Super-Čerenkov Radiations (SČR), as well as the SČR-signatures in dense media are reviewed. Two general SČR coherence conditions as two natural extremes of the same spontaneous particles in (dielectric, nuclear and hadronic) media are underlined. The main results on the quantum theory of the SČR exotic decays as well as the results of the first experimental test of the super-coherence conditions, and also a test of the anomalous Čerenkov rings, obtained by using the experimental data from BNL and CERN, respectively, are presented.

Key words: Čerenkov radiation, Super-Čerenkov effect, anomalous Čerenkov rings, nuclear pionic Čerenkov-like radiation (NPIČR), particle refractive index.

1. ČERENKOV RADIATION (ČR)

The classical theory of the radiation emitted by charged particles moving with superluminal velocities were traced back to Heaviside [1].

Therefore, Heaviside considered the Čerenkov radiation [2] in a nondispersive medium. He considered this topic many times over the next 20 years, deriving most of the formalism of what is now called Čerenkov radiation and which is applied in the particle detectors techniques (e.g., RICH-detectors). So, doing justice (see the papers of Kaiser and Jelley in Nature) to Heaviside [1] and Sommerfeld [3], we must recall that the classical theory of the ČR phenomenon in a dispersive medium was first formulated by Frank and Tamm in 1937 [4]. This theory explained all the main features of the radiation observed experimentally by Čerenkov [2] (see Fig. 1).
The remarkable properties of the Čerenkov radiation find wide applications in practice especially in high energy physics where it is extensively used in experiments for counting and identifying relativistic particles [5] in the fields of elementary particles, nuclear physics and astrophysics.

A quantum theoretical approach of the Čerenkov effect by Ginsburg [6] resulted in only minor modification to the classical theory (see also the books [7-8]). Some interesting discussions about the predictions and experimental discovery of the Čerenkov radiation can be found in the papers of Kaiser [9], Jelley [10], and Tyapkin [11].

In this paper we adopted the usual system of units from particle physics ($\hbar = c = 1$).

In essence, it was revealed by the Heaviside, Čerenkov, Tamm and Frank that a charged particle moving in a transparent medium with an refractive index, $n$, and having a speed $v_i$ greater than phase velocity of light $v_{ph} = c/n = n_i$ in medium will emit electromagnetic radiation, called Čerenkov radiation (ČR) [2], at an polar emission angle $\theta_{CR}$ relative to the direction of motion given by the relation from Fig. 1. Therefore, the essential characteristic features of ČR-photons can be summarized by Eqs. (1)-(6) from Fig. 1.

---

**Fig. 1 – Schematic description of essential characteristics of Čerenkov radiation.**

- **Emission CR-angle**: $\cos \theta_{CR} = \frac{1}{n_i v_i} \leq 1$
- **Maximum CR-angle**: $\delta_{CR} = \arctan \frac{1}{n_i v_i}$
- **Threshold CR-velocity**: $v_{th} = v_{ph}(\alpha) = \frac{1}{n_i}$
- **Threshold CR-energy**: $E_{CR} = m_i \left( \frac{n_i}{\sqrt{n_i^2 - 1}} \right)$
- **CR-photon's polarization**: $\hat{e}_3 \parallel \hat{E}$, $\hat{q} \equiv \text{plane} (\hat{p}_1, \hat{p}_2, \hat{e})$
- **Number of emitted CR-photons**: $\frac{d^2N}{d\Omega d\lambda} = \frac{2\pi \alpha E^2}{\hbar} \left( 1 - \frac{1}{v_n^2 \lambda^2} \right)$
2. SUPER-ČERENKOV RADIATION (SCR)

Recent experimental observations of the subthreshold [12] and anomalous Čerenkov rings [13] as well as multi-ring phenomena have clarified that some fundamental aspects of the ČR-theory can be considered as being still open and that more theoretical and experimental investigations on the ČR are needed. Then, theoretical investigations (see Refs. [14]-[19]) using the ČR correct kinematics lead us to the discovery that Čerenkov radiation is in fact only the low energy component of a more general phenomenon called by us the Super-Čerenkov radiation (SCR) characterized by the Super-Čerenkov (SCR)-decay condition (see Fig. 3).

