A STUDY OF CHARGE-EXCHANGE EXCITATIONS WITH SKYRME-TYPE INTERACTIONS USING THE FINITE RANK SEPARABLE APPROXIMATION

A.P. SEVERYUKHIN\textsuperscript{1,a}, V.V. VORONOV\textsuperscript{1,b}, I.N. BORZOV\textsuperscript{1,c}, NGUYEN VAN GIAI\textsuperscript{2}

\textsuperscript{1} Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
E-mail\textsuperscript{a}: sever@theor.jinr.ru
E-mail\textsuperscript{b}: voronov@theor.jinr.ru
E-mail\textsuperscript{c}: ibor48@mail.ru

\textsuperscript{2} Institut de Physique Nucléaire, CNRS-IN2P3 and Univ. Paris-Sud, 91405 Orsay, France
E-mail: nguyen@ipno.in2p3.fr

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We briefly present the finite rank separable approximation (FRSA) of Skyrme-type residual interactions in the case of charge-exchange modes in nuclei. We show how the FRSA can be used to obtain the equations leading to charge-exchange excitations where the coupling between one- and two-phonon components, as well as pairing effects are treated. Applications are done in the simpler cases of $^{132}\text{Sn}$ and $^{78}\text{Ni}$. First, we check that the FRSA reproduces reasonably well the full charge-exchange RPA results for the spin-dipole resonances in $^{132}\text{Sn}$. Next, the phonon-phonon coupling effects on the $\beta$-decay half-life of $^{78}\text{Ni}$ are studied.

Key words: QRPA, phonon-phonon coupling, Nickel.

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1. INTRODUCTION

The charge-exchange nuclear modes constitute an interesting problem not only from the nuclear structure point of view but they are also very important for the nuclear astrophysics applications. One of the successful tools for studying these modes is the quasiparticle random phase approximation (QRPA) with the self-consistent mean-field derived from a Skyrme-type energy-density functional (EDF), see e.g., [1–4]. These QRPA calculations allow one to describe the properties of the ground and excited states using the same EDF.

A comparison of such calculations with recent experimental data [5] demonstrates that QRPA approach cannot reproduce correctly the strength distribution of the spin-isospin resonances. It is desirable to extend the description beyond the QRPA scheme in order to include damping effects observed experimentally [6–8]. Using the Skyrme EDF and the RPA, such attempts in the past [9, 10] have allowed one to understand the damping of charge-exchange resonances and their particle decay. Recently, the damping of the Gamow-Teller (GT) mode was investigated using the

Skyrme-RPA plus particle-vibration coupling (PVC) [11]. However, the size of the configuration space increases very rapidly and one is limited to a number of typical cases.

It is somewhat simpler to include the PVC in QRPA calculations if one uses separable forces [7, 12]. This idea has led us to develop the finite rank separable approximation (FRSA) for the Skyrme interactions [13, 14], allowing one to perform calculations in large configuration spaces. Applications of the method to study the low-lying states and giant resonances within the QRPA and beyond can be found in Refs [13–16]. Recently, an extension of this approach was proposed for the charge-exchange nuclear excitations [17].

Here, we describe briefly the FRSA method for charge-exchange excitations and we present its extension to the coupling between one- and two-phonon terms in the wave functions of excited states. Before investigating the effects of this coupling we check that the FRSA can reproduce the main characteristics of such nuclear modes. Finally, we illustrate the effects of one- and two-phonon couplings by considering the $\beta$-decay half-life of $^{78}$Ni.

2. FINITE RANK APPROXIMATION FOR CHARGE-EXCHANGE MODES

The RPA and QRPA methods using the FRSA have been introduced in Refs. [13, 14]. In this section we present their extension to the case of charge-exchange excitations [18, 19]. The quasiparticle states are labelled by $a, b, ... (\alpha, \beta, ...)$ for neutron (proton) states. Assuming spherical symmetry for the nuclei considered here, each state $a = (a, m_a)$ is defined by the quantum numbers $a = \{E_a, l_a, j_a\}$ and the angular momentum projection $m_a$. Here, $E_a = (\epsilon^2_a + \Delta^2_a)^{1/2}$ is the corresponding quasiparticle energy expressed in terms of the Hartree-Fock (HF) single-particle energy $\epsilon_a$ and pairing energy $\Delta_a$. The starting point of the method is the HF-BCS description [20] of the parent ground state. The method could be extended to the Hartree-Fock-Bogoliubov (HFB) framework and the corresponding QRPA equations would have the same structure if one expresses them in the canonical quasiparticle basis.

