The deformation energy for $^{180}\text{Hg}$ is investigated in the macroscopic-microscopic approach within the Woods-Saxon two centre shell model. The configuration space includes five generalized coordinates: the elongation, the necking, the mass asymmetry and the fragment deformations. An isomeric state of the parent is obtained. This isomeric state is located at a mass asymmetry compatible with the emission of a nucleus of mass 70 and can explain the asymmetry in the fission mass distribution.

Key words: Fission, macroscopic-microscopic approach, $^{180}\text{Hg}$.

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1. INTRODUCTION

A new type of asymmetric fission was experimentally observed in the neutron rich $^{180}\text{Hg}$ nucleus [1]. The $^{180}\text{Hg}$ is obtained after the electron capture of the $^{180}\text{Tl}$. From a theoretical point of view, a symmetric distribution of fission fragments was expected, that is a maximum yield for two semi-magic $^{90}\text{Zr}$ products. As mentioned already in Ref. [1], the symmetric mode must dominate in the region of $A \approx 200$. Surprisingly, the best yields were obtained for a fragmentation of masses (80,100). The experimental data were interpreted as a new type of asymmetric fission that is not caused by the large shell effects of the fragments. This phenomenon was explained within the macroscopic-microscopic approach [2] by evidencing a so-called "local" minimum in the potential energy surface. Moreover, another analyse within Hartree-Fock approximation was realized at different values of the excitation energy [3,4]. It was found that asymmetric fission is favoured over the symmetric one for excitation of nearly 10 MeV [3]. Motivated by these intriguing aspects, in the following, we will use the macroscopic-microscopic approach based on the two centre Woods-Saxon model [5] in order to investigate the experimental preference for the asymmetric fission.

2. THE MODEL

In the macroscopic-microscopic method, the nuclear system is characterized by some collective coordinates. These variables give approximately the behaviour of
the intrinsic variables [6]. The basic ingredient in the model is the shape parametriza-
tion ruled by the macroscopic degrees of freedom. The deformation energy is a sum
of two terms: the liquid drop part and a microscopic correction. Usually the micro-
scopic correction is evaluated with the Strutinsky procedure [7].

We use an axial symmetric nuclear shape that offers the possibility to obtain a
transition from one initial nucleus to the separated fragments. This parametrization
is obtained by smoothly joining two spheroids of semi-axis \( a_i \) and \( b_i \) \((i=1,2)\) with
a neck surface generated by the rotation of a circle around the axis of symmetry.
By imposing the condition of volume conservation we are left by five independent
generalized coordinates \( \{q_i\} \ (i=1,5) \) that can be associated to five degrees of free-
dom: the elongation \( R \) given by the distance between the centres of the spheroids;
the necking parameter \( C_3 = S/R_3 \) related to the curvature of the neck, the eccen-
tricities \( \varepsilon_i \) associated with the deformations of the nascent fragments and the mass
asymmetry parameter \( \eta = a_1/a_2 \). Alternatively, the mass asymmetry can be charac-
terized also by the mass number of the light fragment \( A_2 \). This number is obtained
by considering that the sum of the volumes of two virtual ellipsoids characterized by
the mass asymmetry parameter \( \eta \) and the eccentricities \( \varepsilon_i \ (i=1,2) \) gives the volume
of the parent. This parametrization was widely used by the Bucharest group in the
calculations addressing the cluster and alpha decay [8–15], the dissipation during the
fission [16–18], the pair breaking [19], the generalization of time dependent pairing
equations [20], the heavy element synthesis [21,22], the fission [23–25] or the cran-
ing inertia [26,27]. It is important to note that the generalized inertia obtained within
the two centre shell model [26] indicated that the diagonal matrix elements of the
time derivative of the Hamiltonian give rise mainly to intrinsic excitations. So, these
diagonal elements must not contribute in principle to the inertia values. Such an idea
intuitively also resorts in Ref. [28]. In contrast to the cluster approximations [29–32],
the two-centre shell model offers the opportunity to treat the alpha decay as a super-
asymmetric fission process. Such a model was assumed by other groups and applied
to characterize the dissipation in a wide range of mass asymmetries [33].

