Under different forms of intensity-dependent coupling (IDC), mixed moving atom entanglement transferred from Bell state (BS) light field is investigated. It’s found that the atomic motion related to the field-mode structure parameter (FMSP) can make the atom-atom entanglement evolve periodically. The periodical constant evolvement of the entanglement can be realized for some atomic initial states without IDC. When FMSP increases, the atom-atom entanglement is severely destroyed and even completely eliminated for larger FMSP irrespective of time and the initial atomic state. But under suitable forms of IDC, the atom-atom entanglement is not only revived, but also enhanced to be maximally entangled even for the mixed initial atomic states. These corresponding forms of IDC can be used as appropriate switchers to turn the entanglement on and off. The periodicity of the entanglement evolvement is also kept.

Key words: entanglement, intensity-dependent coupling, Bell state, mixed moving atom.

1. INTRODUCTION

Entanglement is one of the most striking features of quantum mechanics and has been considered as a physical resource of quantum information processing. It is profoundly important in quantum teleportation [1], quantum computing [2], cryptography [3] and quantum games [4, 5]. The conception of entangled state represented with continuum variables is proposed in Ref [6]. Using Jayness-Cumming-type interaction between atoms and cavity fields [7], entanglement transfer from two-mode squeezed vacuum state to two separable atoms which are prepared in pure state has been studied in [8]. Zou et al. have discussed the entanglement transfer from some entangled two-mode fields to mixed qubits [9]. Deng et al. have studied the periodic death and anabiosis of the entanglement between the two moving atoms [10].

The Jayness-Cumming model was expanded by Buck and Sukumar into the intensity-dependent coupling (IDC) [11] and has attracted many investigations [12, 13, 14] in the studies of light squeezing [12], atomic inversion [13], generation

of nonlinear coherent states in a micromaser [14] and etc. In this article, we focus our attention on the mixed moving atom entanglement transferred from Bell state (BS) light field under given forms of IDC. In the next two sections, the model and its exact solution are presented firstly. And then it is shown that the atomic motion related to large field-mode structure parameter (FMSP) can destroy the atom-atom entanglement and lead to complete disentanglement of the atoms for even larger FMSP without IDC. Suitable forms of IDC function can revive and greatly enhance the entanglement. These forms of IDC can be used as appropriate entanglement switchers. The final section is the conclusions.

2. THE MODEL AND ITS SOLUTION

Now we let the two moving separate atoms simultaneously interact with the two modes of BS light field respectively. The interaction Hamiltonian between one of the moving atoms and one mode of the light field under IDC is [9,10,14] \( \hat{H}_{ij} = \text{gf}(z_j)(|e_j\rangle\langle g_j|a_jF(a_j^*a_j) + F(a_j^*a_j)a_j^*|g_j\rangle\langle e_j|) \), (1)

where \(|g_j\rangle\) and \(|e_j\rangle\) \( (j = 1,2) \) denote respectively the ground and exited states for the \( j \)th atom, \( a_j \) and \( a_j^* \) are the annihilation and creation operators for \( j \)th mode of the cavity field, \( g \) is the atom-field coupling constant. \( F(a_j^*a_j) \) represents an arbitrary function of IDC. When \( F(a_j^*a_j) = + |g_j\rangle\langle e_j| \), it means that there is no IDC. \( f(z_j) \) is the shape function of the atomic motion along the \( z \) axis only considering the \( z \)-dependence of the mode field function. The atomic motion can be incorporated in an unusual way and the transverse electromagnetic modes (TEM\(_{mnlp}\), where \( m, n \) mean the ordinal numbers of the transverse mode) can be defined specifically as [10] \( f(z_j) \rightarrow f(v)t = \sin(p\pi v t/L) \), (2)

