SHELL MODEL WITH REALISTIC LOW-MOMENTUM TWO-BODY EFFECTIVE INTERACTIONS

A. COVELLO

Dipartimento di Fisica, Università di Napoli Federico II, and Istituto Nazionale di Fisica Nucleare, Complesso Universitario di Monte S. Angelo, Via Cintia, I-80126 Napoli, Italy

E-mail: covello@na.infn.it

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In this paper, a brief outline is first given of the theoretical framework for shell-model calculations employing a two-body effective interaction derived from the free nucleon-nucleon potential. Then, some selected results of recent studies of nuclei beyond $^{132}$Sn are presented, which have been obtained starting from the CD-Bonn potential renormalized by use of the $V_{\text{low-k}}$ approach. Attention is focused on the two-valence-particle nuclei $^{134}$Te and $^{134}$Sn and on the odd-neutron nucleus $^{137}$Xe.

Key words: Nuclear structure, shell model, realistic effective interactions.

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1. INTRODUCTION

A major problem with the nuclear shell-model has long been the choice of the two-body effective interaction $V_{\text{eff}}$ between valence nucleons. Since the early 1950s through the mid 1990s, in the majority of shell-model calculations either empirical effective interactions containing adjustable parameters have been used or the two-body matrix elements themselves have been treated as free parameters. Although efforts to derive effective interactions from the free nucleon-nucleon ($NN$) potential started as early as 50 years ago [1, 2], for a long time there was widespread scepticism about the practical value of what had become known as “realistic shell-model calculations” (see, e.g., [3]). A revival of interest in this more fundamental approach to the shell model occurred in the early 1990s and continued to increase in the following years. A comprehensive account of the developments in the field through 2008 is given in the review paper [4]. In summary, however, one may say that the main outcome of these developments is that realistic effective interactions derived from modern $NN$ potentials are able to provide, with no adjustable parameters, an accurate description of nuclear properties. This was indeed found [5] to be particularly true for nuclei in the $^{132}$Sn and $^{208}$Pb regions.

In this paper, after a brief outline of the theoretical framework, some selected results of our studies in the $^{132}$Sn region are presented and discussed, focusing attention on nuclei beyond $^{132}$Sn. Over the past several years, in fact, we have performed...
various shell-model studies [5–7] of trans-tin nuclei, which have all yielded results in very good agreement with experiment. In these studies, a unique shell-model Hamiltonian has been used with single-particle energies taken from experiment and a low-momentum two-body effective interaction derived from the CD-Bonn nucleon-nucleon potential without any adjustable parameter. Based on these results, we have recently used the same Hamiltonian to study the evolution of single-particle states when moving away from the one valence-nucleon systems $^{133}_{\text{Sn}}$ and $^{133}_{\text{Sb}}$. In particular, we have considered nuclei in the close vicinity of $^{132}_{\text{Sn}}$ [8,9] which are within reach of transfer reactions with radioactive ion beams.

2. OUTLINE OF THEORETICAL FRAMEWORK

We give here an outline of the essentials of our derivation of the shell-model effective interaction from the free $NN$ potential. A detailed description including references to original literature can be found in [4].

The Schrödinger equation for a system of A nucleons interacting via two-body forces can be written

$$H\Psi_i = (H_0 + H_1)\Psi_i = E_i\Psi_i,$$  \hspace{1cm} (1)

where

$$H_0 = T + U$$  \hspace{1cm} (2)

and

$$H_1 = V_{NN} - U,$$  \hspace{1cm} (3)

$T$ being the kinetic energy and $U$ an auxiliary potential introduced to define a convenient single-particle basis. The effective interaction $V_{\text{eff}}$ acting only within a reduced model space is then defined through the eigenvalue problem

$$PH_{\text{eff}}P^{|\Psi_\alpha\rangle} = P(H_0 + V_{\text{eff}})P^{|\Psi_\alpha\rangle} = E_\alpha P^{|\Psi_\alpha\rangle},$$  \hspace{1cm} (4)

where the $E_\alpha$ and the corresponding $\Psi_\alpha$ are a subset of the eigenvalues and eigenfunctions of the original Hamiltonian defined in the complete Hilbert space. The $P$ operator projects onto the chosen model space, which is defined in terms of the eigenvectors of the unperturbed Hamiltonian $H_0$.