\[
\cos \theta_{\text{SC}} = \nu_{xph} \cdot \nu_{xph}, \ x = \text{source particle}
\]  

(1)

*Table 1*

Super Čerenkov Radiation

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i-SCR)</td>
<td>The incident particle-source must be coupled to a specific radiation (electromagnetic, mesonic, fermionic-antifermionic, etc) field (RF)</td>
</tr>
<tr>
<td>(ii-SCR)</td>
<td>The propagation properties of the RF (M-quanta) as well as those of particle source must be modified inside the medium according to Foldy-Lax formula</td>
</tr>
</tbody>
</table>
| (iii-SCR)                  | The particle-source (X) must be moving in the medium with a phase velocity such that the SCR-decay condition: \[
\cos \theta_{\text{SCR}} = \nu_{xph} \cdot \nu_{xph} \leq 1
\]  is fulfilled |
Fig. 2 – The principal signatures of the Super-Čerenkov Radiation (SČR), as they are obtained from the quantum SČR-theory, are summarized (see also Fig. 5 in Ref. [14]).

Indeed, let \( [E_1, p_1, \text{Re} n_1(E_1)], [E_2, p_2, \text{Re} n_2(E_2)], [\omega, k, \text{Re} n_1(\omega)] \) be the energies, momenta, and the refractive indices of particles in given medium from the two-body decay in medium described schematically in Fig. 2. Then, using energy-momentum conservation law

\[
E_1 = E_2 + \omega, \quad p_1 = p_2 + k
\]

we obtain

\[
\cos \theta_{\gamma} = \frac{E_1}{k} \quad \frac{p_1}{p_2} = \frac{D_i - D_i - D_j}{2kp_i},
\]

\[
D_i \equiv E_i^2 - p_i^2, D_j \equiv \omega^2 - k^2
\]

Therefore, identifying the phase velocities in medium as

\[
v_{\gamma ph} = \frac{\omega}{k}, \quad v_{1 ph} = \frac{E_1}{p_1}, \quad v_{2 ph} = \frac{E_2}{p_2},
\]

and considering the second term in Eq.(3) as a small quantum correction from (3) we obtain the condition (1). Hence, the SČR-condition (1) is obtained in a natural way from the energy-momentum conservation law when the influence of medium on the propagation properties of the charged particle is also taken into account and when the quantum corrections are neglected.
The signatures of the SČR-effects are schematically described in Fig. 2:

(i) **Low γ−energy SČR-photons** with the usual polarization properties in the decay plane $Q$, but with an emission angle given in Table 2.

(ii) **High γ−energy SČR-photons** with the polarization perpendicular on decay plane $Q$, with the emission SČR-angle given in Table 2.

**Table 2**
The main predictions of SČR-Quantum Theory for the exotic electromagnetic decays of spin $1/2$-particles in (dielectric, nuclear, hadronic)-media.

<table>
<thead>
<tr>
<th>Predictions</th>
<th>Low γ-energy (LE) SČR-sector</th>
<th>High γ-energy (HE) SČR-sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 SCR-decay angle</td>
<td>$\cos \theta_{\gamma} = v_{\gamma \text{ph}} V_{\gamma \text{ph}} \leq 1$</td>
<td>$\cos \theta_{\gamma 2} = v_{\gamma \text{ph}} V_{\gamma \text{ph}} \leq 1$</td>
</tr>
<tr>
<td>2 Maximum SCR-decay angle</td>
<td>$\theta_{\gamma \gamma} = \arccos \frac{1}{n_1 n_2}$</td>
<td>$\theta_{\gamma 2} = \arccos \frac{1}{n_1 n_2}$</td>
</tr>
<tr>
<td>3 SCR-Threshold velocity</td>
<td>$\gamma_{\gamma} = \frac{1}{n_1 n_2}$</td>
<td>$\gamma_{\gamma} = \frac{1}{n_1 n_2}$</td>
</tr>
<tr>
<td>4 SCR-Threshold energy</td>
<td>$E_{\gamma} = m_0 \sqrt{n_1^2 n_2^2 - 1}$</td>
<td>$E_{\gamma} = m_0 \sqrt{n_1^2 n_2^2 - 1}$</td>
</tr>
<tr>
<td>5 SCR-Photon polarizations</td>
<td>100% ($\tilde{\gamma}_k \parallel Q$)</td>
<td>100% ($\tilde{\gamma}_k \perp Q$)</td>
</tr>
<tr>
<td>6 $\frac{d^2 \mathcal{N}_{\gamma 2}}{d \omega d \alpha}$</td>
<td>$\propto \frac{d^2 \mathcal{N}_{\gamma 2}}{d \omega d \alpha}$</td>
<td>$\propto \frac{d^2 \mathcal{N}_{\gamma 2}}{d \omega d \alpha}$</td>
</tr>
<tr>
<td>7 $\propto \frac{d^2 \mathcal{N}_{\gamma 2}}{d \omega d \alpha}$</td>
<td>$\propto \frac{d^2 \mathcal{N}_{\gamma 2}}{d \omega d \alpha}$</td>
<td>$\propto \frac{d^2 \mathcal{N}_{\gamma 2}}{d \omega d \alpha}$</td>
</tr>
<tr>
<td>8 $\propto \frac{d^2 \mathcal{N}_{\gamma 2}}{d \omega d \alpha}$</td>
<td>$\propto \frac{d^2 \mathcal{N}_{\gamma 2}}{d \omega d \alpha}$</td>
<td>$\propto \frac{d^2 \mathcal{N}_{\gamma 2}}{d \omega d \alpha}$</td>
</tr>
</tbody>
</table>

