TRIPARTITE ENTANGLEMENT SWAPPING OF CONTINUOUS VARIABLES

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We propose a scheme for tripartite entanglement swapping of continuous variables, where one EPR pair and one local W-type entangled state are utilized. Because of the co-existence of both bipartite and tripartite entanglement in a W-type entangled state, one optical beams of the EPR state will be entangled with the output modes displaced by Bob and Claire. Actually, the entanglement swapping processing could be viewed as teleclone of entangled state of continuous variables.

Key words: Continuous variables, Tripartite entanglement swapping, W-type entangled state, EPR pair.

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1. INTRODUCTION

Entanglement is one of the most important features in quantum mechanics. Assisted with entangled state, one can complete many impossible tasks within the classical world, e.g. it can be used as quantum channel for quantum teleportation [1] and quantum dense coding [2], and in quantum computation, such as quantum algorithms [3], quantum cryptography [4] and quantum error correction [5].

Quantum teleportation is a good example manifesting the nonlocality of quantum entanglement. It has attracted much attention right after it was proposed and has already been verified experimentally with both photonic [6] and atomic qubits [7, 8]. It has also been generalized to continuous variables (CV) [9]. Then a protocol for teleporting of quadrature amplitudes of a light field was proposed in 1998 [10]. In the same year, teleporting an arbitrary unknown CV state was realized with bipartite entanglement, generated from two single-mode squeezed vacuum states combined by a beam splitter (BS) [11]. So far, the best fidelity achieved, for teleporting a coherent state input, is $F_{coh} = 0.70 \pm 0.02$ [12], but improvements [13, 14] are still on the way.

At the same time, quantum teleportation in another aspect, was generalized to teleport an unknown entangled state rather than a single-qubit superposition state, which is also called entanglement swapping in Ref. [15]. Pan et al. have experimentally realized the entanglement swapping of the polarization entanglement created by
type-II parametric down-conversion [16]. Continuous-variable entanglement swapping is also an important application for quantum communication and has been investigated in theory [17–19] and experiment [20]. Recently, we have proposed a scheme for controlled entanglement swapping via tripartite GHZ-type entangled state [21]. For the three-body entanglement, it falls into two inequivalent classes [22], i.e., the GHZ- and W-type entangled states. W-type entangled state is a special kind of tripartite entangled state, which has bipartite and tripartite quantum entanglement simultaneously. Consequently, when a W-type entangled state is used in a quantum communication protocol, there will be some new results [23, 24].

Motivated by the discussions mentioned-above, we present a scheme for tripartite entanglement swapping via CV W-type entangled state. The schematic setup is shown in Fig. 1. In our scheme, the W-type entangled state, which has both bipartite (modes $A$, $B$ and modes $A$, $C$) and tripartite (modes $A$, $B$, and $C$) entanglement, and a pair of EPR state (modes $in$ and $ref$) are used. One of the modes of the EPR beam is combined with the mode $A$ of the W-type entangled state. Then, the other two modes of the W-type entangled state that are not combined will be displaced according to the received classical information of the combined mode. By so doing, the entanglement between the two output modes and the mode $ref$ are established. And the entanglement swapping processing is accomplished. It is notable that if one wants to accomplish the entanglement swapping, the fidelity of teleportation of coherent state should surpass the no-cloning limit [12], i.e., $F_{coh}>2/3$. As the only limiting factor for this fidelity is the squeezing degree of the used quantum entangled states, we will give the successful teleportation conditions in terms of these squeezing parameters. Actually, in our scheme, we teleclone one beam of the EPR entangled state to two receivers (Bob and Claire) via W-type entangled state, and the other beam of the EPR state are both entangled with the two outputs displaced by Bob and Claire. In a sense, the swapping processing could be viewed as teleclone of the EPR state.

2. THE PROCEDURE FOR TRIPARTITE ENTANGLEMENT SWAPPING

In order to accomplish the entanglement swapping, a local CV W-type entangled state, involving three entangled optical beams, and a local CV EPR state, involving two entangled optical beams, should be utilized. The EPR state can be generated by combining two squeezed vacuum states with a half BS [25] or from nondegenerate optical parametric amplifier process [26]. The W-type entangled state can be generated by first combining two squeezed vacuum states with a 50:50 BS and then one of the output beams is divided into two beams B and C by another 50:50 BS [27]. In our scheme, the local W-type entangled state consists of modes $A$, $B$ and $C$, which belong to Alice, Bob and Claire, respectively. The EPR state consists of modes $in$ and $ref$; the mode $ref$ is used as a reference, while the mode $in$ will be
combined with mode A and measured by Alice.