A well-established approach to the derivation of realistic effective interactions from the free $NN$ potential $V_{NN}$ is provided by the $Q$-box folded-diagram expansion [4]. In our calculations, we have used as initial input the CD-Bonn potential [10]. The existence of a strong repulsive core, however, makes this potential unsuitable for perturbative calculations and hence requires a renormalization procedure. This we do through use of the $V_{\text{low-k}}$ approach [11], which has proved to be an advantageous alternative to the traditional Brueckner $G$-matrix method. More precisely, we construct a smooth low-momentum potential, $V_{\text{low-k}}$, by integrating out the high-momentum
modes of $V_{NN}$ down to a cutoff momentum $\Lambda$. This integration is carried out with the requirement that the deuteron binding energy and phase shifts of $V_{NN}$ up to $\Lambda$ are preserved by $V_{\text{low}-k}$.

Once the $V_{\text{low}-k}$ is obtained, we use it, plus the Coulomb force for protons, as input interaction for the derivation of $V_{\text{eff}}$. The calculation of the $Q$-box, which is a sum of irreducible valence-linked diagrams, is performed at second order in the interaction. That is to say, we include four two body terms: the $V_{\text{low}-k}$, the two core polarization diagrams $V_{1p1h}$ and $V_{2p2h}$, corresponding to one particle-one hole and two particle-two hole excitations, and a ladder diagram accounting for excluded configurations above the chosen model space. The shell-model effective interaction $V_{\text{eff}}$ is finally obtained by summing up the $Q$-box folded diagram series using the Lee-Suzuki iteration method [12].

3. SELECTED FINDINGS

In this section, we report some selected results of our realistic shell-model calculations for nuclei beyond doubly magic $^{132}$Sn, which is assumed to be a closed core. We let the valence protons occupy the five levels $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ of the 50-82 shell, while for neutrons the model space includes the six levels $0h_{9/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $0i_{13/2}$ of the 82-126 shell. As regards the choice of single-proton and -neutron energies, they have been taken from experiment. The adopted values are reported in [13] and [7] for protons and neutrons, respectively.

We consider here the three nuclei $^{134}$Te, $^{134}$Sn and $^{137}$Xe, the latter having been the subject of a recent $(d,p)$ experiment [14]. We first discuss our results for the two nuclei $^{134}$Te and $^{134}$Sn. These nuclei, with two valence protons and neutrons, respectively, allow a direct study of the main features of the proton-proton and neutron-neutron interaction in the presence of the same closed core. This is particularly interesting because the available experimental data have evidenced a large difference between proton and neutron energy gaps. More precisely, while the first $2^+$ state in $^{134}$Te lies at 1.28 MeV, the excitation energy of this state in $^{134}$Sn drops to 726 keV, making it the lowest first-excited $2^+$ level observed in a semi-magic even-even nucleus over the whole chart of nuclides.

In Fig. 1 the experimental energies of the first three excited levels in both nuclei are compared with the calculated ones. We see that the properties of $^{134}$Te, which exhibits a “normal” proton pairing, as well as those of $^{134}$Sn with a weak neutron pairing, are well described by our realistic effective interaction. Especially worthy of note is that the energy of the $2^+$ states, namely the proton and neutron gap, is remarkably well reproduced. This clearly means that our effective interaction possesses good pairing properties, which makes it interesting to clarify the mechanism
that lies behind them. To this end, we note that all states reported in Fig. 1 have a weak configuration mixing. More precisely, the ground state as well as the three excited states are dominated by the \((\pi g_{7/2})^2\) configuration in \(^{134}\text{Te}\) and by the \((\nu f_{7/2})^2\) configuration in \(^{134}\text{Sn}\). This makes it possible to only focus our attention on the matrix elements involving these two configurations. Furthermore, since we are interested in the pairing properties of the nucleon-nucleon force in the nuclear medium, we consider an effective interaction derived without inclusion of the Coulomb force.

In Fig. 2 we show the matrix elements of \(V_{\text{eff}}\), \(V_{\text{low-k}}\), and \(V_{1p1h}\) for the \((\nu f_{7/2})^2\) and \((\pi g_{7/2})^2\) configurations. In this analysis we consider only \(1p1h\) contributions, because the other three diagrams play a minor role in our discussion. From Fig. 2(a) we see that the behaviour of \(V_{\text{eff}}\) as a function of the angular momentum is similar to that of the bare \(V_{\text{low-k}}\) interaction, the latter being only slightly modi-
fied by the $1p1h$ contributions. The $V_{1p1h}$ curve is in fact almost flat around 0 MeV, showing a non negligible decrease only for $J = 0^+$. Actually, the $J = 0^+$ matrix element of $V_{\text{low-k}}$, which is the most affected one by medium effects, is shifted down by only 250 keV including all contributions. In practice, the behaviour exhibited by the $(\nu f_{7/2})^2$ multiplet is essentially determined by the $V_{\text{low-k}}$ interaction, which results in the weakness of the neutron-neutron pairing component.

