12}\text{C})]_{J=0} \phi(\alpha)\}\). On the other hand, as mentioned before, due to the Bayman-Bohr theorem, the doubly-closed-shell wave function,
\[ (0s)^4(0p)^{12}, J = 0 \], can be rewritten in the form of cluster-model wave function, 
\[ C_2A\{R_{4.2}(r)|Y_2(\hat{r})\phi_2(1^{2}C)|\}_{J=0} \phi(\alpha) \}. \] It is to be noted that the cluster-model representation of the doubly-closed-shell wave function adopted here is the type of 
\[ ^{12}C(2^+_1) + \alpha(D) \] while that for the previous \(^{16}O(0^+_2)\) case is the type of \(^{12}C(0^+_1) + \alpha(S)\). The \(E0\) transition between the ground and \(^{0^+_2}\) states is essentially the \(E0\) transition between the \(^{12}C(2^+_1) - \alpha(D)\) relative wave functions of the ground and \(^{0^+_2}\) states;

\[ R_{4.2}(r_{C-\alpha}, 3\nu) \xrightarrow{E0 \text{ transition}} \chi_2(r_{C-\alpha}) \] (23)

As in the previous \(^{0^+_2}\) case, we can show [9] that only the relative-motion part of \(O(E0, ^{16}O)\), \(1/2)(12 \times 4/16)\) \(r_{C-\alpha}^{12}\), contributes to the \(E0\) transition.

By using the cluster-model representation of \(|(0s)^4(0p)^{12}, J = 0\rangle\) of the type of 
\[ ^{12}C(2^+_1) + \alpha(D) \], the analytic formula of the \(E0\) transition matrix element between 
\[ |(0s)^4(0p)^{12}, J = 0\rangle \] and \(A\{\chi_2(r)|Y_2(\hat{r})\phi_2(1^{2}C)|\}_{J=0} \phi(\alpha) \} \) is calculated to be [9]

\[ M(E0, 0^+_2 \leftrightarrow 0^+_1) = \] (24)

\[ = \langle (0s)^4(0p)^{12}, J = 0|O(E0, ^{16}O)|A\{\chi_2(r)|Y_2(\hat{r})\phi_2(1^{2}C)|\}_{J=0} \phi(\alpha) \}\rangle g(X_C, 16\nu) \]

\[ = \frac{1}{2} \sqrt{\frac{\tau_{2.4}}{\tau_{2.6}}} \zeta_6 \langle R_{4.2}(r, \nu) | r^2 | R_{6.2}(r, \nu) \rangle, \] (25)

\[ A\{\chi_2(r)|Y_2(\hat{r})\phi_2(1^{2}C)|\}_{J=0} \phi(\alpha) \} = \sum_{N=6}^{\infty} \zeta_N(D_N A\{\Psi_{2,N} \}), \] (26)

\[ ||D_N A\{\Psi_{2,N} \|| = 1. \] (27)

Like in the \(^{16}O(0^+_2)\) case, in the above analytical formula, quantities \(\tau_{2,N}\) appear in the form of ratio, \(\tau_{2.4}/\tau_{2.6}\), and the magnitude of the ratio is close to unity. \(\zeta_6\) is the coefficient of \(2\)\(\omega\)-jump component of \(A\{\chi_2(r)|Y_2(\hat{r})\phi_2(1^{2}C)|\}_{J=0} \phi(\alpha) \}\), and its magnitude is around 0.4. Note that \(\zeta_6\) is not percentage quantity, \((\zeta_6)^2\). Like in the \(^{16}O(0^+_2)\) case, the \(E0\) matrix element of the relative motion, \(\langle R_{4.2}(r, \nu) | r^2 | R_{6.2}(r, \nu) \rangle\) is larger than the corresponding \(E0\) matrix elements of the single-nucleon motion, \(\langle R_{00}(r, \nu) | r^2 | R_{20}(r, \nu) \rangle\) and \(\langle R_{11}(r, \nu) | r^2 | R_{31}(r, \nu) \rangle\), by about 50%. We thus see that the order of magnitude of \(M(E0, 0^+_2 \rightarrow 0^+_1)\) is also the same as the single-nucleon strength. The numerical value of \(M(E0, 0^+_2 \rightarrow 0^+_1)\) calculated with the above formula with suitable parameter values for the wave functions is 3.89 fm\(^2\) which is surely of the same order of magnitude as the observed value, 4.03 ± 0.09 fm\(^2\).