First, we teleport the mode in to Bob and Claire via tripartite entanglement swapping, simultaneously. The W-type entangled state and the EPR state, in Heisenberg picture, are \[25, 27, 30\],

\[
\begin{align*}
\hat{x}_A &= \frac{1}{\sqrt{2}} e^{r} \hat{x}_1^{(0)} + \frac{1}{\sqrt{2}} e^{-r} \hat{x}_2^{(0)} \\
\hat{p}_A &= \frac{1}{\sqrt{2}} e^{-r} \hat{p}_1^{(0)} + \frac{1}{\sqrt{2}} e^{+r} \hat{p}_2^{(0)} \\
\hat{x}_B &= \frac{1}{2} e^{+r} \hat{x}_1^{(0)} - \frac{1}{2} e^{-r} \hat{x}_2^{(0)} + \frac{1}{\sqrt{2}} \hat{x}_3^{(0)} \\
\hat{p}_B &= \frac{1}{2} e^{-r} \hat{p}_1^{(0)} - \frac{1}{2} e^{+r} \hat{p}_2^{(0)} + \frac{1}{\sqrt{2}} \hat{p}_3^{(0)} \\
\hat{x}_C &= \frac{1}{2} e^{+r} \hat{x}_1^{(0)} - \frac{1}{2} e^{-r} \hat{x}_2^{(0)} - \frac{1}{\sqrt{2}} \hat{x}_3^{(0)} \\
\hat{p}_C &= \frac{1}{2} e^{-r} \hat{p}_1^{(0)} - \frac{1}{2} e^{+r} \hat{p}_2^{(0)} - \frac{1}{\sqrt{2}} \hat{p}_3^{(0)}
\end{align*}
\]

and

\[
\begin{align*}
\hat{x}_{in} &= \frac{1}{\sqrt{2}} e^{s} \hat{x}_4^{(0)} + \frac{1}{\sqrt{2}} e^{-s} \hat{x}_5^{(0)} \\
\hat{p}_{in} &= \frac{1}{\sqrt{2}} e^{-s} \hat{p}_4^{(0)} + \frac{1}{\sqrt{2}} e^{+s} \hat{p}_5^{(0)} \\
\hat{x}_{ref} &= \frac{1}{\sqrt{2}} e^{s} \hat{x}_4^{(0)} - \frac{1}{\sqrt{2}} e^{-s} \hat{x}_5^{(0)} \\
\hat{p}_{ref} &= \frac{1}{\sqrt{2}} e^{-s} \hat{p}_4^{(0)} - \frac{1}{\sqrt{2}} e^{+s} \hat{p}_5^{(0)}
\end{align*}
\]

where the superscript "(0)" denotes the initial vacuum modes, \( r \) and \( s \) are the squeezing parameters, and operators \( \hat{x} \) and \( \hat{p} \) are the electromagnetic field quadrature amplitudes, i.e. the real part and imaginary part of the mode’s annihilation operator.

Using the sufficient inseparability criteria \[28, 29\] for CV entanglement in Eq. (1), one obtains \[30\]

\[
\begin{align*}
\Delta_{A,B} &= \langle [\Delta(\hat{x}_A - \hat{x}_B)]^2 \rangle + \langle [\Delta(\hat{p}_A + \hat{p}_B)]^2 \rangle \\
&= \left( \frac{1 - \sqrt{2}}{2} \right)^2 e^{2r} + \left( \frac{1 + \sqrt{2}}{2} \right)^2 e^{-2r} + \frac{1}{4} < 1
\end{align*}
\]
Fig. 1 – Schematic setup of tripartite entanglement swapping of continuous variables. Modes A, B and C are initially prepared in W-type local entangled state. Before the detections, mode $in$ is entangled with mode $ref$. The broken line is vacuum mode. BS, AM and PM denote beam splitter, amplitude modulator and phase modulator, respectively.

$$\Delta_{A,C} = \langle \Delta(\hat{x}_A - \hat{x}_C)^2 \rangle + \langle \Delta(\hat{p}_A + \hat{p}_C)^2 \rangle$$

$$= \left( \frac{1 - \sqrt{2}}{2} \right)^2 e^{2r} + \left( \frac{1 + \sqrt{2}}{2} \right)^2 e^{-2r} + \frac{1}{4} < 1$$

This means modes A and B and modes A and C are bipartitely entangled, while modes A, B and C are tripartitely entangled. The inequality can be minimized when $e^{-2r} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$, i.e., $-7.7$ dB squeezing. By using these bipartite entangled modes, we can perform the entanglement swapping among involved regions.

