SEISMIC WAVE ATTENUATION IN THE VRANCEA REGION.
PART I. THE APPROACH FOR 1-D $Q$-MODEL ESTIMATION

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Abstract. An approach based on high-frequency waveform modelling is proposed to estimate 1-D (depth-dependent) models of the quality factor of the medium $Q$ in the area located at the bending of the Eastern Carpathians. The algorithm is a non-linear inversion in which the normalised amplitude spectra of local records of low-to-moderate magnitude earthquakes are compared with the synthetic signal spectra, generated for point sources with the same location and mechanism as the recorded events. The best fitting $Q$-models are determined by minimizing the sum of squares of logarithmic residuals between theoretical and observed spectra. Several tests investigating the effect of the uncertainty of earthquake mechanism and hypocenter location on the resolution of the retrieved attenuation structures are presented.

Key words: Seismic wave attenuation, quality factor of the medium, $Q$-models, Vrancea region, waveform modelling.

1. INTRODUCTION

The Vrancea seismic region, located in a complex tectonic setting at the bend of the South-eastern Carpathians, is the main source of seismic hazard in the extra-Carpathian region of Romania.

Numerous investigations, based on the high quality records provided by the National Seismic Network – which has continuously been developed and upgraded (e.g., [1–4]) – and/or collected during the tomographic experiment CALIXTO’99, and the seismic refraction experiments VRANCEA’99 and VRANCEA 2001, resulted in (1-D, 2-D, 3-D) wave velocity models for the lithosphere beneath the Vrancea region and surroundings (e.g., [5–15]).

By contrast, the seismic attenuation in the area was significantly less investigated. Most of the studies are qualitative discussions (e.g., [16, 17]), or they give average estimates of the quality factor of the medium $Q$ along the entire path from the seismic source to the observation point (e.g., [18–20]). Consequently the $Q$-factor is the less known structural parameter in the region.

Nevertheless, the availability of adequate local $Q$-models is important in inverse problems based on waveform modelling (such as source parameter estimation for events in a wide range of magnitudes), as well as in neodeterministic hazard analyses which use numerical simulations of the ground motion (synthetic seismograms).

Since the 1-D models of the medium are still largely used to generate theoretical seismograms (several widely applied methods for synthetic seismogram computation are developed for 1-D structures), the goal of this work is to estimate 1-D (depth-dependent) models of seismic wave attenuation in the Vrancea region and adjacent area.

In the first part of the study we present our approach and test its capability. In the second part we apply the proposed methodology to investigate the crustal attenuation in the extra-Carpathian zone.

2. METHODOLOGY

The approach is based on the modelling of local waveforms of low-to-moderate magnitude earthquakes. The algorithm is a non-linear inversion in spectral domain, similar to the procedure proposed by [21].

The normalised amplitude spectra of the records collected by a local network – the epicentral distances do not exceed a few tens of km – are compared with the synthetic signal spectra, generated for instantaneous point sources with the same location and mechanism as the observed events.

To compute the theoretical waveforms we use the multimodal summation in layered inelastic media, a technique which allows to solve in an exact and complete way the full wave equation in a pre-assigned interval of frequencies and phase velocities [22, 23].

The 1-D models of the elastic parameters of the medium – density and seismic wave velocities – along the focus – recording station paths, which are required in the calculation of the synthetic seismograms, are compiled by integrating and harmonizing published results, after a thorough selection and evaluation. These models are designed to allow an accurate simulation of the observed wave trains (particularly their arrival times).

The input $Q$-structures are generated by keeping the layer configuration of the velocity and density models. The quality factor is allowed to vary in a wide range, between 0 and 1500 units; we adopted a step of 50 units for $Q$-values in the range 0 to 400, a step of 100 units for $Q$ in the range 400 to 600, a step of 200 for the range 600 to 1200, and a step of 300 for $Q$ greater than 1200.