3.2.3. \(^{0^+_6}\) State

As mentioned before, the dominant structure of the calculated \(^{0^+_6}\) state is the \(4\alpha\)-condensed structure which, in microscopic model, is described by the \(4\alpha\) THSR
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\[ \Phi_{4\alpha}^{\text{THSR}} = A \{ \exp[-\beta \sum_{j=1}^{4} (X_j - X_G)^2] \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \} \] (28)

\[ = A \{ \exp[-\beta((8/3)\xi_1^2 + 2\xi_2^2 + 3\xi_3^2)] \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \}. \] (29)

In the actual calculation, this \( \Phi_{4\alpha}^{\text{THSR}} \) needs to be orthogonalized to the low-lying \( 0^+ \) states, \( 0^+_1 \sim 0^+_5 \). In the \( 0^+_6 \) wave function of \( 4\alpha \) OCM calculation [2], this orthogonality to the \( 0^+_1 \sim 0^+_5 \) states is, of course, automatically fulfilled. We mentioned before that the doubly-closed-shell wave function, \( |0^+_s 4\alpha \rangle \), which is the dominant component of the ground state, can be rewritten in the form of \( 4\alpha \)-cluster-model wave function as \( A \{ R_{0,0,0}^{12,J=0}(\xi_1,\xi_2,\xi_3) \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \} \). Therefore the \( E0 \) transition between the ground and \( 0^+_6 \) states is essentially the \( E0 \) transition between the \( 4\alpha \) relative wave functions of the ground and \( 0^+_6 \) states;

\[ R_{0,0,0}^{12,J=0}(\xi_1,\xi_2,\xi_3) \rightarrow E0 \text{ transition} \exp[-\beta((8/3)\xi_1^2 + 2\xi_2^2 + 3\xi_3^2)]. \] (30)

This situation is quite similar to the \( E0 \) transition between the ground and Hoyle \( (0^+_2) \) states in \( ^{12}\text{C} \). Thus it is quite natural that the \( 4\alpha \) OCM calculation [2] gives, as the \( E0 \) transition strength between the ground and \( 0^+_6 \) states, the value \( M(E0)_{\text{cal}} = 1.0 \text{ fm}^2 \) whose order of magnitude is the same as that of the single-nucleon strength.

4. EXISTENCE OF CLUSTER STATES AS AN INEVITABLE CONSEQUENCE OF THE DUALITY OF THE GROUND STATE

The ground states of \( ^{12}\text{C} \) and \( ^{16}\text{O} \) have dual characters of mean-field-type structure and cluster structure. In the previous section we showed that large magnitudes of observed many \( E0 \) transitions between the ground state and cluster states which are puzzling phenomena can be resolved quite naturally when we utilize the dual character, especially clustering character of the ground state which is usually regarded as having only the mean-field-type character. In resolving the \( E0 \) puzzle, we noticed the fact that the \( E0 \) transition between the ground state and a cluster state can be regarded as being the \( E0 \) transition between the inter-cluster relative wave functions of the ground state and the cluster state. This fact can be rephrased that the \( E0 \) excitation of the cluster state from the ground state is achieved by the \( E0 \) excitation of the inter-cluster relative wave function of the ground state to the inter-cluster relative wave function of the cluster state. We consider that this statement implies that the generation of the cluster state from the ground state is nothing but the excitation of the inter-cluster relative wave function of the ground state to the inter-cluster relative wave function of the cluster state. Thus we can regard that the formation of cluster states is due to the excitation of the clustering degree of freedom possessed by the
ground state due to its dual character. This recognition is in parallel with the usually accepted recognition that the formation of mean-field-type excited states is due to the excitation of the mean-field-type degree of freedom possessed by the ground state. In this way, the dual character of the ground state gives rise to the following two types of excitation,

Excitation of mean-field degree of freedom of the ground state

\[
\Rightarrow \text{mean-field-type excited states}
\]

Excitation of clustering degree of freedom of the ground state

\[
\Rightarrow \text{cluster-type excited states}
\]

