A DARK ENERGY MODEL WITH VARIABLE EOS PARAMETER
IN SELF-CREATION THEORY OF GRAVITATION

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We have constructed LRS Bianchi type I dark energy models with variable equation of state (EoS) parameter in Barber’s second self creation theory. We found the equation of state for dark energy is time independent and its existing range for derived models is in good agreement with the recent observations of SNe Ia data. In order to obtain a determinate solution of the field equations we consider the special law of variation of Hubble’s parameter (Berman in Nuovo Cimento B 74, 182, 1983) which yields a cosmological model with negative deceleration parameter. Some physical and geometrical aspects of the models are also discussed.

Key words: Dark energy, EoS parameter, Barber second self creation theory.

1. INTRODUCTION

The cosmological observations of type Ia Supernovae (SNe Ia) indicate the accelerated expansion of the universe in the current era [1,2]. In addition, the combination of results from the large scale distribution of galaxies and the most precise data on the cosmic microwave background (CMB) from the Wilkinson Microwave Anisotropy Probe (WMAP) confirm such a cosmic acceleration [3, 4]. Riess et al. [5] also confirmed the present acceleration of the expansion rate and showed the existence of an deceleration-acceleration transition and red shift around $z_c \sim 0.5$. These results suggest a transition around $z_c$ from a standard matter dominated universe to a universe dominated by a new type of component with negative pressure responsible for the acceleration. This new component is called dark energy which amounts to about 70 percent and permeates homogeneously in the universe. The simplest explanation for this dark energy is the introduction of a cosmological constant, with equation of state $\omega_\Lambda = -1$. In this case the value of the cosmological constant is not tuned to get an static Einstein universe, but instead its value is responsible for an accelerated expansion [6]. There are other candidates to
dark energy apart from the cosmological constant in the literature. Copeland et al. [7] proposed that dark energy could be identified with the energy density of a dynamical scalar field (quintessence). The quintessence [8, 9], phantom field [10, 11], quintom [12, 13], chaplygin gas [14], k-essence [15–18], tachyon field [19, 20], holographic [21–24] and agegraphic [25] are various alternative candidates for dark energy. In spite of these attempts dark energy is still one of the most important open questions in theoretical physics.

In order to understand how the expansion rate of the universe changes over time high-precision measurements of expansion of the universe are required. In general relativity, the evolution of the expansion rate is parameterized by the relationship between temperature, pressure, and combined matter, energy, and vacuum energy density. Today one of the biggest efforts in observational cosmology is the measurement of equation of state for dark energy. The dark energy model is characterized by the equation of state (EoS) parameter \( \omega = \frac{p}{\rho} \) which is not necessarily constant [26]. Here \( p \) is the fluid pressure and \( \rho \) is the energy density. The methods allowing for restoration of the quantity \( \omega(t) \) from experimental data is developed by Sahni and Starobinsky [27], and an analysis of the experimental data is carried out by Sahni et al. [28] to determine this parameter as a function of cosmic time. The simplest candidate for dark energy is the vacuum energy (\( \omega = -1 \)) which is mathematically equivalent to the cosmological constant (\( \Lambda \)). The other conventional alternatives are quintessence (\( \omega > -1 \)), Phantom energy (\( \omega < -1 \)) and quintom (that can across from phantom region to quintessence region) as evolved and have time dependent EoS parameter. Some other limits obtained from observational results coming from SNe Ia data [29] and combination of SNe Ia data with CMBR anisotropy and galaxy clustering statistics [30] are \(-1.67 < \omega < -0.62 \) and \(-1.33 < \omega < -0.79 \), respectively. The latest results obtained by Hinshaw et al. [31], Komatsu et al. [32] after a combination of cosmological data sets coming from CMB anisotropies, luminosity distances of high red shift type Ia supernovae and galaxy clustering, constrain the dark energy EoS to \(-1.44 < \omega < -0.92 \) at 68% confidence level. Usually the EoS parameter is considered as a constant with phase wise value \(-1, 0, -\frac{1}{3} \) and +1 for vacuum field, dust fluid, radiation and stiff fluid dominated universe respectively. It is because of the lack of observational evidence in making a distinction between constant and variable \( \omega \) [33, 34]. But in general \( \omega \) is a function time or red shift [35-37]. Some of the authors [38-41] constructed quintessence models involving scalar fields which give rise to time dependent EoS parameter \( \omega \). Cosmological models with variable EoS parameter in Kaluza-Klein metric and wormholes are available in literature [41–43]. Authors [44, 45] used various form of time dependent \( \omega \) to
construct variable Λ models. Dark energy models [46–57] are constructed with variable EoS parameter in general relativity. Recently Rao et al. [58] studied dark energy model with variable EoS parameter in the Saez and Ballester scalar tensor theory of gravitation.

