THE BACKREACTION OF LOCALIZED SOURCES AND DE SITTER VACUA

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Received January 11, 2012

D-branes and orientifold planes are important ingredients in semi-realistic type II string compactifications. Determining their explicit back-reaction on the compactification background poses generically a difficult computational problem. This problem is greatly simplified if one formally assumes the brane charges and masses to be smeared over the whole internal manifold, which corresponds to taking into account their back-reaction only in an averaged sense. I summarize recent progress in the understanding of the range of validity of this smearing approximation and comment on possible implications for attempts to build "classical" de Sitter vacua.

Key words: de Sitter vacua, flux compactifications, D-branes.

PACS: 11.25.Mj, 11.25.Uv, 11.25.Wx.

1. THE "SMEARING" OF D-BRANES AND O-PLANES

D-branes and orientifold planes (O-planes) are indispensable ingredients of all phenomenologically interesting type II string compactifications. They provide chiral matter, contribute to supersymmetry breaking and play an important role for moduli stabilization, *e.g.* via tadpole cancellation conditions for fluxes or by providing sources for non-perturbative scalar potentials.

On the other hand, the presence of D-branes and O-planes in string compactifications also leads to unwelcome technical complications, because they source warp factors, RR-potentials and generically also a non-trivial dilaton profile. This back-reaction on the compactification background can usually not be completely neglected, as this would in general lead to inconsistencies, *e.g.* with the Bianchi identities of the RR-forms. Taking the full back-reaction into account, however, would require solving a complicated coupled system of non-linear partial differential equations and hence seems out of reach, except possibly in a few highly symmetric situations.

A common approach to this problem therefore is to take into account the back-reaction of localized sources only in an averaged or integrated sense so as to avoid at least the most obvious inconsistencies such as, *e.g.*, a violation of the global tadpole cancellation conditions. At the level of the 10D field equations, this averaging corresponds to "smearing" the delta-function-like mass and charge profiles of the sources along the directions transverse to their world volumes.

With smeared sources it is much easier to construct explicit solutions of the 10D supergravity equations (see *e.g.* [1] for several interesting examples), because the warp factor, as well as the sourced RR-potentials and the dilaton, may often be assumed constant. In this case a drastic simplification also occurs with regard to the dimensional reduction, as one does not have to deal with, *e.g.*, the subtleties of "warped effective field theories" along the lines of [2]. Moreover, on group or coset manifolds, the restriction to the left-invariant modes provides a consistent truncation [3] analogous to the constant Fourier modes for a torus compactification if the sources are smeared appropriately. This ensures that solutions found in the lower-dimensional truncation (*e.g.* de Sitter extrema) uplift to full solutions of the 10D theory with smeared sources.

To summarize, the smearing of D-branes and O-planes is a commonly employed simplification to obtain explicit 10D flux compactifications or consistently truncated 4D effective theories. It takes into account some brane back-reaction in an averaged sense, but ignores the *local* back-reaction on warp factor, dilaton or RR-potentials. As real D-branes and O-planes are, however, *not* smeared, it is important to understand in what circumstances the smearing is actually a good approximation and when it is not. This is a particularly pressing problem for recent attempts to build "classical" de Sitter vacua, as we will now explain.

2. CLASSICAL DE SITTER VACUA AND THE DOUGLAS-KALLOSH PROBLEM

While string theory naturally combines many of the theoretical ingredients we find useful for the description of the world we observe, its ability to produce a positive cosmological constant in four dimensions seems less immediate. In fact, there are surprisingly powerful no-go theorems that rule out de Sitter compactifications at leading order in the α' and g_s-expansion (which we will henceforth refer to as "classical" de Sitter vacua) even if the most general fluxes and arbitrary sets of (anti-)D-branes are included [4]. These simplest no-go theorems no longer apply in the presence of objects that violate the strong energy condition. The best known such objects in string theory are orientifold planes, which have negative tension. However, even if one allows for orientifold planes, more refined no-go theorems against classical de Sitter compactifications can be proven, provided one restricts oneself to compactifications with non-negative integrated curvature $\int d^6x\sqrt{-g^{(6)}}R^{(6)} \geq 0$ [5].

