PLANE SYMMETRIC UNIVERSE WITH COSMIC STRING AND BULK VISCOSITY IN SCALAR TENSOR THEORY OF GRAVITATION

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A plane symmetric cosmological model is obtained within the framework of a scalar-tensor theory of Gravitation proposed by Saez-Ballester [4] when the source for energy momentum tensor is a bulk viscuous fluid containing one dimensional cosmic strings. The physical and kinematical properties of the model have been discussed.

Key words: Plane symmetric space-time, Viscosity, Cosmic String, Saez-Ballester theory.

1. INTRODUCTION

General relativity (GR) passes all present tests with flying colours, however, there are several reasons why it remains very important to consider alternative theories of gravity. The first one is that theoretical attempts at quantizing gravity or unifying it with other interactions generically predict deviation from Einstein's theory, because gravitation is no longer mediated by a pure spin-2 field but also by partners to the graviton. The second reason is that it is any way extremely instructive to contrast GR's predictions with those of alternative models, even if there were no serious theoretical motivation for them. The third reason is the existence of several puzzling experimental issues, which do not contradict GR in a direct way, but may nevertheless suggest that gravity does not behave at large distances exactly as Newton and Einstein predicted.

The scalar tensor theories are the generalizations of Einstein's theory of gravitation in which the metric is generated by a scalar gravitational field together with non-gravitational field (matter). The scalar gravitational field itself is generated by the non-gravitational fields *via* a wave equation in curved space time. The strength of the coupling between gravity and scalar field is determined by an arbitrary coupling function w. The theories of gravitation involving scalar fields have been extensively developed by some authors [1-3]. There are two different types of gravitational theories involving a classical scalar field function f. Brans-Dicke[1] introduced a scalar-tensor theory of gravitation involving a scalar function

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in addition to the familiar general relativistic metric tensor. In this theory the scalar field has a dimension of inverse of the gravitational constant G and its role is confined to its effects on gravitational field equations.

The theory of second type involves a dimensionless scalar field, *e.g.*, Saez - Ballester theory [4]. Saez and Ballester [4] developed a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields. In spite of a dimensionless character of the scalar field, an anti-gravity regime appears. This theory suggests a possible way to solve the Friedmann-Robertson-Walker (FRW) cosmologies. FRW and spatially homogeneous anisotropic models have been widely studied in this theory of gravitation. Singh and Agrawal [5,6], Ram and Singh [7], Ram and Tiwari [8], Singh and Ram [9], Reddy and Rao [10], Reddy [11], Mohanty and Sahu[12], Reddy *et al.* [13], and Rao *et al.* [14, 15] are some of the authors who have investigated the cosmological models in this theory. Tiwari [16] has obtained Bianchi type-V cosmological models with constant deceleration parameter within the framework of scalar-tensor theory of gravitation proposed by Saez and Ballester. Pradhan *et al.* [17] have studied some exact Bianchi type-I cosmological models in scalar-tensor theory of gravitation with time dependent deceleration parameter. Adhav *et al.* [18], Katore *et al.* [19], Sahu [20], Singh [21] and Pradhan and Singh [22] have obtained the solutions in Saez-Ballester scalar-tensor theory of gravitation in different context. Recently, Naidu *et al.* [23] and Reddy *et al.* [24] have studied LRS Bianchi type-II and five dimensional dark energy models in Saez and Ballester scalar tensor theory of gravitation respectively. Recently, Kumar and Singh [25] obtained exact Bianchi type-I cosmological models in Saez and Ballester Scalar-Tensor theory of gravitation by assuming the constant deceleration parameter.

Cosmic strings may have been created during phase transitions in the early era [26] and they act as a source of gravitational field [27]. Relativistic string models in the context of Bianchi space times have been obtained by Krori *et al.* [28], Banarjee *et al.* [29], Tikekar and Patel [30], and Bhattacharjee and Baruah [31]. String cosmological models in scalar-tensor theories of gravitation have been investigated by Sen [32], Barros *et al.* [33], Banerjee *et al.* [34], Gundlach and Ortiz [35], Barros and Romero [36], Pradhan [37], Mohanty *et al.* [38].

Viscosity may be important in cosmology for a number of reasons. Dissipative mechanisms responsible for smoothing out initial isotropies and the observed high entropy per baryon in the present state of the universe can be explained by involving some kind of dissipative mechanisms. Dissipative effects including bulk viscosity are supposed to play a very important role in the early evolution of the universe. Padmanabhan and Chitre [39] have investigated the effect of bulk viscosity on the evolution of the universe at large. Beesham [40] has presented a cosmological model with variable G and Λ in the presence of a bulk viscous fluid. Singh and Kale [41] have discussed Bianchi type-I, Kantowski-sachs and Bianchi type-III cosmological models filled with bulk viscous fluid together with variable G and Λ. Very recently, Rao *et al.* [42] have discussed anisotropic Bianchi type–I universe with cosmic strings and Bulk Viscosity in a scalar tensor theory proposed by Saez and Ballester [4].