where \( v \) denotes the velocity of the \( j \)th atom. The parameter \( p \) which is called field-mode structure parameter (FMSP) represents the number of half-wave lengths of the field mode inside a cavity of a length \( L \). For simplicity, we consider the situation for \( v_1 = v_2 = v \). Assuming that \( B_j = a_jF(a_j^*a_j) \), \( B_j^* = F(a_j^*a_j)a_j^* \), the time evolution operator of the whole system can be derived \( U(t) = u_1(t) \otimes u_2(t) \) in the interaction picture, where
\[ u_j(t) = \exp\left(-i \int_0^t H_{ij} dt'\right) \]

\[
\begin{pmatrix}
\cos[g\theta(t)\sqrt{B_j B_j^+}] & -i\sin[g\theta(t)\sqrt{B_j B_j^+}]B_j^+ \\
-i\sin[g\theta(t)\sqrt{B_j B_j^+}]B_j & \cos[g\theta(t)\sqrt{B_j B_j^+}]
\end{pmatrix},
\]

(3)

with

\[ \theta(t) = \int_0^t f(vt')dt' = \frac{L([1 - \cos(\pi p vt)])}{\pi pv} \]

Choosing the atomic motion velocity as \( v = gL/\pi \), \( \theta(t) \) becomes

\[ \theta(t) = \frac{1 - \cos(pgt)}{pg}. \]

(4)

It is assumed that the two atoms are initially prepared in the following separate state

\[ \rho_A(0) = [x|e_1\rangle\langle e_1| + (1 - x)|g_1\rangle\langle g_1|] \otimes [x|e_2\rangle\langle e_2| + (1 - x)|g_2\rangle\langle g_2|], \]

(5)

where \( 0 \leq x \leq 1 \), which represents the mixedness of the initial atomic state. The mixed property of the state is generated by the environment. When \( x = 0 \), both atoms are in the ground states; when \( x = 1 \), the atoms are in the excited states. The initial density operator of the system can be whiten as

\[ \rho_{FA}(0) = \rho_F(0) \otimes \rho_A(0) = |\psi_F\rangle_F \langle \psi_F| \otimes \rho_A(0), \]

where \( |\psi_F\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle - |0,1\rangle) \) is the initial state of the field(BS) which means that the two modes of the field are initially maximally entangled. Now we can obtain the density operator of the system at the time \( t \):

\[ \rho_{FA}(t) = U(t) \rho_{FA}(0) U^\dagger(t). \]

To get the entanglement between the two atoms, we need to trace over the field variables to obtain the reduced density matrix for the two atoms which is given by
$\rho_{A}(t) = \text{tr}_F \rho_{F,A}(t) = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & e & 0 \\ 0 & e^* & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}.$ \hspace{1cm} (6)

where

\begin{align*}
a &= x^2 \cos^2[\sqrt{2}F(2)g\theta(t)]\cos^2[F(1)g\theta(t)] + \\
&\quad + x(1-x)\sin^2[F(1)g\theta(t)]\cos^2[F(1)g\theta(t)] \\
b &= \frac{x^2}{2} \{\cos^2[\sqrt{2}F(2)g\theta(t)] + \cos^4[F(1)g\theta(t)] + \sin^4[F(1)g\theta(t)]\} \\
&\quad + \frac{(1-x)^2}{2}\sin^2[F(1)g\theta(t)] \\
d &= x^2 \sin^2[\sqrt{2}F(2)g\theta(t)]\sin^2[F(1)g\theta(t)] + x(1-x)\sin^2[\sqrt{2}F(2)g\theta(t)] + \\
&\quad + \frac{1}{4}\sin^2[2F(1)g\theta(t)] \\
&\quad + (1-x)^2 \cos^2[F(1)g\theta(t)] \\
e &= -\frac{x^2}{2}\cos^2[\sqrt{2}F(2)g\theta(t)]\sin^2[F(1)g\theta(t)] + \\
&\quad + x(1-x)\cos[\sqrt{2}F(2)g\theta(t)]\sin^2[F(1)g\theta(t)] \\
&\quad - \frac{1}{2}(1-x)^2 \sin^2[F(1)g\theta(t)]. \hspace{1cm} (7)
\end{align*}