The amplitude spectra of observed and theoretical waveforms are calculated (using FFT) for cosine-tapered time windows centered on the most energetic part
of the signals, then smoothed by averaging the amplitudes, for a bandwidth of 1 Hz. The optimal $Q$-structures are determined by minimizing the sum of squares of logarithmic residuals between theoretical and observed spectra $\chi^2$:

$$\chi^2 = \sum_n [\ln D(f_n) - \ln U(f_n)]^2,$$

where $f_n$ is the frequency, and $D$ and $U$ are the observed and theoretical normalized spectral amplitudes, respectively. To avoid the effect of data noise, $\chi^2$ is evaluated only in the frequency range where the signal-to-noise spectral amplitude ratio is greater than 2.

The steps of the algorithm are illustrated below, using the short period velocity record of the earthquake of February 21, 1983 (Table 1), provided by the telemetered seismic station Popeni (PPE), equipped with vertical S13 seismometer (1s free period and damping of 0.7).

The earthquake parameters and the focal mechanism are taken from [24].

<table>
<thead>
<tr>
<th>Date</th>
<th>Origin time</th>
<th>Lat. [°N]</th>
<th>Lon. [°E]</th>
<th>Depth [km]</th>
<th>Local magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 February 1983</td>
<td>18 : 21 : 05</td>
<td>45.33</td>
<td>26.97</td>
<td>16</td>
<td>3.6</td>
</tr>
</tbody>
</table>

The source-station geometry is shown in Fig. 1, together with the 1-D models adopted for the elastic parameters.

The P- and S-wave velocity models are constructed as follows: for the sedimentary cover we used the local structures for the recording station site, given in [25]; for the crystalline crust we compiled average models along the focus-station path, using the information available from recent literature [26, 27, 14, 15].

The density model is based on the work of [25].

To illustrate the diversity of the considered attenuation models, Fig. 2 displays examples of $Q$-structures generated for this path.

The responses of the input structures to a point source having the same location and mechanism as the recorded event are calculated by modal summation for a maximum frequency of 10 Hz.

The observed and theoretical waveforms are subsequently filtered by low-pass with cut-off frequency at 5 Hz, and the synthetics are convolved with the instrument response.
Fig. 1 – Left: location of epicenter (star) and seismic station (triangle), and earthquake focal mechanism (after [24]). Right: the 1-D models of the elastic parameters, used in the synthetic seismogram computation.

Fig. 2 – Examples of Q-structures constructed for the source-station path.
Figure 3 shows the selection of the time window for spectrum computation (15 seconds in this example), and the normalised amplitude spectra of the record and synthetics, respectively. The choice of the frequency interval for the evaluation of $\chi^2$ is also illustrated.

3. RESOLUTION TESTS

In general, the non-linear inversion of geophysical data does not yield a unique multi-valued solution. A set of models may be compatible with the observed data; these models are equally probable – the discrepancy between the
computed and observed data is sufficiently small with respect to a threshold defined on the basis of the quality of data.

To assess the resolution of the retrieved attenuation structures we carried out synthetic tests in which we investigated the effect of the uncertainty of the available hypocenter locations and earthquake mechanisms.

The synthetic data in these experiments are computed by using seismic source parameters – hypocenter location and focal mechanism – of low-magnitude earthquakes recorded in the study region, and by placing the observation points in the locations of the seismic stations.

The tests investigating the effect of hypocenter mislocation took into account the errors standardly reported by the location routine. Since the error in epicenter location is smaller than the mislocation in depth, we examined only errors in depth amounting to ±5 km.

The synthetic data were computed assuming the ‘correct’ source location – that is 5 km above or below the hypocenter provided by the earthquake catalogue – while the ‘theoretical’ seismograms, generated to determine the optimal Q-model, were calculated considering the kinematic hypocenter.

The experiments simulating depth mislocations of 5 km, performed with source – station configurations of the study region, show that, in general, the ‘real’ Q-models (considered in the computation of the synthetic data) are not the best-fitting structures. The ‘real’ models yield r.m.s. values of the logarithmic residuals of eq. (1) which amount to 0.08.

In the example displayed in Fig. 4, the source parameters of the event of February 21, 1983 (Table 1 and Fig. 1), and the location of the seismic station Popeni (PPE) (Fig. 1) are considered.