Denoting by $c^+_a, c_a$ (resp. $c^*_a, c_a$) the creation and annihilation operators of a neutron (resp. proton) in a single-particle state, the creation and annihilation operators of neutron quasiparticles $-d^+_a, d_a$ - are obtained by the canonical Bogoliubov transformation:

$$c^+_a = u_a d^+_a + (-1)^{j_a-m_a}v_a d_{a'},$$

where $a' \equiv (a, -m_a)$, and $u_a, v_a$ are the BCS amplitudes. Similar relations apply for the proton states.

To each quasiparticle state $a$ corresponds a HF wave function:

$$\varphi_a(r, \sigma) = \frac{f_a(r)}{r} Y_a(\hat{r}, \sigma).$$
In practice, the HF mean field is calculated with a Skyrme-type interaction whereas the BCS equations are solved self-consistently with a density-dependent, zero-range pairing force of the type [21]:

\[ V_{\text{pair}}(r_1, r_2) = V_0 \left( 1 - \eta \left( \frac{\rho(r_1)}{\rho_0} \right)^\alpha \right) \delta (r_1 - r_2) , \]

where \( V_0, \eta, \alpha \) are adjusted parameters, \( \rho(r) \) is the HF-BCS ground state density, \( \rho_0 \) being the nuclear matter saturation density. The parameters are determined by adjusting the empirical odd-even mass differences of the nuclei in the region under study. One thus obtains the sets of proton and neutron quasiparticle states, \( \alpha \) and \( \alpha \), with the corresponding wave functions (\( \varphi_\alpha \) and \( \varphi_\alpha \), quasiparticle energies (\( E_\alpha \) and \( E_\alpha \)) and BCS amplitudes (\( u_\alpha, v_\alpha \) and \( u_\alpha, v_\alpha \)). The continuum part of the quasiparticle spectrum is generally discretized by various methods like harmonic oscillator expansion or box boundary conditions.

To build the QRPA equations on the basis of quasiparticle states as defined above and using consistently the residual interactions (derived from the Skyrme EDF in the particle-hole channel and from the zero-range pairing force in the particle-particle channel) is a standard procedure [22]. This leads to the familiar QRPA equations in configuration space:

\[ \begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix} , \]

where the dimensions of the matrices \( A \) and \( B \) grow very rapidly with the size of the nuclear system unless severe and damaging cut-offs are made to the 2-quasiparticle configuration space. It is well known that, if the matrix elements of the \( A \) and \( B \) matrices take a separable form the eigenvalues of Eq.(4) can be obtained as the roots of a relatively simple secular equation [12, 23]. In the case of the Skyrme EDF this feature has been exploited by different authors [13,24,25]. In particular, a method has been proposed in Refs. [13,14] to calculate non charge-exchange excitations, and we extend here this method to the case of non closed-shell nuclei and charge-exchange excitations. The main step is to replace the Skyrme particle-hole interaction by its Landau-Migdal approximation and to keep only the \( l = 0 \) terms. We recall that the central terms of the Skyrme force do not act in states with relative momentum \( l \geq 2 \). It has been shown [2] that the effect of the two-body spin-orbit residual interaction on the G-T excitations is small, and furthermore, we do not consider here versions of the force having a two-body tensor component. Thus, keeping only \( l = 0 \) might not be that bad, and this will be discussed in the next section.

We write the p-h interaction in the spin-isospin channel in the following form:

\[ V_{\text{res}}(r_1, r_2) = N_0^{-1} \left[ F'_{0}(r_1) + G'_{0}(r_1) \sigma^{(1)} \cdot \sigma^{(2)} \right] \times \tau^{(1)} \cdot \tau^{(2)} \delta (r_1 - r_2) , \]
where \( \sigma^{(i)} \) and \( \tau^{(i)} \) are the spin and isospin operators, and \( N_0 = 2k_Fm^*/\pi^2h^2 \) with \( k_F \) and \( m^* \) standing for the Fermi momentum and nucleon effective mass in nuclear matter. \( F'_0, G'_0 \) are functions of the position \( r \) through their density dependence, and their expressions in terms of the Skyrme force parameters can be found in Ref. [26].