3. THE ATOM ENTANGLEMENT TRANSFERRED FROM BS LIGHT FIELD

It has been proved that for a bipartite system described by the density matrix $\rho$, the negativity criterion for entanglement of the two subsystems is given by the following formula [15,16]
\[ \varepsilon = -2 \sum_{i} \lambda_{-i}^{-}, \]

where the sum is taken over the negative eigenvalues \( \lambda_{-i}^{-} \) of the partial transposition of the density matrix \( \rho \). The value \( \varepsilon = 0 \) indicates that the two subsystems are separable. This function varies between 0 and 1, and monotonically increases as the entanglement grows. For the above system, the entanglement of the atoms can be expressed as

\[
\varepsilon = \sqrt{(a-d)^2 + 4|\varepsilon|^2} - a - d.
\]  

(8)

Firstly, when \( p = 1 \), the atom-atom entanglement transferred from BS light field is presented for \( F(n) = 1 \) (no IDC) and \( F(n) = \sqrt{n} \) respectively. The numerical results for \( \varepsilon \) against scaled time \( gt \) and the parameter \( x \) are plotted in Figure 1.

![Fig. 1](image1.png)

**Fig. 1 – \( \varepsilon \) as a function of time \( gt \) and \( x \) for \( p = 1 \) when the initial field is at Bell state \( |1,0\rangle - |0,1\rangle \). Fig (a),(b) correspond to \( F(n) = 1, \sqrt{n} \) respectively.**

It can be seen in Figure 1(a) that the atom-atom entanglement evolves periodically irrespective of the values of \( x \) for \( F(n) = 1 \). When \( x = 0 \), there are two peaks in a period. At \( gt = 2.18 + 2k\pi, 4.10 + 2k\pi \) (\( k \) integer), \( \varepsilon \) reaches its maximum value 1. At the time, maximally entangled atomic states can be derived in this way. When \( x \) increases, \( \varepsilon_{\text{max}} \) decreases and the time of entanglement is shortened. One reason is that when environment intervenes, the decoherence phenomenon takes place and the entanglement is impaired. When \( x = 0.79 \), the atom-atom entanglement evolves steadily with a constant value (\( \varepsilon_{\text{max}} = 0.55 \)) for \( 2.79 + 2k\pi \leq gt \leq 3.49 + 2k\pi \). When \( x \) further increases, this phenomenon also holds but the time of constant entanglement is decreased.
When $F(n) = \sqrt{n}$, it can be seen in Figure 1(b) that the atom-atom entanglement keeps its periodical evolvement. When $x = 0$, the time evolution of $\varepsilon$ is the same as the one for $F(n) = 1$. At $gt = 2.18 + 2k\pi$, $4.10 + 2k\pi$ (k integer), $\varepsilon$ also reaches its maximum value 1 and the time of entanglement is also shortened as $x$ increases. But $\varepsilon_{\text{max}}$ keeps unchanged for different $x$. Compared with the results in Figure 1(a), it’s obvious that the atom-atom entanglement is enhanced and the decoherence phenomenon arising from the environment can be reduced by the IDC function. It also can be seen that the middle part between the two peaks caves in and decreases to zero and the two peaks in each period become much sharper as $x$ increases which is different from the results in Figure 1(a). So it can be noted that the periodical constant evolvement of the atom-atom entanglement which emerges in Fig. 1(a) is destroyed by the IDC function $F(n) = \sqrt{n}$.

Fig. 2 – $\varepsilon$ as a function of $gt$ and $x$ for $p = 2$. Fig (a),(b),(c),(d) correspond to $F(n) = 1, \sqrt{n}, \sqrt{3n}, \sqrt{\sqrt{n}}$ respectively.
When \( x = 0 \), it can be seen in Figure 2(a),(b) that the atom-atom entanglement evolves similarly for \( F(n) = 1, \sqrt{n} \) just like the above results in Figure 1 and \( \varepsilon \) is much smaller (\( \varepsilon_{\text{max}} = 0.47 \) at \( gt = 1.57 + k\pi \)) compared with the results in Figure 1. The entanglement disappears completely for \( x \geq 0.3 \) (\( F(n) = 1 \)) and \( x \geq 0.5 \) (\( F(n) = \sqrt{n} \)) which is shown in Figure 2(a),(b) respectively. So it can be noted that as \( x \) increases, \( \varepsilon \) decreases rapidly as a result of the decoherence phenomenon caused by the environment for these two situations. Compared with the results in Figure 1, it can be concluded that when \( p \) increases the atom-atom entanglement is severely destroyed by the atomic motion without IDC and the IDC function \( F(n) = \sqrt{n} \) has little effect in enhancing the atom-atom entanglement.