Barber [59] proposed two self creation theories modifying the Brans and Dicke [60] theory and general relativity theory. These modified theories create the universe out of self contained gravitational and matter fields. Brans [61] pointed out that first theory of Barber is unsatisfactory as it violates equivalence principle. The second theory is a modification of general theory of relativity to a variable G theory and predicts local effects, which are within the observational limits. The field equations in Barber’s second theory are

\[ G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = -8\pi \frac{T_{ij}}{\varphi} \]  

and

\[ \Box \varphi = \frac{8}{3\pi} \lambda T \]

where λ is coupling constant to be evaluated from experiment and ϕ is the Barber scalar. In this theory the scalar field does not directly gravitate, but simply divides the matter, acting as a reciprocal gravitational constant. In the limit λ → 0 the theory approaches Einstein’s theory in every respect. In view of consistency of this theory several authors [62–66] constructed different cosmological models in this theory and studied various aspects of this theory in presence of different gravitating fields.

In this paper we construct LRS Bianchi type I dark energy model with variable EoS parameter in Barber’s second self creation theory of gravitation.

2. METRIC AND FIELD EQUATIONS

We consider LRS Bianchi type I space-time given by

\[ ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) \]

where the metric potentials A and B are functions of t alone. This ensures that the model is spatially homogeneous.

The simplest generalization of EoS parameter of perfect fluid may be to determine the EoS parameter separately on each spatial axis by preserving the diagonal form of the energy momentum tensor in a consistence way with the considered metric. Therefore the energy momentum tensor of fluid is taken as

\[ T_i^j = \text{diag}[T_1^1, T_2^2, T_3^3, T_4^4] \]
Thus, one may parameterize it as follows,

\[
T_i' = \text{diag}[-p_x, -p_y, -p_z, \rho]
\]

\[
= \text{diag}[-\omega_x, -\omega_y, -\omega_z, 1] \rho
\]

\[
= \text{diag}[-\omega, -(\omega + \delta), -(\omega + \delta), 1] \rho
\]  

(5)

Here \( \rho \) is the energy density of the fluid, \( p_x, p_y \) and \( p_z \) are the pressures, and \( \omega_x, \omega_y \) and \( \omega_z \) are the directional EoS parameters along the x, y and z axes respectively. \( \omega(t) = \frac{\rho}{\rho} \) is the deviation free EoS parameter of the fluid. We have parameterized the deviation from isotropy by setting \( \omega_x = \omega \) and then introducing skewness parameter \( \delta \) which is the deviation from \( \omega \) along both y and z axes.

In a commoving co-ordinate system, Barber’s field equations (1) and (2) for the metric (3) in case of (5) lead to the following system of equations:

\[
2 \left( \frac{B_{44}}{B} + \left( \frac{B_4}{B} \right)^2 \right) = \frac{8\pi}{\varphi} \omega \rho
\]

(6)

\[
\frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{A_{44}}{A} = \frac{8\pi}{\varphi} (\omega + \delta) \rho
\]

(7)

\[
2 \left( \frac{A_4 B_4}{AB} + \left( \frac{B_4}{B} \right)^2 \right) = \frac{8\pi}{\varphi} \rho
\]

(8)

\[
\varphi_{44} + \left( \frac{A_4}{A} + 2 \frac{B_4}{B} \right) \varphi = \frac{8}{3} \pi \lambda (3 \omega \rho + 2 \delta \rho - \rho)
\]

(9)

Here the sub indices 4 in A, B, \( \varphi \) and elsewhere denote ordinary differentiation with respect to \( t \).

**3. SOLUTIONS OF THE FIELD EQUATIONS**

The system of field equations (6)–(9) is an under determined system containing six unknowns viz. A, B, \( \rho \), \( \omega \), \( \delta \), and \( \varphi \). Therefore, in order to obtain explicit exact solutions two more equations are required.

We solve the above system of highly nonlinear differential equations using the special law of variations of Hubble’s parameter proposed by Berman [67] which yields constant deceleration parameter of the models of the universe. We consider
Dark energy model in self-creation theory of gravitation

\[ q = -\frac{RR_{44}}{R_{4}^{2}} = \text{constant} \quad (10) \]

where \( R = (AB^{2})^{1/3} \) is the overall scale factor. For an accelerating model of the universe we take the constant as negative.

From (10) we obtain

\[ R = (1 - q)^{\frac{1}{1-q}} (at + b)^{\frac{1}{1-q}} \quad (11) \]

where \( a \neq 0 \) and \( b \) are constants of integration. Further, in view of the anisotropy of the space time, we assume that the expansion \( (\theta) \) in the model is proportional to the shear which leads to

\[ A = R^{n} \quad (12) \]

where \( n \neq 0 \) is a constant.