Hence, the main signatures of the SČR-phenomenon are as follows:

- The SČR-effect in the low energy sector with the SČR-coherence condition (6) instead of ČR-condition (1). So, the subthreshold ČR-effects will be observed down to the SČR-threshold (7). The usual ČR is a limiting process when $\text{Re} n_1 \rightarrow 1$. 
• The SČR-effect in the high energy sector (see Fig. 2) have two main signatures, namely, the secondary SČR-effects responsible for anomalous ČR-rings (see Fig. 2), and possible secondary SČR-effects (produced by high-energy gamma) responsible for the concentric ČR-rings.

3. EXPERIMENTAL EVIDENCES OF SUPER-ČERENKOV RADIATION

3.1. EXPERIMENTAL TESTS OF SUPER-ČERENKOV DECAY CONDITION (1)

Čerenkov radiation is extensively used in experiments for counting and identifying relativistic particles in the fields of elementary particles, nuclear physics and astrophysics.

![Fig. 3 – Čerenkov-ring radii of electrons, muons, pions kaons in a C4F10-Ar (75:25) RICH-counter [12]. The solid curves show the expected ČR-radii for an index of refraction of \( n = 1.00113 \). The shaded regions represent a 5% uncertainty in the absolute momentum scale.](image)

Therefore, the problem of the experimental test of Super-Čerenkov coherence condition (1) is of great interest not only for the fundamental physics but also for practical applications to the particle detection. Such a test was performed by us [14] by using the experimental data of Debbe et al. [12] obtained at BNL with a ring imaging Čerenkov Detector (RICH). The results are presented in Fig. 6.
Fig. 4 – Experimental Čerenkov ring radii of the particles e, μ, π, K, obtained by Debbe et al. [12] with RICH detector, are compared with the theoretical [14] Super-Čerenkov predictions (solid curves), and also with the Čerenkov predictions (dashed curves).

3.2. OBSERVATION OF ANOMALOUS ČERENKOV RINGS AS EVIDENCE OF SUPER-ČERENKOV RADIATION

The Čerenkov radiation caused by relativistic lead ions was studied at SPS CERN [13]. A beam of $^{208}_{82}$Pb$^+$ ions with the energy of 157.7 A GeV was going along the axis of the Čerenkov detector. The Čerenkov light emitted in the radiator (its length along the optical axis is 405 mm) got into the objective of a photocamera
after its reflection in the mirror inclined under 45 degree angle relative to the axis of the radiator (see the original paper [13] for details). A bright narrow ring of the Čerenkov radiation seen on the picture in Fig. 5 is caused by relativistic lead ions. Besides, in this picture we have found hardly noticeable narrow Čerenkov radiation rings of the particles flying out under small angles to the direction of the beam. The calculation of the velocity of these particles in usual way has shown that it corresponds to those of particles moving faster than the light velocity in the vacuum.