The ‘real’ source is assumed to be located 5 km above the hypocenter provided by the location routine. The attenuation model adopted to construct the synthetic data is the best-fitting Q-structure displayed in Fig. 3.

The results show that the preferred Q-model is not the ‘real’ one. The r.m.s. value yielded by the ‘real’ model is 0.06, the minimum being 0.05.

The tests analysing the effect of the focal mechanism uncertainty were also performed with source – station configurations of the study region.

The experiments simulating reasonably large errors of the available fault plane solutions pointed out that the ‘real’ attenuation structures (used to generate the synthetic data) are recovered by accepting a threshold of the r.m.s. of the differences of eq. (1), which amounts to 0.08.

In the example of Fig. 5, the earthquake hypocenter and station location displayed in Table 1 and Fig. 1, respectively, are considered. The synthetic data are constructed with the focal mechanism of Fig. 1 (considered as the ‘real’ mechanism), while the ‘theoretical’ seismograms used to retrieve the optimal Q-model are computed with the ‘estimated’ fault plane solution displayed in Fig. 5.
The results show that the preferred $Q$-structure (r.m.s. of the logarithmic residuals – 0.05) differs from the ‘real’ attenuation model – the best-fitting $Q$-structure of Fig. 3 – which yields a r.m.s. value of 0.07.

On the basis of the synthetic tests described above we set the maximum value of the r.m.s. of the logarithmic residuals of the acceptable models to 0.08 – threshold which allows to recover the real $Q$-structure when a hypocenter depth error of 5 km is present. This threshold permits also the recovery of the real $Q$-structure when the available fault plane solution deviates noticeably from the real mechanism.

We tested our acceptance criterion in synthetic experiments simulating hypocenter mislocation and error of the mechanism, simultaneously.

In the tests where the simulated errors resulted in r.m.s. values of the best-fitting $Q$-structures larger than the fixed threshold, the results were considered unreliable, and discarded.

The successful experiments emphasized that the ensembles of accepted $Q$-models always cover the ‘real’ $Q$-structures – admit the $Q$-values of the ‘real’ structures (even in cases in which large simulated errors lead for the ‘real’ models to r.m.s. values that exceed the threshold).

In the example presented in Fig. 6 the ‘real’ source is assumed to display the focal mechanism of Fig. 1, and to be located 5 km above the kinematic hypocenter of Table 1. The recording point has the location of PPE station (Fig. 1), and the ‘real’ attenuation model is the best-fitting $Q$-structure of Fig. 3. The ‘available’ (‘estimated’) fault plane solution – taken into account in the search for the optimal $Q$-structure – deviates significantly from the ‘real’ mechanism.

The outcome of the reported synthetic tests is an ensemble of acceptable attenuation structures whose variability provides information about the $Q$-model uncertainty.

With the adopted acceptance criterion, the resolution of the $Q$-factor is quite good over the depth range between the Earth’s surface and the seismic source, while in the deeper layers the parameter is very poorly constrained.

In the previous synthetic experiments, we assumed an exact modelling of the density and velocity structures.

In practice, however, the 1-D structural models are first approximations only. In regions with complex tectonics, the extent of the mismodelling of the real medium by oversimplified 1-D structures for individual source-to-station paths is hard to quantify. On the other hand, the mismodelling may be strong enough to distort significantly the theoretical waveforms constructed with the simplified 1-D models.