The macroscopic part of the deformation energy is computed within the finite
range liquid drop model [34] as presented in Ref. [35]. It is worth to mention that
recently, the macroscopic model was extended also for ternary fission [36]. The shell
effects are obtained as a sum between the shell and the pairing microscopic cor-
rections. In this context, the Strutinsky procedure [7] was used. These corrections
represent the varying parts of the total binding energy caused by the internal quan-
tum structure. A microscopic potential must be constructed to be consistent within
our nuclear shape parametrization. In this context, a two-centre shell model with
a Woods-Saxon potential was developed recently [5]. The mean field potential is
defined in the frame of the Woods-Saxon model:

\[ V_0(\rho, z) = -\frac{V_c}{1 + \exp\left(\frac{\Delta(\rho, z)}{a}\right)} \]  

(1)

where \( \Delta(\rho, z) \) represents the distance between a point \((\rho, z)\) and the nuclear surface. This distance is measured only along the normal direction on the surface and it is negative if the point \((\rho, z)\) is located in the interior of the nucleus. \( V_c \) is the depth of the potential while \( a \) is the diffuseness parameter. In our work, the depth is \( V_c = V_{0c}[1 \pm \kappa(N_0 - Z_0)/(N_0 + Z_0)] \) with plus sign for protons and minus sign for neutrons, \( V_{0c} = 51 \text{ MeV}, a=0.67 \text{ fm}, \kappa=0.67 \). Here \( A_0, N_0 \) and \( Z_0 \) represent the mass number, the neutron number and the charge number of the parent, respectively. This parametrization, referred as the Blomqvist-Walhlborn in the literature, is adopted because it provides the same radius constant \( r_0 \) for the mean field and the pairing field. That ensures a consistency of the shapes of the two fields at hyper deformations, i.e., two tangent ellipsoids. The Hamiltonian is obtained by adding the spin-orbit and the Coulomb terms to the Woods-Saxon potential. The eigenvalues are obtained by diagonalization of the Hamiltonian in the semi-symmetric harmonic two centre basis [37, 38]. In this work, the major quantum number used is \( N_{\text{max}}=11 \). The two centre Woods-Saxon model will be used to compute shell and pairing corrections that contribute to the total energy of the nucleus. There are many versions of two centre shell models in the literature, some of them being elaborated very recently [39, 40].

3. RESULTS

The possibility that at a given deformation the potential energy exhibits a minimum that can be a cause for the asymmetric fission is investigated. For this purpose, we calculated the potential energy surface for different values of the necking parameter \( C_3 \): -0.07, -0.05, -0.03, -0.01, 0.01, 0.03, 0.05, 0.1, 1. and 10000 fm\(^{-1}\). When the necking parameter has negative values, the shapes are swollen in the median surface and when this parameter reaches positive values, the shapes are necked. The necked shapes characterizes the configurations of the outer fission barrier in the case of actinides and those close to the scission point. In the region of the ground state configuration, usually the shapes are swollen in the interior. The pertinent values of the elongation \( R \) that must be investigated are smaller than 20 fm, because for larger values the system is already split in two separated fragments. In order to scan all the asymmetries, the possible mass partitions ranging from \( A_2=4 \) (alpha decay) and \( A_2=90 \) (symmetric fission) are analysed in this work. As mentioned, \( A_2 \) is the mass number of the light fragment. The potential energy landscapes are calculated and the results are plotted in Fig. 1. In order to have a representation of the interrelation between the different values of \( C_3 \) and the parametrization involved, in Fig. 2 the
Fig. 1 – Deformation energy as function of the mass number of the light fragment $A_2$ and the elongation $R$. Each panel is associated to a fixed value of the neck parameter $C_3$. $C_3=-0.07, -0.05, -0.03, -0.01, 0.01, 0.03, 0.05, 0.1, 1.$ and $10000$ fm$^{-1}$ for the panels (a), (b), (c), (d), (e), (f), (g), (h), (i), and (j), respectively.