When calculating particle-hole matrix elements of the interaction (5) one obtains a sum of products of angular momentum coefficients times one-dimensional radial integrals. These integrals are numerically calculated by choosing a large enough cutoff radius \( R \) and using an \( N \)-point integration Gauss formula with abscissas \( r_k \) and weights \( w_k \) [13]. Thus, one is led to deal with a problem where the \( V_{ph,p'h'} \) matrix elements are sums of products, \( \Sigma_n V_{ph}(n)V_{p'h'}(n) \), and the number \( \tilde{N} \) of terms in the sums depends only on \( N (\tilde{N} = 4N \) for the cases studied here). One can call it a separable approximation of finite rank \( \tilde{N} \) since finding the roots of the secular equation amounts to find the zeros of a \( \tilde{N} \)-determinant, and the dimensions of the determinant are independent of the size of the configuration space, i.e., of the nucleus considered.

Let us briefly explain the main steps leading to the FRSA solutions of the QRPA equations. Our goal is to express the matrix elements of the angular momentum coupled matrices \( A_{a\alpha,b\beta} \) and \( B_{a\alpha,b\beta} \) appearing in the QRPA equations (4) as sums of products of terms depending on \((a,\alpha)\) or \((b,\beta)\). We separate in the calculations the cases of natural parity states \((L = J)\) and unnatural parity states \((L = J \pm 1)\), where \( L \) and \( J \) are the orbital and total angular momenta of the excitations considered.

First, we introduce the interaction strengths \( \kappa^{(k)}_F, \kappa^{(k)}_G \) at points \( r_k \) by the following definitions:

\[
\begin{pmatrix}
\kappa^{(k)}_F \\
\kappa^{(k)}_G
\end{pmatrix} = -N_0^{-1} R w_k \frac{F'_0(r_k)}{2r_k} \begin{pmatrix} F'_0(r_k) \\ G'_0(r_k) \end{pmatrix},
\]

where all quantities are defined in the preceding paragraph. Next, we introduce the quantities \( h^{Jk}_{a\alpha} \) and \( g^{JLk}_{a\alpha} \) which are related to reduced matrix elements of the operators \( Y_J \) and \( T_{LJ} \) by:

\[
\begin{align*}
h^{Jk}_{a\alpha} &= f_a(r_k)f_a(r_k)(a||i^JY_J||a), \\
g^{JLk}_{a\alpha} &= f_a(r_k)f_a(r_k)(a||i^LT_{LJ}||a).
\end{align*}
\]

Now, the matrix elements of the QRPA matrices take the form:
The coefficients $\kappa^{(n)}$ (resp. $d^{(n)}_{aa}$) appearing in Eq.(8) expressed in terms of the quantities defined in Eq.(6) (resp. Eq.(7). The index $k$ runs from 1 to the number of Gauss points $N$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\kappa^{(n)}$</th>
<th>$d^{(n)}_{aa}$</th>
<th>$\kappa^{(n)}$</th>
<th>$d^{(n)}_{aa}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$\kappa^{(k)}_{G}$</td>
<td>$g_{aa}$</td>
<td>$\kappa^{(k)}_{G}$</td>
<td>$g_{aa}$</td>
</tr>
<tr>
<td>$k+N$</td>
<td>$\kappa^{(k)}_{F}$</td>
<td>$h_{aa}$</td>
<td>$\kappa^{(k)}_{F}$</td>
<td>$h_{aa}$</td>
</tr>
</tbody>
</table>

Equation (8) becomes:

$$
A_{aa,b\beta} = -2\hat{J}^{-2} (u_{a}v_{\alpha}v_{\beta} + v_{a}u_{\alpha}v_{\beta})
\times \sum_{n=1}^{2N} \kappa^{(n)} d_{aa}^{(n)} \delta_{a\alpha} \delta_{b\beta},
$$

$$
B_{aa,b\beta} = -2\hat{J}^{-2} (u_{a}v_{\alpha}v_{\beta} + v_{a}u_{\alpha}v_{\beta})
\times \sum_{n=1}^{2N} \kappa^{(n)} d_{aa}^{(n)} \delta_{a\beta} .
$$

Here, $\hat{J} = \sqrt{2J+1}$ and the 2-quasiparticle energies are $\varepsilon_{aa} = E_{a} + E_{a}$. The $\kappa^{(n)}$ interaction strengths and $d_{aa}^{(n)}$ matrix elements in Eq. (8) are defined in Table I.