The conclusion of Vodopianov et al. [13] is that the large Čerenkov radiation ring, shown as Ring number 7 in Fig. 5, corresponds to a tachyon velocity approximately equal to \( \beta \approx 1.0008 \). The ring diameter of its radiation is approximately two times larger than the ring diameter of the proton radiation at the velocity of motion \( \beta \to 1 \). Totally, seven rings of the anomalous Čerenkov radiation have been found in the three photos [13] (see Fig. 5).

The rings were analyzed by Vodopianov et al. [13] using the standard approximation expression:

\[
\cos \theta = 1/n_\gamma v
\]

instead of the true complete formula

\[
\cos \theta = \frac{1}{n_{Pb^+}(E)n_\gamma(\omega)v}
\]

where \( n_{Pb^+}(E) \) is the refractive index of the lead ions \( ^{208}\text{Pb}^+ \) in medium. So, the authors of Ref. [13] obtained in their table the values for \( \beta n_{Pb^+} \) instead of the values of \( \beta \).

The intensity of the beam Čerenkov radiation was measured and compared with the intensity of the radiation concentrated in anomalous ring. It was shown that the radiation energy concentrated in the anomalous ring is quite large and approximately 0.01 of the energy of the Čerenkov radiation from 208 lead nuclei, which is about 2.7 erg. Totally, seven rings of the anomalous radiation have been found in the three photos. The kinematic parameters for each ring of the anomalous radiation are given in the Table in Fig. 5 (points 1-7), namely: the mean ring diameter, radiation angle, the axis angle (\( \alpha \)) of the radiation cone relative to the direction of the lead ion beam and a relative velocity of the hypothetical particle expressed in the units of the light velocity in the vacuum. For comparison the first line of the Table gives the parameters for the Čerenkov radiation ring caused by the
As is seen from Fig. 5, the slope angle of the particle path towards the beam direction is identified as SCR-angle of the particle source.

Therefore, the observation of the anomalous Čerenkov ring can be interpreted as one of the most important signature of the HE-component of Super-Čerenkov radiation (see again Fig. 5) produced by lead ions in the radiator medium.

4. ASTROPHYSICAL IMPLICATIONS OF SUPER-ČERENKOV PHOTONS

Then, we known in advance that the astonishingly similar character of the pulse profile (see that of the Crab Pulsar) in very different spectral regions of Pulsars can be well accounted for by the two-component SCR model of pulsar emission. So, different strongly correlated (high and low) SCR-bands can be emitted by the same beam of charged particles simultaneously in the same time and space region of a pulsar (see Fig. 6). Therefore, the same “pulse profile” will be obtained as a simple
consequence of the simultaneous emission of all SČR-bands. We also investigated the possibility of SČR-emission from Pulsars in the paper [20]. So, very high energy gamma radiation (VHEGR) ($E_\gamma \geq 100$ GeV) and ultrahigh energy gamma radiations (UHEGR) ($E_\gamma \geq 100$ TeV) are produced by ultrarelativistic charged particles (e.g. electrons, muons, protons, light nuclei, etc.) during their interaction with ambient medium (atoms, nuclei, etc.). Then, we obtained that the SČR predictions are satisfied experimentally to a surprising accuracy by the data from some important VHEGR/UHEGR sources such as Crab and Vela Pulsars, Cygnus X-3 and Vela X-1 binary systems, etc.

![High energy pulse profiles of the Crab pulsar from 0.1 keV up to 10 GeV](image)

Fig. 6 – High energy pulse profiles of the Crab pulsar from 0.1 keV up to 10 GeV (from Kuiper et al., 2001 [21]).
5. MESONIC ČERENKOV-LIKE EFFECTS IN HADRONIC MEDIA

The classical variant of the theory of the mesonic Čerenkov-like radiation in hadronic media [22] was applied to the study of single meson production in the hadron-hadron interactions at high energy. This variant is based on the usual assumption that hadrons are composed from a central core (in which the hadron mass is concentrated) surrounded by a large and more diffuse mesonic cloud (hadronic medium). Then it was shown [22-30] that a hadronic mesonic Čerenkov-like radiation (HMČR) mechanism, with an mesonic refractive index in hadronic medium given by pole approximation, is able to describe with high accuracy the integrated cross section of the single meson production in the hadron-hadron interactions.