We know that the ground state wave function of the self-conjugate 4n nucleus ("N = Z = even" nucleus) with mass number up to at least around 40 has a large component with SU(3) symmetry. The wave function with SU(3) symmetry has dual character of mean-field-type structure and cluster structure, which is due to the Bayman-Bohr theorem. Thus the ground states of these self-conjugate 4n nuclei have both mean-field degree of freedom and clustering degree of freedom. It means that we can expect that there should exist cluster states which are formed by the excitation of clustering degree of freedom in the ground state. Namely the existence of cluster states is an inevitable consequence of the dual character of the ground state. This understanding is ensured to be actually true by the present-day nuclear-structure study [12]. Below we explain actual situations of several self-conjugate 4n nuclei as typical examples which verify our understanding.

4.1. CLUSTER STATES IN $^{16}$O

As shown in (12) and (13), the doubly-closed-shell wave function has $^{12}$C + $\alpha$ nature and $4\alpha$ nature. In section 3.2, we discussed that the $0^+_2$ and $0^+_3$ states can be regarded as being formed by the excitation of the $^{12}$C - $\alpha$ relative motion embedded in the ground state. Structure studies of $^{16}$O have revealed that not only these $0^+_2$ and $0^+_3$ states but also most of excited states of $^{16}$O below about 15 MeV excitation energy can be considered to have $^{12}$C + $\alpha$ clustering structures [6, 11, 12]. This statement is nicely confirmed by Fig. 1 which shows good reproduction of most low-lying levels of $^{16}$O by a microscopic $^{12}$C + $\alpha$ cluster model [6, 11].

4.2. CLUSTER STATES IN $^{20}$Ne

It is known that the ground state wave function of $^{20}$Ne has as its dominant component the SU(3) wave function $|(1s, 0d)^4; [4](\lambda, \mu) = (8, 0) L = 0\rangle$ which has
the following dual character,

\[ |(1s, 0d)^4; [4] (8, 0) L \rangle = \]
\[ C_L A \{ R_{8L} (r_{O-\alpha}, (16/5) \nu) Y_L (r_{O-\alpha}) \phi (16O) \phi (\alpha) \} g(X_G, 20 \nu). \]  

(31)

Therefore we can expect the existence of excited states with \(^{16}\text{O} + \alpha\) cluster structure which are formed by the excitation of the \(^{16}\text{O} - \alpha\) relative motion embedded in the ground state. Actually the excited rotational band states with \(K^\pi = 0^-\) and \(K^\pi = 0^+\) are considered to have \(^{16}\text{O} + \alpha\) cluster structure [6, 12]. Fig. 2 shows good reproduction of \(^{20}\text{Ne}\) spectra by AMD [13] including \(K^\pi = 0^-\) and \(K^\pi = 0^+\) bands with \(^{16}\text{O} + \alpha\) cluster structure. In this figure the \(K^\pi = 2^-\) band is of mean-field-type structure.