Now, we show how to construct the secular equation in a determinantal form. In the $4N$-dimensional space we introduce a vector \( \begin{pmatrix} D_+ \\ D_- \end{pmatrix} \) by its components:

$$
D^{\nu n}_{\pm} = \sum_{aa} d^{(n)}_{aa} \psi_{\nu n}^{(\pm)} \left( X^{\nu}_{aa} \pm Y^{\nu}_{aa} \right),
$$

where $u_{aa}^{(\pm)} = u_{a}v_{\alpha} v_{b} u_{a}, n = 1, 2, ... 2N$, and the index $\nu$ refers to the $\nu$-th QRPA state. The QRPA amplitudes $(X, Y)$ can be expressed in terms of the $D_{\pm}$ as:

$$
X^{\nu}_{aa} = \frac{1}{\varepsilon_{aa} - \omega_{\nu}} \sum_{n=1}^{2N} d^{(n)}_{aa} \kappa^{(n)}
\times \left( D^{\nu n}_{+} u_{aa}^{(+)\nu} + D^{\nu n}_{-} u_{aa}^{(-)\nu} \right),
$$

$$
Y^{\nu}_{aa} = \frac{1}{\varepsilon_{aa} + \omega_{\nu}} \sum_{n=1}^{2N} d^{(n)}_{aa} \kappa^{(n)}
\times \left( D^{\nu n}_{+} u_{aa}^{(+)\nu} - D^{\nu n}_{-} u_{aa}^{(-)\nu} \right).
$$
Now, the QRPA equations (4) become:

$$\begin{pmatrix} M_1 - \frac{1}{2} I & M_2 \\ M_2 & M_3 - \frac{1}{2} I \end{pmatrix} \begin{pmatrix} D_+ \\ D_- \end{pmatrix} = 0 ,$$

(12)

where the matrix elements of the $2N \times 2N$ matrices $M_k$ ($k=1, 2, 3$) have the following expressions:

$$M_{1}^{nn'} = \frac{\kappa(n')}{J^2} \sum_{\alpha \alpha'} d_{\alpha \alpha'}^{(n')} d_{\alpha' \alpha}^{(n)} (u_{\alpha \alpha}^{(+)})^2 \varepsilon_{\alpha \alpha} ,$$

(13)

$$M_{2}^{nn'} = \frac{\kappa(n')}{J^2} \sum_{\alpha \alpha'} d_{\alpha \alpha'}^{(n')} d_{\alpha' \alpha}^{(n)} (u_{\alpha \alpha}^{(+)})(u_{\alpha \alpha}^{(-)}) \varepsilon_{\alpha \alpha} ,$$

(14)

$$M_{3}^{nn'} = \frac{\kappa(n')}{J^2} \sum_{\alpha \alpha'} d_{\alpha \alpha'}^{(n')} d_{\alpha' \alpha}^{(n)} (u_{\alpha \alpha}^{(-)})^2 \varepsilon_{\alpha \alpha} ,$$

(15)

The eigenvalues $\omega$ of Eq.(4) are the roots of the equation:

$$\det \begin{pmatrix} M_1 - \frac{1}{2} I & M_2 \\ M_2 & M_3 - \frac{1}{2} I \end{pmatrix} = 0$$

(16)

Since the vector $\begin{pmatrix} D_+ \\ D_- \end{pmatrix}$ satisfies Eq.(12), the QRPA amplitudes $X_{\alpha \alpha}^{\nu}, Y_{\alpha \alpha}^{\nu}$ are obtained by Eqs.(10), (11) and the normalization condition:

$$\sum_{\alpha \alpha'} |X_{\alpha \alpha'}^{\nu}|^2 - |Y_{\alpha \alpha'}^{\nu}|^2 = 1 .$$

(17)

The excitation energies with respect to the parent ground state are

$$\Omega_{J_i}^{\pm} = \omega_{J_i} \mp (\lambda_n - \lambda_p) ,$$

(18)

where $\omega_{J_i}$ denotes the QRPA energies in the $T_z$ channels, $\lambda_n$ and $\lambda_p$ being the neutron and the proton chemical potentials, respectively.