The illustration of these important results in Fig. 7 we presented the measured integrated cross section for the process: \( pp \rightarrow pp\pi^0 \) compared [22-30], with the prediction of mesonic Čerenkov-like radiation (HMČR-mechanism). This result was very encouraging for the extension of the Čerenkov-pions analysis (HMČR-variant) to all processes of single meson production in hadron-hadron interaction. Collecting \( \chi^2 / \text{dof} \) for all 139 reactions fitted with the HMČR approach [22-30] we obtained the important results presented in Fig. 7b.

![Fig. 7 – Experimental evidences for Čerenkov mesons in hadronic media: (a) measured integrated cross section for the process \( pp \rightarrow pp\pi^0 \) compared with the prediction of mesonic Čerenkov-like radiation in hadronic media (HMČR-mechanism) [22]; (b) The collection of \( \chi^2 / \text{dof} \) for all 139 reactions fitted with the HMČR approach (see Refs. [22-30] quoted in [19]).](image-url)


Fig. 8 – Experimental evidences for NPIČR-pions in the first pionic-SCR band predicted in Ref. [38].
In 1990–1995, we have extended these ideas to the nuclear media [31-39] where the nuclear pionic (NPIČR) and nuclear gamma Čerenkov radiations (NGČR) should be possible to be emitted from charged particles moving through nuclei with a velocity larger than the phase velocity of photons or/and pions in the nuclear media. The refractive indices of the gamma ($n_\gamma$), meson ($n_\pi$), nucleon ($N_n$), was calculated by using Foldy-Lax formula [39-40] and the experimental pion-nucleon cross sections combined with the dispersion relations predictions, the refractive index of pions in the nuclear media has been calculated [31-35]. Then, the detailed predictions for the spontaneous pion emission as nuclear pionic Čerenkov radiation (NPIČR) inside the nuclear medium are obtained and published in Refs. [37-38].

Moreover, it is important to note that in 1999, G.L.Gogiberidze, E.K. Sarkisyan and L.K. Gelovani [26] performed the first experimental test of the pionic Čerenkov-like effect (NPIČR) in Mg-Mg collisions at 4.3 GeV/c/nucleon by processing the pictures from 2m Streamer Chamber SKM-200. So, after processing a total of 14218 events, which were found to meet the centrality criterion, the following experimental results [41]-[42] presented in Fig. 8.

6. CONCLUSIONS

Theoretical investigations using the ČR correct kinematics lead us to the discovery that ČR is in fact only the low energy component of a more general phenomenon called by us the Super-Čerenkov radiation (SČR) [14]-[18] characterized by the Super-Čerenkov (SČR)-decay condition presented in Fig. 2 and Table 2.

Our theoretical investigations have shown [14]-[18] that the SČR-phenomenon includes in a unified way:
(i) Gamma-Čerenkov radiation including subthreshold ČR, (*LE-component*) (see Fig. 2);
(ii) “Source particle” Čerenkov-like effect (see Fig. 2 and Table 2, *HE-component*);
(iii) Anomalous Čerenkov radiation (secondary anomalous SČR-rings (see Fig. 5)).
The observation of the anomalous Čerenkov rings [32] was interpreted as one of the most important signature of the HE-component of Super-Čerenkov radiation;
(iv) The experimental test of the SČR prediction (i)-(iii) were described in sections 3.1 and 3.2 (see Figs. 4 and 5);
(v) SČR-model can exply the astonishingly similar character of the Crab pulsar pulse profile at all energies since different spectral Super-Čerenkov bands can be simultaneously produced by the same beam of accelerated particles in the same space region of the pulsar (see Fig. 6).
(vi) The experimental evidence [41]-[42] for NPIČR-pions in the first pionic SČR-band (predicted in ref. [38]) is described in section 5 (see Fig. 8).

Finally, we remark that new theoretical investigations as well as accurate experimental measurements of the Čerenkov ring radii (both from LE and HE components of SČR) are needed.
Acknowledgments. This paper is dedicated to Professor Aureliu Sândulescu’s 80 Anniversary, who contributed so much to development of quantum physics on various topics in nuclear physics. I am grateful to Prof. Aurel Sândulescu since he was not only one of the referees of my PhD thesis [22] but also one the active supporters of the developing of SCR theory.

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