Here, we introduce "Alice", "Bob", "Claire" for illustrating the whole protocol of tripartite entanglement swapping of continuous variables. Initially, Alice, Bob and Claire share the W-type entangled state of modes A, B and C. Alice possesses the modes $in$ and mode $ref$. In order to accomplish the entanglement swapping, Alice should teleport her mode $in$ to Bob and C Claire simultaneously. Thus she first
combines her mode \( \hat{a}_\text{in} \) with mode \( \hat{a}_A \) at a 50/50 BS and obtains the quadratures [25]

\[
\hat{x}_u = \frac{1}{\sqrt{2}} \hat{x}_\text{in} - \frac{1}{\sqrt{2}} \hat{x}_A \\
\hat{p}_v = \frac{1}{\sqrt{2}} \hat{p}_\text{in} + \frac{1}{\sqrt{2}} \hat{p}_A
\]  

(4a) (4b)

Meanwhile, Bob’s mode \( \hat{a}_B \) and Claire’s mode \( \hat{a}_C \) become

\[
\hat{x}_B = \hat{x}_\text{in} - (\hat{x}_A - \hat{x}_B) - \sqrt{2} \hat{x}_u \\
\hat{p}_B = \hat{p}_\text{in} + (\hat{p}_A + \hat{p}_B) - \sqrt{2} \hat{p}_v \\
\hat{x}_C = \hat{x}_\text{in} - (\hat{x}_A - \hat{x}_C) - \sqrt{2} \hat{x}_u \\
\hat{p}_C = \hat{p}_\text{in} + (\hat{p}_A + \hat{p}_C) - \sqrt{2} \hat{p}_v
\]  

(5a) (5b) (5c) (5d)

Then Alice measures the \( x_u, p_v \) for operators \( \hat{x}_u \) and \( \hat{p}_v \) by homodyne detectors and informs Bob and Claire of the measurement results by classical channels. After receiving the classical information from Alice, Bob and Claire can displace his and her modes as

\[
\hat{x}_B \to \hat{x}_\text{out1} = \hat{x}_B + \sqrt{2} g_x \hat{x}_u \\
\hat{p}_B \to \hat{p}_\text{out1} = \hat{p}_B + \sqrt{2} g_p \hat{p}_v \\
\hat{x}_C \to \hat{x}_\text{out2} = \hat{x}_C + \sqrt{2} g_x \hat{x}_u \\
\hat{p}_C \to \hat{p}_\text{out2} = \hat{p}_C + \sqrt{2} g_p \hat{p}_v
\]  

(6a) (6b) (6c) (6d)

where \( g_x \) and \( g_p \) are information gains of classical channels from Alice to Bob and Claire, which we assume to be unity, i.e., \( g_x = g_p = 1 \). Therefore, Bob’s and Claire’s output mode are [25, 30]

\[
\hat{x}_\text{out1} = \frac{1}{\sqrt{2}} e^{+s} \hat{x}_4^{(0)} + \frac{1}{\sqrt{2}} e^{-s} \hat{x}_5^{(0)} + \frac{1}{2} e^{r} \hat{x}_1^{(0)} \\
- \frac{1 + \sqrt{2}}{2} e^{-r} \hat{x}_2^{(0)} + \frac{1}{\sqrt{2}} \hat{x}_3^{(0)}
\]  

(7a)

\[
\hat{p}_\text{out1} = \frac{1}{\sqrt{2}} e^{-s} \hat{p}_4^{(0)} + \frac{1}{\sqrt{2}} e^{+s} \hat{p}_5^{(0)} + \frac{1 + \sqrt{2}}{2} e^{-r} \hat{p}_1^{(0)} \\
- \frac{1 - \sqrt{2}}{2} e^{r} \hat{p}_2^{(0)} + \frac{1}{\sqrt{2}} \hat{p}_3^{(0)}
\]  