In the next step, we construct the wave functions from a linear combination of one-phonon and two-phonon configurations

$$\Psi_{\nu}(JM) = \left( \sum_i R_i(J\nu) Q_{iJM}^{+} \right) + \sum_{\lambda_1 \lambda_2} P_{\lambda_1 \lambda_2}^{+}(J\nu) \left[ Q_{\lambda_1 \lambda_2}^{+} \bar{Q}_{\lambda_2 \lambda_2}^{+} \right]_{JM} |0\rangle .$$

(19)

$Q_{\lambda \lambda}^{+} |0\rangle$ is the QRPA state having energy $\bar{\Omega}_{\lambda i}$. Non-charge-exchange QRPA is performed in the same way as charge-exchange QRPA [14].
The normalization condition for the wave functions (19) is

\[ \sum_i R_i^2(J\nu) + \sum_{\lambda_1i_1,\lambda_2i_2} (P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu))^2 = 1. \]  

(20)

Using the variational principle we obtain a set of linear equations

\[ (\Omega_{\lambda_1} - E_{\nu}) R_i(J\nu) + \sum_{\lambda_1i_1,\lambda_2i_2} U_{\lambda_1i_1,\lambda_2i_2}^{\lambda_1i_1}(Ji_1) P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu) = 0, \]  

(21)

\[ (\Omega_{\lambda_1} + \Omega_{\lambda_2} - E_{\nu}) P_{\lambda_2i_2}^{\lambda_1i_1}(J\nu) + \sum_i U_{\lambda_2i_2}^{\lambda_1i_1}(Ji_1) R_i(J\nu) = 0. \]  

(22)

Its solution requires to compute the matrix elements coupling one- and two-phonon configurations

\[ U_{\lambda_2i_2}^{\lambda_1i_1}(Ji_1) = \langle 0|Q_{\lambda_1i_1}^+ H_{\lambda_2i_2}^{+1} Q_{\lambda_2i_2}^{1+} |0 \rangle. \]  

(23)

Eqs. (21), (22) have the same form as the equations of the quasiparticle-phonon model [7, 12], but the single-particle spectrum and the residual interaction are derived from the same Skyrme EDF.

3. RESULTS AND DISCUSSION

We first evaluate the accuracy of the FRSA by comparing results obtained using this separable approximation with those from a full treatment of the Skyrme-type p-h residual interaction. In the literature one can find full charge-exchange TDA [2] and RPA [27] calculations of spin-dipole (SD) states built on $^{90}$Zr. We select as illustrative cases the SD transitions in the $T_-$ and $T_+$ channels from the parent ground states of $^{90}$Zr and $^{132}$Sn. For the sake of simplicity the check is done within the TDA without pairing effects. The RPA results would be similar because the backward-going graphs are somewhat blocked in these charge-exchange channels, and in any case we are just interested here in assessing the validity of the FRSA.

The calculations are done with the Skyrme parameter set SGII [26] which was designed to give reasonable values for the spin and spin-isospin Landau parameters ($G_0=0.01, G_0'=0.50$ at saturation density). The HF mean field is first calculated in coordinate space, then the single-particle spectra necessary for the TDA calculations (energies and wave functions) are obtained by diagonalizing the HF mean field in a 12-shells harmonic oscillator basis and keeping all states below 100 MeV. This is sufficient to exhaust the Ikeda sum rule $3(N-Z)$ [28] for the G-T strength as well as the SD sum rule [29, 30]:

\[ S_- - S_+ = \frac{9}{4\pi} [N\langle r^2 \rangle_n - Z\langle r^2 \rangle_p], \]

(24)
SD sum rule values in $^{90}$Zr and $^{132}$Sn, expressed in fm$^2$. $S(r_n, r_p)$ stands for the r.h.s. of Eq.(24). The experimental data are taken from Refs. [27, 30].

<table>
<thead>
<tr>
<th></th>
<th>$r_n$ (fm)</th>
<th>$r_p$ (fm)</th>
<th>$S(r_n, r_p)$</th>
<th>$S_-$</th>
<th>$S_+$</th>
<th>$S_- - S_+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{90}$Zr</td>
<td>full</td>
<td>4.253</td>
<td>4.198</td>
<td>142.9</td>
<td>279.2</td>
<td>135.7</td>
</tr>
<tr>
<td></td>
<td>FRSA</td>
<td>4.253</td>
<td>4.198</td>
<td>142.9</td>
<td>279.6</td>
<td>136.0</td>
</tr>
<tr>
<td></td>
<td>Expt.</td>
<td></td>
<td></td>
<td>271±14</td>
<td>124±11</td>
<td>147±13</td>
</tr>
<tr>
<td>$^{132}$Sn</td>
<td>full</td>
<td>4.856</td>
<td>4.658</td>
<td>607.7</td>
<td>694.3</td>
<td>87.3</td>
</tr>
<tr>
<td></td>
<td>FRSA</td>
<td>4.856</td>
<td>4.658</td>
<td>607.7</td>
<td>694.3</td>
<td>87.3</td>
</tr>
</tbody>
</table>

where

$$ S_+ = \sum_\nu |\langle N, Z|\hat{O}_\pm|N \mp 1, Z \pm 1; \nu \rangle|^2 $$

are the total SD transition strengths to the neighbouring daughter nuclei induced by the operators

$$ \hat{O}_\pm = \sum_{i,m,\mu} r_i t_\pm (i) \sigma_m (i) Y_1^\mu (\hat{r}_i) . $$