The situation is just reversed when we look at the curves of Fig. 2(b) for the $(\pi g_{7/2})^2$ configuration, which evidence the crucial role of the $1p1h$ core polarization in determining the proton-proton pairing. In other words, $V_{\text{low-k}}$ does not possess the right pairing properties, the $J = 0^+$ and $2^+$ matrix elements being almost equal and quite close to those with $J = 4^+$ and $6^+$. On the contrary, $V_{1p1h}$ has a $J = 0^+$ matrix element much larger than those with $J \neq 0$, which explains the gap existing between the ground and the $2^+$ states in the spectrum of $^{134}$Te.

In summary, the effective pairing interaction between two protons and two neutrons outside $^{132}$Sn may be mainly traced to different core renormalizations of the $V_{\text{low-k}}$ NN potential. For protons, the dominant role is played by the one particle-one hole excitations, which produces a sizeable energy gap. These core excitations play instead a minor role for neutrons, which results in a reduced neutron pairing. This important point is discussed in greater detail in [15].

Let us now come to the odd-mass nucleus $^{137}$Xe. As already mentioned above, the single-neutron structure of this nucleus was recently investigated via the $^{136}$Xe $(d,p)$ reaction. In particular, spectroscopic factors for several states in $^{137}$Xe were extracted from the data by DWBA calculations, providing valuable information on the evolution of single-neutron states outside $^{132}$Sn, which is a subject of great current interest [16, 17].

In Fig. 3, the calculated excitation energies and spectroscopic factors are compared with those obtained in [14], including, for each $J^\pi$, only the state with the largest spectroscopic factor. The experimental energies are well reproduced by the theory, with discrepancies not exceeding 150 keV, the only exception being the $13/2^+$ state, whose energy is overestimated by about 330 keV.

As regards the spectroscopic factors, an overall good agreement is found between our values and those obtained in [14]. In fact, the discrepancies do not exceed 23%, the only exception concerning the $9/2^-$ state for which we overestimate the value of [14] by 67%. We see that the predicted single-particle strength, in agreement with experiment, is strongly concentrated in the yrast states only for $J^\pi = 7/2^-$ and $13/2^+$. This is not the case for the $J^\pi = 1/2^-, 3/2^-$ and $5/2^-$ states. However, we find that for the first two angular momenta the spectroscopic factors of the yrast states are still the largest ones with the single-particle strength mainly distributed over the lowest-lying states (see [8] for more details).
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Fig. 3 – Experimental and calculated energies and spectroscopic factors for $^{137}$Xe.

4. CONCLUDING REMARKS

In this paper, I have given a very brief survey of realistic shell-model calculations and presented, by way of illustration, some selected results of our recent calculations for nuclei beyond doubly-magic $^{132}$Sn. I have first focused attention on the two-valence-nucleon systems $^{134}$Te and $^{134}$Sn, which offer the opportunity to test directly the proton-proton and neutron-neutron effective interaction in this region. Our results for both nuclei are in very good agreement with experiment. In particular, they reproduce remarkably well the large difference between the proton and neutron energy gap, the latter being significantly smaller than the former. From an analysis of our effective interaction, we have brought out the key role of core polarization in producing this difference.

I have then reported on our very recent study [8] of the single-neutron structure of the nucleus $^{137}$Xe. Our results reproduce very well the experimental excitation energies and yield spectroscopic factors that come close to those extracted from the data [14].

In closing, it is well worth emphasizing that in all our studies [5–9] of nuclei in the $^{132}$Sn region we have consistently employed a unique Hamiltonian with a low-momentum two-body effective interaction derived from the CD-Bonn $NN$ potential without any adjustable parameter.

Acknowledgements. This paper is dedicated to my friend Apolodor Raduta on the occasion of his 70th birthday. I have known Apolodor for more than 20 years and remember with pleasure the good times we had together in Naples as well as at various Predeal Summer Schools. Dear Apolodor, I wish you many more years of health and fruitful activity.
REFERENCES

5. L. Coraggio, A. Covello, A. Gargano, N. Itaco, Phys. Rev. C 80, 021305(R) (2009), and references therein.