In order to construct de Sitter compactifications, one obvious avenue is to go beyond the leading order approximation and invoke stringy and/or quantum corrections. This has by now led to many interesting scenarios, starting with the work [6], which, as most of these approaches, uses non-perturbative quantum corrections as an essential ingredient. The advantage of this approach is that the use of small corrections might make it easier to generate small scales for the breaking of supersymmetry or the cosmological constant. The "price" one has to pay, on the other hand, is that the explicit computation of the relevant perturbative or non-perturbative corrections usually poses a hard problem. Because of this, the evidence that these vacua really exist is also rather indirect.

The other possibility would be to try to construct de Sitter vacua at leading order by violating – already at the classical level – some of the assumptions that underlie the above-mentioned no-go theorems [7]. Although it may make it more difficult to generate small scale supersymmetry breaking or tiny cosmological constants, this purely "classical" approach certainly promises a gain in computational control.

2.1. THE DOUGLAS-KALLOSH PROBLEM

Looking at the possible caveats of the existing no-go theorems, the best-controlled classical ingredients that may evade them are a combination of orientifold planes and fluxes on spaces of *negative* integrated curvature $\int d^6x\sqrt{-g^{(6)}}R^{(6)} < 0$.* As the dimensional reduction on a generic negatively curved space is much less well understood as for Ricci-flat spaces such as Calab-Yau manifolds, most attempts have focused on group or coset manifolds, where the left-invariant modes provide a consistent truncation when the O-planes are smeared.

It is here where the Douglas-Kallosh problem [8] becomes particularly obvious. On a group or coset manifold, the curvature scalar of left-invariant metrics is in general a constant, *i.e.* the manifold has negative curvature *at every point*. As was pointed out by Douglas and Kallosh, however, the Einstein equation then requires that there also be a source of negative energy density *at every point* of the internal space. Clearly, a smeared orientifold plane provides precisely that, and in fact there are exact de Sitter solutions of the 10D field equations with such smeared oerientifold planes on group and coset spaces [9] (the known examples are not yet satisfactorily: they all have at least one tachyon, and there are possible issues with flux quantization). But the problem is that true O-planes are *not* smeared, and hence it seems unclear how a negative curvature space could possibly be supported at all. One should note that this is a general problem of compactifications on spaces of negative curvature that is not necessarily related to de Sitter vacua.

*The use of negative internal curvature here stems from the fact that the dimensional reduction of the internal part of the Einstein Hilbert term gives rise to a *positive* contribution to the scalar potential when the integrated internal curvature is negative. This positive contribution can then effectively act as an uplifting potential that may help to reach a positive vacuum energy.

Douglas and Kallosh suggest two possible ways around this problem. One is that negative curvature might be supported if there is an everywhere strongly varying warp factor, and another possibility could be effects from higher curvature terms. For lowest order solutions, only the first option could be viable, and indeed, a localized O-plane does in general lead to a varying warp factor. The important question then is, though, whether a solution with an everywhere *strongly* varying warp factor sourced by a localized O-plane can reliably be approximated by a solution with a smeared O-plane and a *constant* warp factor.

This motivated the works [10–12] where the validity of the smearing approximation is studied in simple setups that admit some control also over the localized solutions. The two main questions in this context then are:
1) Do smeared solutions always have a localized counterpart?
2) If yes, how much do the physical properties of the smeared and the localized solution differ from one another (*e.g.* with respect to the moduli values, the cosmological constant etc.)?

3. SMEARING IN THE BPS-CASE I

There is one known type of flux compactification where the back-reaction effects of localized brane sources are well enough understood to allow for an explicit comparison with the corresponding smeared solutions. These are the type IIB compactifications first discussed by Giddings, Kachru and Polchinski (GKP) [13]. In their simplest version they feature conformally Calabi-Yau spaces with H_3 and F_3 flux and spacetime-filling O3-planes. These source a non-trivial warp factor and a non-trivial profile for the C_4-potential, as follows from the F_5 Bianchi identity

$$dF_5 = H_3 \wedge F_3 - \mu_3 \delta_6(O3), \tag{1}$$

where μ_3 is a positive constant and $\delta_6(O3)$ denotes a 6-form with delta-function support at the position of the O3-planes on the internal 6D manifold.