From our previous studies of the FRSA applied to non charge-exchange excitations [15, 16] we could conclude that a value $N=45$ for the number of Gauss points in the radial integrals is sufficient for the desired accuracy in all nuclei with $A \leq 208$. For example, choosing $N=60$ would change the calculated energies and transition probabilities by less than 1%. Thus, we adopt here the value $N=45$. All calculations are without any quenching factor. In the figures, the calculated strength distributions are folded out with a Lorentzian distribution of 1 MeV width.

In Table II, we compare the SD sum rule (24) calculated in FRSA using the full SGII force. To compare with experimental data in $^{90}$Zr we choose the energy interval $E \leq 50$ MeV for the $T_-$ channel and $E \leq 26$ MeV for the $T_+$ channel [27, 30]. One can see that the FRSA sum rules are quite close to those of the full calculations. The calculated values $S_-, S_+$ agree well with experimental data [27,30] in $^{90}$Zr. In $^{132}$Sn a perfect agreement is obtained between the r.h.s. of Eq.(24) and the l.h.s. calculated either in FRSA or full Skyrme TDA.

The SD sum rule (24) is an integral characteristic and it is less sensitive to the details than the SD strength distribution. As an example we consider the spin-dipole (SD) excitations in $^{132}$Sn [17]. Results of our calculations for the SD states with different values of the spin and parity for the $T_-$ channel are shown in Fig.1. The strength distributions are folded out with a Lorentzian distribution of 1 MeV width. One can conclude that the FRSA can reliably be used for the study of charge-exchange modes.
Fig. 1 – Spin-dipole strength distributions in the $T_-$ channel of the parent nucleus $^{132}$Sn, calculated by using the force SGII [26]. The results with the FRSA for the p-h interaction (dashed lines), and with the full p-h interaction (solid lines) are shown.

Let us now discuss the extension of the configuration space to one- and two-phonon terms. As an application of the method, we evaluate the influence on the GT strength distribution in $^{78}$Ni of the $2_1^+$ phonon contributing to the coupling terms in Eq. (19). In particular, we focus on describing the $\beta$-decay half-life since this integrated nuclear quantity is sensitive to the inclusion of the $[1_2^+ \otimes 2^+_1]$ terms. Its experimentally known value puts an indirect constrain on the calculated GT strength distributions within the $Q_\beta$-window. In the allowed GT approximation, the $\beta^-$-decay rate is expressed by summing the probabilities of the energetically allowed transitions (in units of $G_A^2/4\pi$) weighted with the integrated Fermi function $f_0$:

$$T_{1/2}^{-1} = D^{-1} \left( \frac{G_A}{G_V} \right)^2 \sum_\nu f_0(Z, A, -E_\nu) B(GT)_\nu,$$

(27)
where $D=6147$ s and $G_{A}/G_{V}=1.25$ [31]. We employ the Skyrme interaction SLyIII.0.7 [32] which describes isotopic properties of nuclei from the $\beta$-stability line to the drip lines. As a result, we find the half-life equal to 370 ms within the RPA. At the same time, taking into account the phonon coupling leads to a decrease of the half-life, $T_{1/2}=285$ ms. Our results are in a reasonable agreement with the experimental value, $(T_{1/2})_{EXP} = 110^{+100}_{-60}$ ms [33]. The inclusion of other coupling terms is presently in progress.

4. CONCLUSIONS

A finite rank separable approximation for the charge-exchange QRPA calculations with Skyrme interactions is extended to take into account the coupling between one- and two-phonon configurations in the wave functions of excited states. The suggested approach enables one to perform the calculations in very large configuration spaces. Preliminary results of our studies for the phonon-phonon coupling effect on the $\beta$-decay half-life $T_{1/2}$ of $^{78}$Ni are reported. There is a clear influence of the $2^+_1$ phonon on $T_{1/2}$. A systematical study of the effects of the two-phonon terms on the $\beta^-$-decay rates is still underway.

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