In this paper, we have investigated Plane Symmetric cosmological model in the presence of cosmic string and Bulk Viscosity in Saez-Ballester scalar tensor theory of gravitation.

2. METRIC AND FIELD EQUATIONS

Consider a plane symmetric space-time described by the line element is given by

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2(\mathrm{d}x^2 + \mathrm{d}y^2) - B^2\mathrm{d}z, \tag{1}$$

where *A, B* are the functions of cosmic time *t* only.

The field equations given by Saez and Ballester [4] for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^n\left(\phi_{,i}\phi_{;j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{'k}\right) = -T_{ij,} \tag{2}$$

and the scalar field ϕ satisfies the equation,

$$2\phi^n\phi^i_{;i} + n\phi^{n-1}\phi_{,k}\phi^{'k} = 0, \tag{3}$$

here ω and *n* are constant, T_{ij} is the energy momentum tensor of the matter and comma and semicolon denote partial and covariant differentiation, respectively.

Also

$$T^{ij}_{;i} = 0, \tag{4}$$

is consequence of the field equations of equations (2) and (3).

We consider the energy momentum tensor for bulk viscous fluid containing one dimensional string as

$$T_{ij} = (\rho + \overline{p})u_i u_j - \overline{p}g_{ij} - \lambda x_i x_j, \tag{5}$$

where ρ is the rest energy density of the system, λ be the tension in the string and

$$\overline{p} = p - 3\eta H, \tag{6}$$

Is the total pressure which includes the proper pressure, $\eta(t)$ is the coefficient of the bulk viscosity, $3\eta H$ is usually known as bulk viscous pressure,

H is the Hubble parameter, $u^i = \delta^i_4$ is the four velocity and x^i is a space like vector which represents the anisotropic direction of the string.

$$g_{ij}u^i u^j = 1, g_{ij}x^i x^j = -1, u^i x_i = 0 . \tag{7}$$

We assume the string to be lying along the Z-axis. The one dimensional string are assumed to be loaded with particles and the particles energy density is

$$\rho_p = \rho - \lambda .$$

We also consider $\rho, \lambda, \overline{p}$ and ϕ are functions of time t only.

By adopting commoving coordinates the field equations (2), (3) and (4) for metric (1) yields the following independent equations

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -\overline{p}, \tag{8}$$

$$2\frac{A_{44}}{A} + \frac{A_4^2}{A^2} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -\overline{p} - \lambda, \tag{9}$$

$$2\frac{A_4 B_4}{AB} + \frac{{A_4}^2}{A^2} + \frac{\omega}{2}\phi^n {\phi_4}^2 = \rho, \tag{10}$$

$$\phi_{44} + \phi_4\left(2\frac{A_4}{A} + \frac{B_4}{B}\right) + \frac{n}{2}\frac{{\phi_4}^2}{\phi} = 0, \tag{11}$$

Here the subscript 4 denotes differentiation with respect to *t.*

3. SOLUTIONS OF FIELD EQUATIONS

The field equations (8) – (11) are four independent equations in six unknowns $A, B, \overline{p}, p, \phi, \lambda$. We can introduce more conditions either by an assumption corresponding to some physical situation or an arbitrary mathematical supposition however these procedure have some drawbacks. Physical situation may lead to differential equations which will be difficult to integrate and mathematical supposition may leads to a non-physical situation. Two additional conditions relating these unknowns are required to obtain explicit solutions of the systems.
We solve the above set of highly non-linear equations with the help of special law of variation of Hubble's Parameter proposed by Berman [44] which yields constant deceleration parameter of the models of the universe. We consider the constant deceleration parameter model define by

$$q = -\frac{RR_{44}}{\left(R_4\right)^2} = \text{constant}. \tag{12}$$

where the scale factor R is given by

$$R = \left(A^2 B\right)^{\frac{1}{3}}. \tag{13}$$

Here the constant is taken as negative (*i.e.* it is an accelerating model of the universe).

The solution of (12), we get

$$R = (at+b)^{\frac{1}{1+q}}, \tag{14}$$

where $a \neq 0$, and b are constant of integration.
This equation implies that the condition of expansion is $1+q>0$.
Secondly we assume that the expansion θ is proportional to the shear scalar σ which leads to

$$B = A^m, \tag{15}$$

where m is a constant.

Solving the field equations (8)–(11) with the help of equations (14) and (15), we obtain the expansion for metric coefficients as follows

$$A = (at+b)^{\frac{3}{(1+q)(m+2)}}, \tag{16}$$

$$B = (at+b)^{\frac{3m}{(1+q)(m+2)}}, \tag{17}$$

By suitable choice of constant and co–ordinates, the metric (1) can be written as

$$\mathrm{d}s^2 = \mathrm{d}T^2 - T^{\frac{6}{(1+q)(m+2)}}\left[\mathrm{d}X^2 + \mathrm{d}Y^2\right] - T^{\frac{6}{(1+q)(m+2)}}\mathrm{d}Z^2, \tag{18}$$

This represent plane symmetric universe with bulk viscosity in the presence of cosmic string in Saez-Ballester scalar tensor theory of gravitation.