4.3. CLUSTER STATES IN \(^{28}\text{Si}\)

It is well-known that there exists the coexistence of oblate and prolate deformations in the low excitation energy region of \(^{28}\text{Si}\) [14]. The ground state is oblate and the dominant component of its wave function is \(|(1s0d)^{12}[444] (0, 12) J \rangle\). This SU(3) wave function with \((\lambda, \mu) = (0, 12)\) has dual character and can be equivalently
rewritten by the $^{24}\text{Mg} + \alpha$ cluster wave function,
\[
|\{(1s0d)^{12}[444]\}(0,12)J\rangle = a_J A \left\{ |R_8(r_{\text{Mg-\alpha}},(24/7)\nu)\phi_{(8,4)}(^{24}\text{Mg})\rangle_{(0,12),J}\phi(\alpha)\right\} g(X_G, 28\nu),
\]
where $|R_8(r)\phi_{(8,4)}\rangle_{(0,12)}$ stands for the SU(3) coupling of $(8,0) \times (8,4) \rightarrow (0,12)$. On the other hand, the prolate deformation appears in the $K^\pi = 0^+$ excited band with the band-head $0^+$ state at 6.7 MeV. The dominant component of its wave function is $|\{(1s0d)^{12}[444]\}(12,0)J\rangle$. This SU(3) wave function with $(\lambda,\mu) = (12,0)$ has dual character and can be equivalently rewritten by the $^{16}\text{O} + ^{12}\text{C}$ cluster wave function,
\[
|\{(1s0d)^{12}[444]\}(12,0)J\rangle = b_J A \left\{ |R_{16}(r_{\text{O-C}},(48/7)\nu)\phi_{(0,4)}(^{12}\text{C})\rangle_{(12,0),J}\phi(16\text{O})\right\} g(X_G, 28\nu).
\]
Experimental studies have reported the existence of many states of $^{24}\text{Mg} + \alpha$ clustering around and above 10 MeV excitation energy. The AMD calculation [15] gives many $^{24}\text{Mg} + \alpha$ cluster states in the same excitation-energy region. We can regard that these cluster states are formed by the excitation of the $^{24}\text{Mg} - \alpha$ relative motion embedded in the ground state with oblate deformation. Very important cluster states in $^{28}\text{Si}$ are the $^{16}\text{O} + ^{12}\text{C}$ molecular resonance states which are observed around the grazing trajectory of $^{16}\text{O} + ^{12}\text{C}$. We can regard that these $^{16}\text{O} + ^{12}\text{C}$ cluster states are formed by the excitation of the $^{16}\text{O} - ^{12}\text{C}$ relative motion embedded in the excited band states with prolate deformation with the band-head $0^+$ state at 6.7 MeV. Actually the microscopic calculations of $^{16}\text{O} + ^{12}\text{C}$ cluster model in [16, 17] reproduce simultaneously $^{16}\text{O} + ^{12}\text{C}$ molecular resonances and prolate-deformation band upon the $0^+$ state at 6.7 MeV.
4.4. CLUSTER STATES IN $^{32}$S

Cluster states studied extensively by experiments in $^{32}$S are $^{16}$O + $^{16}$O molecular resonance states. These cluster states, however, cannot be considered as being formed by the excitation of the clustering degree of freedom from the ground state. It is because the ground state of $^{32}$S does not contain the $^{16}$O + $^{16}$O clustering character, which can be easily verified by noting that the lowest total number of oscillator quanta of the $^{16}$O + $^{16}$O clustering wave function is $4\hbar\omega$ higher than that of the $1s0d$-shell configuration ($1s0d$)\textsuperscript{16}. On the other hand, although not yet confirmed experimentally, Hartree-Fock calculations all have predicted the existence of superdeformed rotational band with the band-head around 9 MeV excitation energy [18]. AMD calculation of Ref. [19] has given almost the same result as Hartree-Fock calculations for the superdeformed states.

The superdeformed states are $4\hbar\omega$-jump states with 4 particles raised from $1s0d$-shell to $1p0f$-shell. Their wave functions have dominantly the intrinsic configuration, $(1,1,0)^{-4}(0,0,3)^4 = (1,0,1)^4(0,1,1)^4(0,0,2)^4(0,0,3)^4$. Here $(n_x,n_y,n_z)$ stands for the Cartesian harmonic oscillator single-particle function with $n_k$ being the number of oscillator quanta along $k$ direction ($k = x$, $y$, or $z$). This intrinsic wave function $(1,1,0)^{-4}(0,0,3)^4$ has dual character and can be equivalently rewritten by intrinsic wave function of $^{16}$O + $^{16}$O cluster model,

\[
(0,0,0)^4(1,0,0)^4(0,1,0)^4(0,0,1)^4(0,1,1)^4(0,0,2)^4(0,0,3)^4 = n_A\{X_{(0,0,24)}(r_O\rightarrow O, 8\nu)\phi^{(16)}(16)\}\ g(X_G, 32\nu), \quad (34)
\]

where $X_{(n_x,n_y,n_z)}(r,\gamma)$ stands for the Cartesian harmonic oscillator function with size parameter $\gamma$. From this dual character of the superdeformed state, we consider that the $^{16}$O + $^{16}$O molecular resonance states are formed by the excitation of the $^{16}$O - $^{16}$O relative motion embedded in the superdeformed band states. This understanding of the formation mechanism of the $^{16}$O + $^{16}$O molecular resonance states was verified to be true by the AMD calculation of Ref. [19]. Fig. 3 shows that the GCM calculation along the deformation-energy curve covering the superdeformation region gives rise to the $^{16}$O + $^{16}$O molecular resonance band as the second-excited rotational band indicated as ”$N=28$” in the figure. In this figure there is also shown the result obtained by including $^{16}$O + $^{16}$O Brink wave functions in the GCM basis states. Inclusion of Brink wave functions gives only small change for the energy but the percentage of the $^{16}$O + $^{16}$O component in the wave function increases from 73 \% to 98 \% for the molecular band ($N = 28$). For the superdeformed band ($N = 24$) the percentage increase is as small as from 42 \% to 44 \%.
4.5. CLUSTER STATES IN $^{40}$Ca