(7b)

\[
\hat{x}_\text{out2} = \frac{1}{\sqrt{2}} e^{+s} \hat{x}_4^{(0)} + \frac{1}{\sqrt{2}} e^{-s} \hat{x}_5^{(0)} + \frac{1}{2} e^{r} \hat{x}_1^{(0)} \\
- \frac{1 + \sqrt{2}}{2} e^{-r} \hat{x}_2^{(0)} - \frac{1}{\sqrt{2}} \hat{x}_3^{(0)}
\]  

(7c)
\[ \hat{p}_{\text{out}2} = \frac{1}{\sqrt{2}} e^{-s \hat{p}_4(0)} + \frac{1}{\sqrt{2}} e^{+s \hat{p}_5(0)} + \frac{1 + \sqrt{2}}{2} e^{-r \hat{p}_1(0)} - \frac{1 - \sqrt{2}}{2} e^r \hat{p}_2(0) - \frac{1}{\sqrt{2}} \hat{p}_3(0) \]  

(7d)

Thus, via W-type entangled state, the mode in is teleported to two outputs at Bob’s and Claire’s locations and the two output modes are both entangled with Alice’s mode ref, and mode ref, mode out1 and mode out2 share a tripartite entanglement which could be described as:

\[ \hat{x}_{\text{ref}} = \frac{1}{\sqrt{2}} e^{+s \hat{x}_4(0)} - \frac{1}{\sqrt{2}} e^{-s \hat{x}_5(0)} \]  

(8a)

\[ \hat{p}_{\text{ref}} = \frac{1}{\sqrt{2}} e^{-s \hat{p}_4(0)} - \frac{1}{\sqrt{2}} e^{+s \hat{p}_5(0)} \]  

(8b)

\[ \hat{x}_{\text{out}1} = \frac{1}{\sqrt{2}} e^{+s \hat{x}_4(0)} + \frac{1}{\sqrt{2}} e^{-s \hat{x}_5(0)} + \frac{1 - \sqrt{2}}{2} e^r \hat{x}_1(0) - \frac{1 + \sqrt{2}}{2} e^{-r} \hat{x}_2(0) + \frac{1}{\sqrt{2}} \hat{x}_3(0) \]  

(8c)

\[ \hat{p}_{\text{out}1} = \frac{1}{\sqrt{2}} e^{-s \hat{p}_4(0)} + \frac{1}{\sqrt{2}} e^{+s \hat{p}_5(0)} + \frac{1 + \sqrt{2}}{2} e^{-r \hat{p}_1(0)} - \frac{1 - \sqrt{2}}{2} e^r \hat{p}_2(0) + \frac{1}{\sqrt{2}} \hat{p}_3(0) \]  

(8d)

\[ \hat{x}_{\text{out}2} = \frac{1}{\sqrt{2}} e^{+s \hat{x}_4(0)} + \frac{1}{\sqrt{2}} e^{-s \hat{x}_5(0)} + \frac{1 - \sqrt{2}}{2} e^r \hat{x}_1(0) - \frac{1 + \sqrt{2}}{2} e^{-r} \hat{x}_2(0) - \frac{1}{\sqrt{2}} \hat{x}_3(0) \]  

(8e)

\[ \hat{p}_{\text{out}2} = \frac{1}{\sqrt{2}} e^{-s \hat{p}_4(0)} + \frac{1}{\sqrt{2}} e^{+s \hat{p}_5(0)} + \frac{1 + \sqrt{2}}{2} e^{-r \hat{p}_1(0)} - \frac{1 - \sqrt{2}}{2} e^r \hat{p}_2(0) - \frac{1}{\sqrt{2}} \hat{p}_3(0) \]  

(8f)

Next, we prove the newly set entanglement in detail. If two modes A and B are entangled, they should satisfy \([28–30]\)

\[ \Delta_{A,B} = \langle [\Delta(\hat{x}_A - \hat{x}_B)]^2 \rangle + \langle [\Delta(\hat{p}_A + \hat{p}_B)]^2 \rangle < 1 \]  

(9)