This setup admits 4D Minkowski solutions provided the following BPS-type equations are satisfied

$$e^{4A} - \alpha = \text{const.} \tag{2}$$

$$F_3 + e^{-\phi} \tilde{*}_6 H_3 = 0. \tag{3}$$

Here, e^{2A} denotes the warp factor (*i.e.* the 10D metric is $ds_{10}^2 = e^{2A} d\tilde{s}_4^2 + e^{-2A} d\tilde{s}_6^2$), the function α determines the F_5-field strength via $F_5 = -(1 + *_{10}) e^{-4A} *_6 d\alpha$, and $\tilde{*}_6$ is the internal Hodge star but with respect to the Calabi-Yau metric $d\tilde{s}_6^2$ without the conformal factor e^{-2A}.

We now want to see what happens to this solution when the $O3$-planes are smeared over the full internal space. This means that the delta-functions in the source terms such as, *e.g.*, the one in (1) are replaced by constants. As a consequence, the functions A and α become constants (in fact, they can be chosen to be zero), so that the first BPS-condition (2) becomes trivial. The form of the second BPS-condition, on the other hand, stays exactly the same, and the solution still gives rise to a 4D Minkowski metric.

These observations have two important consequences:

1) For every smeared Minkowski solution satisfying the second BPS-condition (3) there is a localized Minkowski solution that also satisfies (3) (along with (2)).

2) The localized and the smeared solution have the same 4D cosmological constant (because they are both Minkowski compactifications) and they stabilize the complex structure moduli and the dilaton at the same values (because these moduli are stabilized by the condition (3), which is identical in both cases).

We thus see that for BPS-type solutions of the above type the smearing procedure provides a remarkably robust approximation. Intuitively this can be understood from the fact that the O3-planes and the fluxes do not exert any forces upon one another as they are mutually BPS. The fluxes should thus not be much influenced by a redistribution of the O3-brane charge and energy density.

4. SMEARING IN THE BPS-CASE II

The simple GKP-setup in the previous example does not directly address the Douglas-Kallosh problem as it is based on Ricci-flat internal manifolds in the smeared case. It is well known, however, that these smeared solutions (assuming a suitable $U(1)$ isometry) can formally be T-dualised to compactifications that have negative internal curvature [14]. Starting from a 6-torus, for example, T-duality along a circle that is threaded by part of the H_3 flux, yields a nilmanifold (twisted torus) with constant negative curvature and wrapped O4-planes. The localization of these smeared solutions should then again be possible due to the T-dual version of the above BPS conditions, and one should obtain a very concrete setup that directly addresses the Douglas-Kallosh problem.

For this reason we have explicitly constructed the smeared and the localized version of this negative curvature compactification and also studied the fate of the integrated internal curvature scalar [10]. The result is that, for this BPS-solution, the localized solution always exists, and it has again rather similar properties (cosmological constant, moduli vevs etc.) as the smeared solution. Moreover, the localized solution satisfies the 10D Einstein equations, and the Douglas-Kallosh problem is indeed taken care of by a careful interplay of the warp factor and two different conformal factors for the internal dimensions.

It is this interplay of the various warp and conformal factors that also has an important consequence for the integrated internal curvature. More precisely, one should look at the contribution, $V_{\rm curv}$, of the internal curvature to the effective 4D scalar potential, which must include also the determinant of the 4D metric. One then finds[†]

$$-V_{\rm curv} = \int d^6y \sqrt{g^{(10)}} R^{(6)} = \sqrt{\tilde{g}^{(4)}} \int d^6y \sqrt{\tilde{g}^{(6)}} \left(-\frac{40}{3} (\tilde{\nabla} A)^2 + \frac{1}{4} e^{\frac{16}{3}A} \tilde{R}^{(6)} \right) < 0. \qquad (4)$$

Thus, also in the localized version, the sign of the properly integrated internal curvature stays negative, and the above term can in principle be used as an uplift term (although in this case we know that the final result will be a Minkowski vacuum).

5. SMEARING IN THE NON-BPS CASE

In the above examples we have seen that, for BPS-type setups, the smearing procedure can be a remarkably harmless modification of the solution, and there doesn't seem to be an obvious obstacle to obtain a localized solution from a smeared one. If this is due to the no-force property of BPS-like configurations, however, this maybe entirely different for *non*-BPS solutions. In order to explore this issue, we therefore also studied the localization of a given smeared solution for a particularly simple non-BPS example that is in fact closely related to the above GKP-type solutions.