An important feature can be observed in the panel (d) of the Fig. 1. An isomeric minimum exists at a very large value of the elongation, the shapes being swollen in

family of shapes associated to the light fragment of mass $A_2=80$ are displayed for each value of $C_3$.
the median part of the parent nucleus. The location of this minimum is marked with an arrow. Therefore, the system must proceed through this minimum in its way to fission in order to minimize the action and, therefore, to increase the penetrability of the fission barrier. A larger penetrability means an increased yield. This minimum is at an asymmetry $A_2 \approx 68$ and $R \approx 14$ fm. On the other hand, it is possible to guess by corroborating Fig. 1 and Fig. 2 that for positive values of $C_3$, at $R = 16$ fm, the scission could be produced. However, between negative and positive values of $C_3$ a ridge is produced in the potential landscape. This ridge is responsible for an outer barrier. Therefore the following scenario can be postulated. The parent

Fig. 2 – Families of nuclear shapes for a mass asymmetry given by $A_2=80$ and different values of the neck parameter $C_3$. The labels (a)-(j) corresponds to the same values of $C_3$ as those given in Fig. 1. The values of the elongation are marked on the plot.
Fig. 3 – The predicted macroscopic-microscopic fission barrier for the fragmentation of the $^{180}$Hg with light fragment $A_2=80$ is displayed with a full line. The corresponding liquid drop barrier is plotted with a dashed line.

nucleus acquires deformed shapes in its path towards fission. But, in order to have the minimal deformation energies it is mandatory that the system reaches a region in the configuration space with small values of the energy after $R \approx 14$ fm, that is the isomeric state. This isomeric state is characterized by a non-zero mass asymmetry parameter. After that, the system must penetrate the outer barrier in order to split into two separated fragments. Such a scenario is compatible with the experimental results. The predicted barrier for the asymmetric fission of $^{180}$Hg is plotted in Fig. 3. The isomeric state can be identified at an elongation $R \approx 14$ fm that is characterized by an asymmetry of $A_2=68$. The outer barrier is located in the interval 15-18 fm. The macroscopic barrier is plotted with a dashed line. The macroscopic calculations does not exhibit an isomeric configuration. Therefore, the shell effects are responsible for the minimum of the isomeric well and for the deviation of the fission path from the symmetric configurations. The system tries to acquire the symmetric configuration by a penetration of the outer barrier. This external barrier is lowered for symmetric
Potential energy landscape for $^{180}$Hg

fragmentation. However, as observed in Fig. 2, by increasing the neck parameter $C_3$, the scission is rapidly obtained and the symmetric configuration is not reached, the experimental values being for mass $A_2=80$.

After a more careful analysis of the Fig. 1 it can be also assessed that the cluster emission could be a probable process. A valley in the potential energy surface corresponding to $A_2=14-16$ was identified in the panels (e)-(j). These clusters could be isotopes of C, N or O. This behaviour resemble to that of the "magic" valley obtained in the microscopic description of cluster decays [12–14]. However, in the latter case the daughter disintegration product is a magic nucleus.

4. CONCLUSION

In conclusion, using the macroscopic-microscopic approach based on the Woods-Saxon two-centre shell model, it is possible to explain the asymmetric fission fragment distribution of $^{180}$Hg. A local minimum of the potential energy surface in a given region of configuration space characterized by a large mass asymmetry could be a cause for the experimental behaviour of this distribution. The fission is produced after the passage from this region. A prediction for the fission barrier is offered. Recent studies that predict the existence of this minimum [2] are confirmed. Furthermore, a valley in the potential surface located at very large mass asymmetries indicates that the cluster emission from $^{180}$Hg could be a very probable process.

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