QUINTOM COSMOLOGY WITH GENERALIZED GALILEON CORRECTIONS

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Abstract. In this work we have considered a quintom cosmological model with generalised Galileon corrections in the scalar tensor theories of gravitation. After proposing the corresponding action of the cosmological model we have deduced the Klein–Gordon equations and the modified Friedmann relations. By introducing the auxiliary variables we have described the evolution of the model as an autonomous dynamical system of differential equations for generalised power–law coupling functions. Assuming various initial conditions we have analyzed the dynamical features of the phase space, obtaining the numerical evolution of the effective equation of state and the density parameters, respectively. It is observed that for different initial conditions the system evolves from an initial phantom epoch towards a de–Sitter stage in which the model behaves closely to a cosmological constant, approaching the phantom divide line boundary from bellow asymptotically.

Key words: General relativity and cosmology, Scalar fields in curved space–time.

1. INTRODUCTION

The nature and the particularities of the dark energy phenomenon which is responsible to the accelerated expansion of the Universe [1, 2] are currently unknown questions in the modern cosmological theories. After the discovery of the accelerated expansion at the end of the last millennium by different independent collaborations, many studies have confirmed the validity of such observations. The discovery of the accelerated expansion lead to a fruitful development of the scalar tensor theories of gravitation [3] which can explain the dynamics of the Universe and the large scale behavior of the expansion rate. The ΛCDM cosmological model represents the most simple solution to the problem of the accelerated expansion, by embedding a cosmological constant Λ into the action [4], leading to a constant equation of state. Although this model represents a possible solution to the dark energy problem [5], it suffers from a series of inconsistencies [6–8] and cannot explain the dynamical evolution of the dark energy equation of state and the redshift dependence. In this regard, various observational studies [9, 10] have showed that the barotropic parameter associated to the dark energy equation of state presents a redshift sensitivity, crossing the phantom divide line or the cosmological constant boundary. The crossing [11] over the cosmological constant boundary by the dark energy equation of state represents a fundamental effect associated to the quintom cosmologies which might add an unstable feature to the theoretical aspects [12].

A specific attempt which can explain the accelerated expansion as a dynamical effect in the context of scalar tensor theories of gravitation is represented by the quintom cosmologies [10, 13, 14]. Within these theories the dark energy sector is represented by the inclusion in the corresponding action of two scalar fields [15–17] with opposite kinetic terms, a canonical field called quintessence [18], together with a negative kinetic one (non-canonical), a phantom model [19, 20] with a pathological construction [21] which violates the null energy conditions. The cosmological theories based on quintessence models represent a well established direction [22, 23], leading to a non–constant dark energy equation of state above the phantom divide line (cosmological constant boundary) which can explain the evolutionary aspects of the dark energy equation of state in a non–pathological manner. Despite the inclusion of a pathological feature [24] due to the existence of the phantom scalar field leading to the violation of the null energy condition, the quintom scenarios are consistent with astrophysical observations [10, 13, 14]. For the quintom cosmologies [14] a no-go theorem have been developed recently [25], which can explain the specific crossing over the cosmological constant boundary as a physical effect of the superposition between two scalar fields associated, as two degrees of freedom. The conceptual approach based on quintom scenarios have been an active direction of study in the recent years [26–34], leading to the construction of various non–minimal coupling scalar tensor theories [35–41].

In Marciu *et al.* [42], in a theoretical framework based on scalar tensor theories, a new quintom dark energy model have been studied by linear stability methods, assuming specific Galileon corrections terms [43] added to the scalar fields. It was shown that the trajectories in the phase space structure can explain the existence of the accelerated expansion and the late time evolution as a cosmological constant. In the scalar tensor theories of gravitation the implications of the non–minimal Galileon correction terms have been studied in various analyses [44–47].

In the present paper, we further generalize our previous work [42] which analyzed a quintom model non–minimally coupled with Galileon corrections terms. The model and the corresponding Klein–Gordon equations with the modified Friedmann relations are presented in Section 2. Then, in Section 3 we introduce specific auxiliary variables and discuss the dynamical properties of the model by adopting a numerical approach. Finally, the physical interpretations of the results and the final concluding remarks are presented in the last section.

2. THE GENERALIZED QUINTOM MODEL AND THE FIELD EQUATIONS

In this work we shall extend our previous quintom cosmological model [42] with Galileon corrections to generalised nonminimal coupling functions, proposing a scenario with the following action:

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2}g_{\mu\nu}\nabla^{\mu}\phi\nabla^{\nu}\phi+\frac{1}{2}g_{\mu\nu}\nabla^{\mu}\sigma\nabla^{\nu}\sigma\right.$$
$$\left.-g(\phi)(\nabla\phi)^2\Box\phi-h(\sigma)(\nabla\sigma)^2\Box\sigma-V_1(\phi)-V_2(\sigma)\right]+S_m, \quad (1)$$

where we have defined the operators: $(\nabla\phi)^2=g_{\mu\nu}\nabla^{\mu}\phi\nabla^{\nu}\phi$ and $\Box\phi=g_{\mu\nu}\nabla_{\mu}\nabla_{\nu}\phi$. In what follows we consider the case of the Friedmann-Robertson-Walker-Lemaître metric (with $a(t)$ the scale factor, t the cosmic time and $H=\dot{a}/a$ the Hubble parameter):

$$ds^2=-dt^2+a^2(t)\left[\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta d\phi^2)\right], \quad (2)$$

assuming the case of a flat space ($k=0$) [48]. If we consider the variation of the relation (1) which describes the corresponding action for our model with respect to the inverse metric $g^{\mu\nu}$ we can obtain the following modified Friedmann equations [42, 46, 47]:

$$3H^2=\rho_m+\frac{1}{2}\dot{\phi}^2-\frac{1}{2}\dot{\sigma}^2+V_1(\phi)+V_2(\sigma)-3gH\dot{\phi}^3-3hH\dot{\sigma}^3$$
$$+\frac{1}{2}\frac{dg}{d\phi}\dot{\phi}^4+\frac{1}{2}\frac{dh}{d\sigma}\dot{\sigma}^4, \quad (3)$$

$$2\dot{H}+3H^2=-p_m-\frac{1}{2}\dot{\phi}^2+\frac{1}{2}\dot{\sigma}^2+V_1(\phi)+V_2(\sigma)-g\dot{\phi}^2\ddot{\phi}-h\dot{\sigma}^2\ddot{\sigma}$$
$$-\frac{1}{2}\frac{dg}{d\phi}\dot{\phi}^4-\frac{1}{2}\frac{dg}{d\sigma}\dot{\sigma}^4, \quad (4)$$

where the dot(s) represents differentiation with respect to the cosmic time.

For this particular action in eq. (1) the evolution of the scalar fields (the quintessence and the phantom, respectively) is described by the following Klein–Gordon equations [42, 46, 47]:

$$\ddot{\phi}+3H\dot{\phi}+2\frac{dg}{d\phi}\dot{\phi}^2\ddot{\phi}+\frac{1}{2}\frac{d^2g}{d\phi^2}\dot{\phi}^4-3g\dot{\phi}(3H^2\dot{\phi}+\dot{H}\dot{\phi}+2H\ddot{\phi})+\frac{dV_1}{d\phi}=0, \quad (5)$$

$$-\ddot{\sigma}-3H\dot{\sigma}+2\frac{dh}{d\sigma}\dot{\sigma}^2\ddot{\sigma}+\frac{1}{2}\frac{d^2h}{d\sigma^2}\dot{\sigma}^4-3h\dot{\sigma}(3H^2\dot{\sigma}+\dot{H}\dot{\sigma}+2H\ddot{\sigma})+\frac{dV_2}{d\sigma}=0. \quad (6)$$

In our representation ϕ is the quintessence (canonical) field and σ the non–canonical (phantom) scalar field with a pathological behavior from the negative kinetic term.

For the matter sector we assume the case of the perfect fluid [48] with the energy momentum tensor:

$$T_{\mu\nu}=(\rho_m+p_m)u_\mu u_\nu+p_m g_{\mu\nu}, \quad (7)$$

which implies that the evolution is dictated by the following continuity equation:

$$\dot{\rho}_m+3H(\rho_m+p_m)=0, \quad (8)$$

taking into account a barotropic equation of state:

$$p_m=w_m\rho_m. \quad (9)$$

In our calculations we shall consider that the matter component represents a pressure–less fluid $(w_m=0)$, a viable scenario from an astrophysical point of view which will simplify further many computations.

Furthermore, from the modified Friedmann identities (the constraint equation (3) and the acceleration relation (4), respectively) we can define the energy density for the dark energy field:

$$\rho_{de}=\frac{1}{2}\dot{\phi}^2-\frac{1}{2}\dot{\sigma}^2+V_1(\phi)+V_2(\sigma)-3gH\dot{\phi}^3-3hH\dot{\sigma}^3+\frac{1}{2}\frac{dg}{d\phi}\dot{\phi}^4+\frac{1}{2}\frac{dh}{d\sigma}\dot{\sigma}^4, \quad (10)$$

and the corresponding pressure:

$$p_{de}=\frac{1}{2}\dot{\phi}^2-\frac{1}{2}\dot{\sigma}^2-V_1(\phi)-V_2(\sigma)+g\dot{\phi}^2\ddot{\phi}+h\dot{\sigma}^2\ddot{\sigma}+\frac{1}{2}\frac{dg}{d\phi}\dot{\phi}^4+\frac{1}{2}\frac{dh}{d\sigma}\dot{\sigma}^4. \quad (11)$$