4. SOME PHYSICAL PROPERTIES OF THE MODEL

Equation (18) represents a bulk viscous string cosmological model.

The spatial volume in the model is given by

$$V^3 = A^2 B = T^{\frac{3}{1+q}}. \tag{19}$$

The Scalar field in the model is given by

$$\phi = \left[\frac{(n+2)}{2} \phi_0 T^{\frac{(q-2)}{q+1}} \right]^{\frac{2}{n+2}} \tag{20}$$

The physical quantities that are important in cosmology are

$$\bar{p} = \frac{k_1}{(1+q)^2 (m+2)^2 T^2} + \frac{w \left\{ \left[\frac{2a-(q-2)}{(1+q)(n+2)} \right]^2 \left[\left(\frac{n+2}{2} \right) \phi_0 T^{q-2/q+1} \right]^{\frac{-2n(n-1)}{(n+2)}} \right\}}{2T^6}, \tag{21}$$

where $k_1 = -(qa^2 - 15a^2qm + 6ma^2 - 12a^2q + 6m^2a^2 - 2 - 3a^2qm^2)$.
The String tension density

$$\lambda = \frac{k_2}{(1+q)^2 (m+2)^2 T^{2,}} \tag{22}$$

$$k_2 = -(18a^2 - a^2qm - 12ma^2 - 12a^2q - 6m^2a^2 + 3a^2qm^2).$$

Energy density

$$\rho = \frac{k_3}{(1+q)^2 (m+2)^2 T^2} + \frac{w \left\{ \left[\frac{2a(q-2)}{(1+q)(n+2)} \right]^2 \left[\left(\frac{n+2}{2} \right) \phi_0 T^{q-2/q+1} \right]^{\frac{-2n(n-1)}{(n+2)}} \right\}}{2T^6}. \tag{23}$$

Proper Pressure

$p = \in_0 \rho$,

$$= \in_0 \left\{ \frac{k_3}{(1+q)^2 (m+2)^2 T^2} + \frac{w \left\{ \left[\frac{2a(q-2)}{(1+q)(n+2)} \right]^2 \left[\left(\frac{n+2}{2} \right) \phi_0 T^{q-2/q+1} \right]^{\frac{-2n(n-1)}{(n+2)}} \right\}}{2T^6} \right\}, \tag{24}$$

where $\in_0$ is constant, $(0 \le \in_0 \le 1)$.
Also, $\bar{p} = p - 3\eta H$, where $\eta(t)$ is the coefficient of bulk viscosity

i.e. $3\eta H = p - \overline{p}$

$$\eta = \left(\frac{1}{3H}\right)\left\{\frac{k_1 + \in_0 k_3}{(1+q)^2 (m+2)^2 T^2} + \frac{w(\in_0 - 1)\left\{\left[\frac{2a(q-2)}{(1+q)(n+2)}\right]^2 \left[\left(\frac{n+2}{2}\right)\phi_0 T^{q-2/q+1}\right]^{\frac{-2n(n-1)}{(n+2)}}\right\}}{2T^6}\right\} \tag{25}$$

The Scalar of expansion θ and shear scalar σ^2 for the model (18) are given by

$$\theta = \frac{9}{(1+q)T}. \tag{26}$$

$$\sigma^2 = \frac{27}{2}\frac{1}{(1+q)^2 T^2} \tag{27}$$

It is observed that from (19) for large T and $1+q > 0$ spatial volume increases and the universe has accelerated expansion. Also for large values of T the scalar of expansion θ, shear scalar σ^2 tend to zero and they diverge at the initial epoch. As $T \to \infty$, λ, ρ, η, p vanish. The model (18) has no initial singularity. The scalar field in the model diverges for large of T while the tension density in the string vanishes. Also $\frac{\sigma^2}{\theta^2} \neq 0$, this shows that the universe does not approach isotropy for large values of T.

5. CONCLUSIONS

Scalar fields play a vital role in discussing dark energy models and early stages of evolution of the universe. Hence in this paper we have studied plane symmetric cosmological model in the presence of cosmic string and Bulk Viscosity in frame work of Saez-Ballester [4] scalar-tensor theory of gravitation. The model obtained is expanding and non-singular. Also, we observe that they do not approach isotropy for large values of time. The spatial volume is increasing as T increases, which contradicts to the concept of decrement of spatial volume obtained by Rao *et al.* [45]. The properties of the model all decrease with the growth of cosmic time. When $T \to \infty$, all of these properties identically vanish leading to a vacuum universe which resembles with the results of Tripathy *et al.* [46]. At the beginning of the universe, the coefficient of bulk viscosity assumes an infinitely large value and gradually decreases with the advancement of the arrow of time. It is interesting to note that our investigations resembles to the result obtained by

Naidu *et.al.* [43].Also the universe does not approach isotropy for large values of *T* since $\frac{\sigma^2}{\theta^2} \neq 0$, which resembles with the investigations of Naidu *et al.* [43] and Bali *et al.* [47].

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