The mislocation of the hypocenter is also a common effect of an inexact velocity model, while an imperfect location of the seismic source results in an incorrect focal mechanism.
Fig. 4 – Experiment – performed with the seismic event and seismic station displayed in Table 1 and Fig. 1 – which simulates a source depth mislocation of 5 km. (a) The \( Q \)-structures which yielded r.m.s. values of the logarithmic residuals \(< 0.08 \). (b) Selected time window for spectrum calculation: synthetic data (up) and ‘theoretical’ seismograms computed using the ‘real’ and the best-fitting \( Q \)-model, respectively; the synthetics are calculated for a scalar moment \( M_0 = 4 \times 10^{12} \text{Nm} \). (c) Normalised amplitude spectra (smoothed by averaging the spectral amplitudes over a bandwidth of 1 Hz) calculated for the synthetic data, and for the ‘theoretical’ signals generated with the ‘real’ and the best-fitting \( Q \)-model, respectively (up), and with all \( Q \)-models which yielded r.m.s. values of the logarithmic residuals \(< 0.08 \) (down).
Fig. 5 – Experiment – performed with the seismic event and seismic station displayed in Table 1 and Fig. 1 – which simulates an incorrect focal mechanism. (a) The fault plane solutions: the ‘real’ focal mechanism (used to generate the synthetic data) and the ‘available’ focal mechanism (used to determine the optimal Q-model). (b) The Q-structures which yielded r.m.s. values of the logarithmic residuals < 0.08. (c) Selected time window for spectrum calculation: synthetic data (up), and ‘theoretical’ seismograms computed using the ‘real’ and the best-fitting Q-model, respectively; the synthetics are calculated for a scalar moment $M_0 = 4 \times 10^{12}$ Nm. (d) Normalised amplitude spectra (smoothed by averaging the spectral amplitudes over a bandwidth of 1 Hz) calculated for the synthetic data, and for the ‘theoretical’ signals generated with the ‘real’ and the best-fitting Q-model, respectively (up), and with all Q-models which yielded r.m.s. values of the logarithmic residuals < 0.08 (down).
All these kinds of perturbations, frequently encountered in practice, may distort essentially the synthetic seismograms generated to search for the optimal attenuation model.

Therefore we checked how the adopted acceptance criterion is satisfied when the synthetic data are replaced with observed data.

Figure 7 shows the result of the application of the criterion to the example displayed to illustrate the inversion procedure (Table 1, and Figs. 1 and 3).
Fig. 7 – (a) The $Q$-structures which yielded r.m.s. values of the logarithmic residuals < 0.08. (b) Selected time window for spectrum calculation: observed data (up) and theoretical seismograms computed using the best-fitting and the last accepted $Q$-model, respectively; the synthetics are calculated for a scalar moment $M_0 = 4 \times 10^{12}$ Nm. (c) Normalised amplitude spectra (smoothed by averaging the spectral amplitudes over a bandwidth of 1 Hz) calculated for the observed data, and for the theoretical signals generated with the best-fitting and the last accepted $Q$-model, respectively (up), and with all $Q$-models which yielded r.m.s. values of the logarithmic residuals < 0.08 (down).
The r.m.s. of the logarithmic residuals of eq. (1) yielded by the best-fitting model is 0.07.

We observe that the resolution of the attenuation parameter is quite good at depths from the Earth’s surface to the earthquake hypocenter – i.e. in the sedimentary cover and the upper crust – while in the lower crust and subcrustal lithosphere (half-space) the parameter is poorly constrained.

To define the representative attenuation model from the set of acceptable structures, we choose the middle of the ensemble – the ‘Median Model’ of all the solutions, similarly to [28]. At each depth, the half-width of the corridor of acceptable $Q$-values defined by the ensemble specifies the model uncertainty.

Figure 8 exhibits the representative $Q$-structure – the Median Model – and the uncertainty estimate, for the example which illustrates the inversion procedure (Table 1, and Figs. 1 and 3).

4. CONCLUDING REMARKS

The experiments performed with both synthetic and observed data tested the capability of the proposed methodology to retrieve optimal 1-D attenuation structures along specified source–station paths from the study region.

We defined the acceptance criterion for the $Q$-models taking into account the uncertainties of the available hypocenter locations and focal mechanisms, mainly effects of an inexact velocity model.

The tests pointed out the good resolution of the attenuation parameter over the layers situated above the earthquake foci, and, by contrast, the large uncertainty of the $Q$-factor at deeper depths.
On the basis of these results we conclude that the proposed approach provides reliable estimates of the attenuation factor in the depth range delimited by the seismic source and the receiver.

In the second part of the study we apply the inversion procedure to investigate the attenuation of the seismic waves – to estimate 1-D $Q$-models – in the crust in the extra-Carpathian region.

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