The wave function of the ground state of $^{40}$Ca has, as its dominant component, $\left| (0s)^4 (0p)^{12} (1s0d)^{24}, J = 0 \right\rangle$ which is the doubly-closed-shell wave function of 0s, 0p, and 1s0d shells. According to the Bayman-Bohr theorem, this shell-model wave function has dual character and it can be equivalently rewritten by $^{36}$Ar + $\alpha$ cluster-model wave functions and also by $^{28}$Si + $^{12}$C cluster-model wave functions as follows,

$$\left| (0s)^4 (0p)^{12} (1s0d)^{24}, J = 0 \right\rangle = \left( \frac{1}{\sqrt{40!}} \right) \det \left| (0s)^4 (0p)^{12} (1s0d)^{24} \right| \tag{35}$$

$$= E_L A \left\{ R_{8,L}(r_{36Ar-\alpha}, (18/5)\nu) [Y_L(\hat{r}_{36Ar-\alpha})\phi_L(36Ar)]_{J=0} \phi(\alpha) \right\} g(X_G, 40\nu),$$

$$= F(L_{1},L_{2},L) A \left[ R_{16,L}(r_{28Si-12C}, (42/5)\nu) [Y_L(\hat{r}_{28Si-12C})\phi_L(28Si)\phi_{L_{2}}(12C)]_{L} \right]_{J=0}$$

$$\times g(X_G, 40\nu). \tag{36}$$

Therefore we can expect the existence of excited states with $^{36}$Ar + $\alpha$ clustering and those with $^{28}$Si + $^{12}$C clustering which are respectively formed by the excitation of the $^{36}$Ar - $\alpha$ relative motion and by that of $^{28}$Si - $^{12}$C relative motion embedded in the ground state. The existence of $^{36}$Ar + $\alpha$ cluster states is supported experimentally especially by the $\alpha$-transfer experiments of Ref. [20]. This Ref. [20] assigned $^{36}$Ar + $\alpha$ clustering to the $K^\pi = 0^+$ band upon $0^+_2$ state at 3.35 MeV and also to the $K^\pi = 0^-$ band upon $1^-_2$ state at 8.63 MeV whose band-member states are fragmented. As for the excited states with $^{28}$Si + $^{12}$C clustering, the AMD study of Ref. [21] reports that the calculated wave functions corresponding to the superdeformed band upon $0^+_3$ state at 5.21 MeV have large overlap with $^{28}$Si + $^{12}$C cluster wave functions although density distribution of intrinsic state does not show clear spatial clustering.
4.6. CLUSTER STATES IN $^{44}$Ti

The ground-state wave function of $^{44}$Ti has a large component of SU(3) wave function $|\langle 1p0f \rangle^4[4](\lambda, \mu) = (12, 0, L = 0)\rangle$. This SU(3) wave function has dual character and it can be equivalently rewritten by $^{40}$Ca + $\alpha$ cluster-model wave function as follows,

$$|\langle 1p0f \rangle^4[4](12, 0, L = 0)\rangle = D_L A \{ R_{12,L}(r_{Ca-\alpha}, (40/11)\nu) Y_L (\hat{r}_{Ca-\alpha}) \phi(40Ca) \phi(\alpha) \} g(X_G, 40\nu). \quad (37)$$

Therefore we can expect the existence of excited states with $^{40}$Ca + $\alpha$ clustering which are formed by the excitation of the $^{40}$Ca - $\alpha$ relative motion embedded in the ground state. Actually the $\alpha$-transfer experiments of Ref. [20] assigned $^{40}$Ca + $\alpha$ clustering to the $K^\pi = 0^-$ band upon $1^-$ state at 7.16 MeV and also to the $N = 14$, 15 higher-nodal bands where $N$ stands for the number of $2n + L$ ($n =$ number of nodes) of the $^{40}$Ca - $\alpha$ relative motion. The AMD calculation of $^{44}$Ti of Ref. [22] supports these experimental assignments as shown in Fig. 4. It is to be noted that the AMD calculation, on the other hand, supports the usual assignment for the low-lying $K^\pi = 3^-$ band that it has mean-field-type structure with 1-particle 1-hole excitation of the ground state and also the assignment for the $K^\pi = 0^+2$ and $K^\pi = 2^+$ bands that they have superdeformed structure formed by 4-particle jump from the ground state.