More precisely, we now consider a type IIB compactification in which the 3-form fluxes do not satisfy the BPS-condition (3), but instead

$$F_3 - e^{-\phi} \tilde{*}_6 H_3 = 0. \qquad (5)$$

One can show that these fluxes admit an AdS_4 solution with smeared D3-branes and a positively curved internal space, the simplest example being $S^3 \times S^3$. Due to the sign change in (5), this smeared solution does not satisfy BPS-like conditions, but it nevertheless can be shown to be a stable solution in the entire left-invariant sector.

When one now tries to turn this solution into a solution with *localized* D3-branes, using the most general ansatz for F_5 and the warp factor compatible with the field equations, one finds that the smeared flux relation (5) can *not* be maintained in

[†]The 4D metric determinant in fact contributes powers of the warp factor that are necessary in order to make the whole expression manifestly negative definite (after a partial integration). What is also important for this result is the precise form of the two different conformal factors of the internal metric (see [10] for details). Without these contributions, a definite sign of the integrated internal curvature is not apparent [8].

the localized solution, as it would lead to the following contradiction

$$e^{-2A}\tilde{R}_4 = -2\mu_3\delta(D3), \tag{6}$$

where $\tilde{R}_4 < 0$ is the curvature of the unwarped AdS-metric, $\mu_3 > 0$, and $\delta(D3)$ has support only at the positions of the D3-branes and vanishes elsewhere, in contradiction with the left hand side (the warp factor e^{-2A} must be finite away from the branes). Note that in the smeared limit $\delta(D3) \to 1$, $e^{-2A} \to 1$ this contradiction disappears.[‡] This implies that, should a localized version of this non-BPS solution really exist, it must have a relation between the 3-form fluxes different from the smeared relation (5), which then makes it unlikely that the moduli that are stabilized by (5) are stabilized at the same values as in the smeared situation.

In order to make further progress, we considered the analogous smeared compactification on $AdS_7 \times S^3$ in type IIA theory with spacetime-filling D6-branes and H_3-flux on the S^3 and a nontrivial Romans mass parameter F_0, playing the role of F_3 in the previous example. Assuming the most general ansatz for the relevant fields, we could then prove [11] that there is no continuous interpolation between the fully smeared solution and a hypothetical localized solution. Thus there can at best be a localized solution that is disconnected from the smeared one in parameter space. This again makes it unlikely that physical quantities such as moduli vevs or the cosmological constant take on the same or nearby values in the smeared and the localized solution (if one exists).

In [11] and [12] we could furthermore show that if a localized solution exists, it must have some very unusual boundary behavior near the D-branes, as it must involve diverging H_3-flux energy density. As H_3 is not directly sourced by the D6-branes, this divergence might indicate that a global static solution does not exist, or it could be a sign of the D6-branes being unstable in this non-BPS background due to some version of the Myers effect [15]. It would certainly be interesting to understand the physical meaning of this singularity and what it implies for the existence of a localized solution.

6. CONCLUSION

The smearing of D-branes and O-planes is a common and very helpful simplification for finding explicit compactification backgrounds or for deriving the corresponding effective actions. For BPS-type flux-brane configurations, we found this approximation to be remarkably robust and showed that the Douglas-Kallosh problem for negatively curved internal spaces may indeed be taken care of by an interplay of warp and conformal factors.

‡For the BPS-Minkowski solutions of section 2, the right hand side of (6) has the prefactor 0 instead of (-2), and no analogous contradiction occurs in the localized solution.

For non-BPS configurations, however, the general validity of smearing is much less clear and could not yet be confirmed. In fact, our investigation of a few simple examples raised a number of open questions and potential problems with this approximation.

Unfortunately, de Sitter vacua are non-BPS in nature, so it is still unclear whether smearing really makes sense here. As the attempts to build classical de Sitter vacua also have met with other problems, one might take this as an indication that de Sitter vacua are more likely to hide elsewhere in the parameter space of string theory.

In this context, it might be interesting to explore also some parallels with the recent work [16] on the back-reaction of anti-branes in warped throats.

Acknowledgments. This article summarizes much of the work of [10–12], and it is my pleasure to thank Johan Blåbäck, Ulf Danielsson, Daniel Junghans, Thomas Van Riet and Timm Wrase for the interesting and fruitful collaboration on these topics, and the organizers of the BW2011 Workshop for setting up this stimulating and enjoyable meeting. This work is supported by the German Research Foundation (DFG) within the Cluster of Excellence QUEST.

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