Thus we get the exact solutions of the field equations (6)–(9) as

\[ A = (1 - q)^{\frac{n}{1-q}} (at + b)^{\frac{n}{1-q}} \quad (13) \]

\[ B = (1 - q)^{\frac{3}{2(1-q)}} (at + b)^{\frac{3}{2(1-q)}} \quad (14) \]

\[ \varphi = c_{1} (at + b)^{m_{1}} + c_{2} (at + b)^{m_{2}} \quad (15) \]

\[ \rho = \frac{3\alpha^{2}}{32\pi} \left[ \frac{2n - n^{2} + 3}{(1 - q)^{2} (at + b)^{2}} \right] \left[ c_{1} (at + b)^{m_{1}} + c_{2} (at + b)^{m_{2}} \right] \quad (16) \]

\[ \omega = \frac{5 - 3n + 4q}{3n + 3} \quad (17) \]

\[ \delta = \frac{2qn - 2q - 4 + 4n}{2n - n^{2} + 3} \quad (18) \]

Here \( c_{1} \) and \( c_{2} \) are constants of integration and

\[ m_{1} = \frac{(1 - \alpha) + \sqrt{(a - 1)^{2} + 4\beta\lambda}}{2} \quad (19) \]

\[ m_{2} = \frac{(1 - \alpha) - \sqrt{(a - 1)^{2} + 4\beta\lambda}}{2} \quad (20) \]
Thus with suitable transformation the geometry of the universe described by the line element can be expressed as

\[ ds^2 = -\frac{dT^2}{a^2} + (1 - q)^{\frac{2n}{1+q}} T^{\frac{2a}{1+q}} dX^2 + (1 - q)^{\frac{3-n}{1+q}} T^{\frac{3-n}{1+q}} (dY^2 + dZ^2) \]  

(21)

which represents LRS Bianchi type I dark energy cosmological model in Barber’s second self creation theory.

4. SOME PHYSICAL AND GEOMETRICAL PROPERTIES OF THE MODEL

The expressions for Hubble’s parameter $H$, spatial volume $V^3$, expansion scalar $\theta$, shear scalar $\sigma$ for the model (21) are given by

\[ H = \frac{1}{3} \left( \frac{A_4}{A} + 2 \frac{B_4}{B} \right) = \frac{1}{1 - q} \frac{1}{T} \]

\[ V^3 = AB^2 = \left( 1 - q \right)^{\frac{3}{1+q}} T^{\frac{3}{1+q}} \]

\[ \sigma^2 = \frac{1}{3} \left[ \left( \frac{A_4}{A} \right)^2 + \left( \frac{B_4}{B} \right)^2 - 2 \frac{A_4 B_4}{AB} \right] = \frac{3}{4} \frac{(n-1)^2}{(1-q)^2} \frac{1}{T^2} \]

\[ \theta = \frac{A_4}{A} + 2 \frac{B_4}{B} = \frac{3}{1-q} \frac{1}{T} \]

Here we observe that the spatial volume is zero at $T = 0$ and volume increases with the increase of cosmic time $T$. The kinematical parameters $H$, $\theta$ and $\sigma$ diverge at initial epoch and tend to zero as $T \to \infty$. Since $\sigma^2 = \text{constant}$, the model does not approach isotropy through the whole evolution of the universe. Further the Barber scalar $\phi$ is obtained as $\phi = c_1 T^{m_1} + c_2 T^{m_2}$. When the coupling constant $\lambda \to 0$ one mode of $\phi$ i.e. $T^{m_1} \to 0$ whereas the other one $T^{m_2}$ does not approaches zero. Therefore the later mode of $\phi$ is not acceptable.
5. CONCLUDING REMARKS

In this paper, we studied LRS Bianchi type I dark energy model with variable EoS parameter in Barber’s second self creation theory of gravitation. From (17) we have \( \omega = -1 \) if \( q = -2 \), which represents cosmological constant dominated universe. When \( -2 < q < 0 \) (since \( q \) is considered negative), we have \( \omega > -1 \) and for \( -\infty < q < -2 \) we have \( \omega < -1 \) which represent quintessence and phantom fluid dominated universes respectively. We find \( -1.67 < \omega < 0.62 \), when \((-0.5025 - 2.5025) < q < (0.285n - 1.715) \) and \( n \in (-4.98, 6.02) \) which is in good agreement with the limit obtained from observational results coming from SNe Ia data [29]. Further we obtain \( -1.33 < \omega < -0.79 \) when \(-\frac{1}{4}(0.99n + 8.99) < q < -\frac{1}{4}(0.63n - 7.37) \) and \( n \in (-9.08081, 11.6984) \), which is in good agreement with the limit obtained from observational results coming from SNe Ia data with CMB anisotropy and galaxy clustering statistics [30]. In addition, \(-1.44 < \omega < -0.92 \) if \(-\frac{1}{4}(1.32n + 9.32) < q < -\frac{1}{4}(0.24n - 7.76) \) and \( n \in (-7.0606, 32.3333) \) which is in good agreement with the limit of latest observational results [31, 32].

REFERENCES