Fig. 4 – Energy spectra of $^{44}$Ti by AMD [22]. $\langle E_{\text{threshold}} \rangle_{\text{exp}}$ for $^{40}$Ca + $\alpha$ breakup is 5.14 MeV.
5. THE DUALITY OF THE GROUND STATE APPEARING IN NUCLEAR REACTIONS

The dual character of the ground state has been used as an ordinary knowledge in the description of cluster transfer reactions. In the case of the α-transfer reactions of \(^6\text{Li}, d\) and \(^7\text{Li}, t\), the ground states of \(^6\text{Li}\) and \(^7\text{Li}\) are treated as having clustering character, α + d and α + t, respectively, but at the same time they are regarded as having mean-field-type character, \((0s)^4(0p)^2\) and \((0s)^4(0p)^3\), respectively. The mathematical proof of this dual character is of course given by Bayman-Bohr theorem. When we consider, as another example of cluster transfer reaction, the α-transfer reaction of \(^{16}\text{O}, ^{12}\text{C}\), we notice that the doubly-closed-shell wave function of \(^{16}\text{O}\) is treated as having equivalently clustering character of \(^{12}\text{C} + \alpha\).

In the α-transfer reaction, for example \(A + ^6\text{Li} \rightarrow B + d\), the initial state of the α cluster in the ground state of \(^6\text{Li}\) is excited into the final state of the α cluster in the nucleus \(B = A + \alpha\). This process is somewhat similar to the \(E0\) excitation from the ground state to excited α-clustering states where the initial state of the α cluster in the ground state is excited into the final state of the α cluster in excited α-clustering states.

There are many other reaction processes in which the dual character of the ground state of a colliding nucleus play an important role. Examples are the breakup reaction into α + others, the knockout reaction of α(s) such as quasi-elastic process, the dynamical (non-statistical) fragmentation reaction, and so on. In Ref. [23] the AMD calculations of \(^{12}\text{C}\) fragmentation are reported, where it is discussed that, for non-high incident energy lower than about 50 MeV/nucleon, the \(^{12}\text{C}\) nucleus is fragmented into \(3\alpha\)'s via the excitation process of the ground state into the \(3\alpha\) clustering excited states around 10 MeV excitation energy.

6. CLUSTER-GAS STATE, LIQUID-GAS PHASE TRANSITION, AND THE DUALITY OF COMPACT NUCLEAR STATES

We now know that the Hoyle state has a \(3\alpha\)-condensate-like structure, the lowest-energy state of cluster-gas-like states in \(^{12}\text{C}\). It has long been discussed that the nucleus undergoes the liquid-gas phase transition, where the gas phase means the gas of nucleons. The excitation energy of the nucleon-gas state of mass-number \(A\) nucleus is about 8.4 MeV and this high excitation energy makes the nucleon-gas state as the subject of nuclear matter and nuclear reaction rather than nuclear structure. On the other hand, the gas state of clusters is not so highly excited, and can be a discrete state accessible spectroscopically. This situation is shown in Fig. 5 for \(^{12}\text{C}\).