According to Eq. (8), we can obtain

\[ \hat{x}_{\text{ref}} - \hat{x}_{\text{out}1} = \frac{\sqrt{2} - 1}{2} e^{r} \hat{x}_1(0) + \frac{\sqrt{2} + 1}{2} e^{-r} \hat{x}_2(0) - \frac{1}{\sqrt{2}} \hat{x}_3(0) - \sqrt{2} e^{-s} \hat{x}_5(0) \]  

(10a)
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Fig. 2 – $\Delta_{\text{ref, out}}$ versus the squeezing parameters $r$ and $s$, which are of the tripartite entanglement and the EPR states, respectively.

$$\hat{p}_{\text{ref}} + \hat{p}_{\text{out}} = \frac{\sqrt{2} + 1}{2} e^{-r\hat{p}_1} + \frac{\sqrt{2} - 1}{2} e^{r\hat{p}_2} + \frac{1}{\sqrt{2}} \hat{p}_3 + \sqrt{2} e^{-s\hat{p}_4}$$

Assuming all modes have zero mean values and given the fact that $\langle (\Delta \hat{x}_i^{(0)})^2 \rangle = \langle (\Delta \hat{p}_i^{(0)})^2 \rangle = \frac{1}{4}$, we obtain

$$\Delta_{\text{ref, out}} = \langle [\Delta (\hat{\xi}_A - \hat{x}_{\text{out}1})]^2 \rangle + \langle [\Delta (\hat{\xi}_{\text{out}1} + \hat{\xi}_{\text{out}2})]^2 \rangle = \left( \frac{1 - \sqrt{2}}{2} \right)^2 e^{2r} + \left( \frac{1 + \sqrt{2}}{2} \right)^2 e^{-2r} + e^{-2s} + \frac{1}{4}$$

In Fig. 2, we plot $\Delta_{\text{ref, out}}$ versus the squeezing parameters $r$ and $s$. It shows that $\Delta_{\text{ref, out}}$ goes down with both increased squeezing of the tripartite entanglement and the EPR state. We now illustrate it by considering the following two cases for Eq. (11):

1. In the case of infinite squeezing for the EPR state ($s \rightarrow \infty$), $\Delta_{\text{ref, out}(1(out2))} = \left( \frac{1 - \sqrt{2}}{2} \right)^2 e^{2r} + \left( \frac{1 + \sqrt{2}}{2} \right)^2 e^{-2r} + \frac{1}{4} < 1$, $\Delta_{\text{ref, out}(1(out2))}$ can be minimized when $e^{-2r} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$ (-7.7 dB squeezing), which is shown in Fig.3. Namely, the mode $\text{ref}$ and $\text{out1(out2)}$ are entangled, and the entanglement swapping processing is completed.

2. If the tripartite entanglement resource is the best entangled state (corresponds to -7.7 dB squeezing), $\Delta_{\text{ref, out}(out2)} = \frac{1}{2} + e^{-2s}$. $\Delta_{\text{A, out}(out2)}$ decreases with the squeezing parameter of the input EPR state, which is shown in Fig.4. From Fig.4, we find that only in the case of $e^{-2s} < \frac{1}{2}$ or the squeezing parameter $s > \ln 4$, $\Delta_{\text{ref, out}(out2)} < 1$. So the entanglement swapping can be realized for squeezed
Fig. 3 – The relationship of $\Delta_{\text{ref,out}}$ with the quantum channel parameter $r$.

Fig. 4 – The relationship of $\Delta_{\text{ref,out1}}$ with the squeezing parameter $s$.

parameter $s > \ln 4$.

That is to say, we could accomplish tripartite entanglement swapping via W-type CV entangled state. In this course, mode $\text{out1}$ and mode $\text{out2}$ are telecloned from mode $\text{in}$. In a sense, this processing could be viewed as teleclone of the CV EPR state (the entangled state mode $\text{ref}$ and mode $\text{in}$ teleclone to mode $\text{ref}$, mode $\text{out1}$ and mode $\text{ref}$, mode $\text{out2}$).

3. CONCLUSION

In summary, we have proposed a scheme for tripartite entanglement swapping of continuous variables. If the input EPR state is the best entanglement, the swapping processing will be realized. While the EPR state is not perfect, swapping processing will be realized in the condition of $s > \ln 4$. When the W-type entanglement is also not perfect, the swapping will be realized with conditional parameters as plotted in Fig. 2.

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