At \( gt = 1.26 + k\pi, 1.88 + k\pi \) (for \( F(n) = 3n \)) and at \( gt = 1.09 + k\pi, 2.05 + k\pi \) (for \( F(n) = 4n \)), \( \varepsilon \) reaches its maximum value 1 in the above two cases respectively and keeps unchanged as \( x \) increases which can be seen in Fig. 2(c), (d). So it can be concluded that the IDC functions \( F(n) = \sqrt{3n}, \sqrt{4n} \) can greatly enhance the atom-atom entanglement to be maximally entangled and reduce the effect of both the environment and FMPS for \( p = 2 \). It also can be seen that the effect of these two IDC functions for enhancing the atom-atom entanglement is much more obvious compared with the results in Figure 1.

![Fig. 3 – \( \varepsilon \) as a function of \( gt \) and \( x \) for \( p = 10 \). Fig (a),(b) correspond to \( F(n) = 1, \sqrt{58n} \) respectively.](image)

Other numerical calculations show that the atom-atom entanglement is further weakened as \( p \) increases. When \( p = 10 \), the moving atoms are completely disentangled for all values of \( gt \) and \( x \) without IDC which can be seen in Figure 3(a).
It’s obvious that the atom-atom entanglement is severely destroyed and completely eliminated in this case. But when we set \( F(n) = \sqrt{58n} \), it can be seen in Figure 3(b) that the maximum atom-atom entanglement is not only revived but also reaches unit at \( gt = (0.1 + 0.2k)\pi \) and keeps unchanged and evolves also periodically as \( x \) increases. So it can be concluded that the IDC function \( F(n) = \sqrt{58n} \) can also make the entanglement revived and greatly enhanced to be maximally entangled. With such an effect, this IDC function can be used as an appropriate switcher to turn the entanglement on and off in this case. When \( p \) further increases, the atoms are still disentangled for all \( gt \) and \( x \) without IDC. When we set \( F(n) = \sqrt{m^{n}} \) \( (m \geq m_{0}) \) where \( m \) and \( m_{0} \) are positive integers, the atom-atom entanglement is greatly enhanced \( (\varepsilon_{\text{max}} = 1 \) and keeps unchanged as \( x \) increases) just like the result in Figure 3 while other forms of \( F(n) \) can’t do that. The value of \( m_{0} \) varies for different value of the parameter \( p \). When \( p \) increases, \( m_{0} \) increases as well, as can be seen in Table.

<table>
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4. CONCLUSIONS

The entanglement of initially separable and mixed moving atoms transferred from BS light field is investigated. Considering the atomic motion related to FMSP, the atom-atom entanglement evolves periodically. When \( x = 0 \), maximally entangled atomic states can be derived at special values of \( gt \) for \( p = 1 \). The periodical constant evolvement of the atom-atom entanglement can be realized for \( x \geq 0.79 \). When \( p \) further increases, the atom-atom entanglement is severely destroyed. And it is completely eliminated for \( p \geq 10 \). Taking into account suitable forms of the IDC function \( F(n) = \sqrt{m^{n}} \) \( (m \geq m_{0}) \), the atom-atom entanglement is not only revived, but also greatly enhanced to be maximally entangled and kept unchanged for different \( x \), where \( m_{0} \) increases with \( p \). These forms of IDC function can be used as appropriate switchers to turn the entanglement on and off, which are useful for the studies of quantum communication and computation. It is also shown that the effect of suitable IDC function for enhancing the atom-atom entanglement is much more obvious for larger \( p \).
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