The \(\alpha\) cluster gas state discussed above is a state of almost zero temperature. We consider that, in general, the lowest-energy spatially-localized cluster states, \(C_1 + C_2 + \cdots C_n\), are states of almost zero temperature. If we can assume weak inter-cluster interaction, the lowest-energy spatially-localized state formed by \(n\) clusters
Fig. 5 – Excitation energies of $\alpha$-cluster gas state and nucleon gas state in the case of $^{12}$C. C$_1$, C$_2$, ⋅⋅⋅ C$_n$ is located near the breakup threshold of C$_1$ + C$_2$ + ⋅⋅⋅ C$_n$. The highest-energy breakup threshold is of course the full dissociation threshold into $A$ nucleons. The lowest-energy cluster structure of C$_1$ + C$_2$ + ⋅⋅⋅ C$_n$ can not be liquid-like, because a liquid-like compact cluster structure is just equivalent to a mean-field-type structure and hence is not spatially-localized cluster structure. This is just the dual character of nuclear wave function of liquid-like compact cluster structure. When the temperature is raised from zero, two types of excitation occur: one is the mean-field-type excitation of individual clusters with liquid nature and the other is the excitation of inter-cluster relative motions. For the understanding of the configuration of nuclear system at finite temperature, the investigation of the caloric relation is useful.

Fig. 6 gives the constant-pressure caloric curve of $^{36}$Ar ($N = Z = 18$) calculated with AMD [24]. We see that the figure shows clearly the existence of nuclear liquid-gas phase transition in finite nucleus. We see the existence of the region of negative heat capacity which is a characteristic feature of finite systems [25] and which corresponds to the plateau region of zero heat capacity of the constant-pressure caloric curve in the case of infinite nuclear matter. Fig. 7 shows fragment mass distribution for the ensembles along the $P = 0.05$ MeV/fm$^3$ line. There are drawn two lines corresponding to two choices of inter-nucleon distance $r_{\text{cluster}}$ (2.5 fm and 3.0 fm) for identifying clusters. Namely, if the distance between two nucleons is smaller than $r_{\text{cluster}}$, they are regarded as belonging to the same cluster. When the energy is low ($E^*/A = 8$ MeV), the distribution shows U-shape with two peaks so that the typical configuration at this energy is a large nucleus coexisting with a few gaseous nucleons. When the energy is increased ($E^*/A = 12 \sim 16$ MeV), the peak at the large fragment becomes smaller and the distribution changes into shoulder-like and power-law-like shapes. Thus complex configurations with many intermediate and light mass frag-
ments are typical at these energies, and the proportion of light fragments increases as the energy increases (from $E^*/A = 12$ to 16 MeV). When the energy is sufficiently high ($E^*/A = 20$ MeV), the distribution changes into the exponential shape, which can be interpreted as the nucleons are moving almost freely at this energy although a few nucleons come close and make compounds with small probability. It is to be noted that, while the plateau region of the infinite nuclear matter is composed of simple mixture of liquid and nucleon-gas, the transition region of finite nucleus with negative heat capacity is composed of various fragments.

When the pressure $P$ becomes smaller, the caloric curve (before reaching the nucleon-gas phase with $E^*/A = (E^* + E_{\text{gas}})/A = (3/2)T$) comes down towards the zero temperature line (abscissa) Therefore the nuclear configuration along the caloric curve for low pressure has close relation with the nuclear configuration along the zero temperature line. We see that the multiplicity distributions of clusters along a caloric curve shown in Fig. 7 already looks consistent with the distribution of breakup thresholds into clusters.

![Caloric curve](image.png)

Fig. 6 – The constant pressure caloric curve for $^{36}$Ar obtained by AMD. The lines correspond to the pressure $P = 0.02, 0.03, 0.05, 0.07, 0.10, 0.15, 0.20, 0.25, 0.30, \text{ and } 0.40 \text{ MeV/fm}^3$ from the bottom. $E^*$ stands for the excitation energy and $A = 36$. The curves $E/A = T^2/(8 \text{ MeV})$ and $E/A = T^2/(13 \text{ MeV})$, and the line $E/A = (E^* + E_{\text{gas}})/A = (3/2)T$, are drawn for comparison.
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7. SUMMARY

Although cluster states are very different from the ground state in structure, $E^0$ transitions between cluster and ground states are strong in general having comparable magnitude with single-nucleon strength.

Duality of mean-field-type structure and cluster structure possessed by the ground state explains the larger $E^0$ transitions of cluster states to the ground state.

Existence of cluster states is an inevitable consequence of the duality of the ground state and some excited mean-field-type states. This statement is supported by AMD studies of many nuclei.

The duality of the ground state shows up in several types of nuclear reactions.

The formation of cluster-gas states and other spatially localized cluster states is closely related to the phase transition from the liquid phase to the transition region (with negative heat capacity) composed of various fragments.

REFERENCES