We can then define the dark energy equation of state as a barotropic parameter:

$$w_{de}=\frac{p_{de}}{\rho_{de}}=\frac{\frac{1}{2}\dot{\phi}^2-\frac{1}{2}\dot{\sigma}^2-V_1(\phi)-V_2(\sigma)+g\dot{\phi}^2\ddot{\phi}+h\dot{\sigma}^2\ddot{\sigma}+\frac{1}{2}\frac{dg}{d\phi}\dot{\phi}^4+\frac{1}{2}\frac{dh}{d\sigma}\dot{\sigma}^4}{\frac{1}{2}\dot{\phi}^2-\frac{1}{2}\dot{\sigma}^2+V_1(\phi)+V_2(\sigma)-3gH\dot{\phi}^3-3hH\dot{\sigma}^3+\frac{1}{2}\frac{dg}{d\phi}\dot{\phi}^4+\frac{1}{2}\frac{dh}{d\sigma}\dot{\sigma}^4}, \quad (12)$$

and the total (effective) equation of state:

$$w_{\mathbf{eff}}=\frac{p_m+p_{de}}{\rho_m+\rho_{de}}. \quad (13)$$

By introducing the relations for the matter density parameter

$$\Omega_m=\frac{\rho_m}{3H^2}, \quad (14)$$

and the dark energy density parameter:

$$\Omega_{de} = \frac{\rho_{de}}{3H^2}, \tag{15}$$

we can obtain the Friedmann constraint equation in the following standard form:

$$\Omega_m + \Omega_{de} = 1. \tag{16}$$

Finally we introduce the definition of the deceleration parameter

$$q = -\frac{\ddot{a}}{aH^2} \tag{17}$$

which will be subsequently analyzed in the next section.

3. DYNAMICAL EVOLUTION OF THE QUINTOM MODEL

In this section we shall investigate the dynamical properties of the model in the case where the coupling functions have a power law behavior [47], $g(\phi) = g_0\phi^n$, $h(\sigma) = h_0\sigma^m$ with g_0, h_0, m, n constant parameters. For this specific case it is then convenient to introduce the following auxiliary variables [47]:

$$a = \frac{\dot{\phi}}{H\sqrt{6}}, \tag{18}$$

$$b = \frac{\sqrt{V_1(\phi)}}{H\sqrt{3}}, \tag{19}$$

$$c = g(\phi)H\dot{\phi}, \tag{20}$$

$$d = \frac{1}{\phi}, \tag{21}$$

$$e = \frac{\dot{\sigma}}{H\sqrt{6}}, \tag{22}$$

$$f = \frac{\sqrt{V_2(\sigma)}}{H\sqrt{3}}, \tag{23}$$

$$p = h(\sigma)H\dot{\sigma}, \tag{24}$$

$$r = \frac{1}{\sigma}. \tag{25}$$

Concerning the potential energy we shall take into account an usual decomposition $V(\phi,\sigma) = V_1(\phi) + V_2(\sigma)$ into exponential functions, $V_1(\phi) = V_{10}e^{\lambda_1\phi}$, $V_2(\sigma) =$

$V_{20}e^{\lambda_2\sigma}$, where $V_{10}, V_{20}, \lambda_1, \lambda_2$ are positive constant parameters. This specific decomposition of the potential energy is in general usually considered in many recent quintom models [14]. If we further change the time variable to N, where $N = log(a)$, then by linearization we can approximate the evolution of the field equations (3), (4), (5), (6) with the following autonomous dynamical system:

$$\frac{\partial a}{\partial N} = -\frac{a\dot{H}}{H^2} + \frac{\ddot{\phi}}{\sqrt{6}H^2}, \tag{26}$$

$$\frac{\partial b}{\partial N} = -\sqrt{\frac{3}{2}}\lambda_1 ab - \frac{b\dot{H}}{H^2}, \tag{27}$$

$$\frac{\partial c}{\partial N} = \sqrt{6}nacd + \frac{c\dot{H}}{H^2} + \frac{c\ddot{\phi}}{\sqrt{6}aH^2}, \tag{28}$$

$$\frac{\partial d}{\partial N} = -\sqrt{6}ad^2, \tag{29}$$

$$\frac{\partial e}{\partial N} = -\frac{e\dot{H}}{H^2} + \frac{\ddot{\sigma}}{\sqrt{6}H^2}, \tag{30}$$

$$\frac{\partial f}{\partial N} = -\sqrt{\frac{3}{2}}\lambda_2 ef - \frac{f\dot{H}}{H^2}, \tag{31}$$

$$\frac{\partial p}{\partial N} = \sqrt{6}mepr + \frac{p\dot{H}}{H^2} + \frac{p\ddot{\sigma}}{\sqrt{6}aH^2}, \tag{32}$$

$$\frac{\partial r}{\partial N} = -\sqrt{6}er^2. \tag{33}$$

Within these equations, $\ddot{\phi}, \ddot{\sigma}, \dot{H}$ are determined by the Klein–Gordon relations (5), (6) and the acceleration equation (4), respectively, in the case of a pressure–less matter component ($w_m = 0$):

$$\begin{aligned} 0 = 3\sqrt{6}a^3cd^2H^2(n-1)n + 2\sqrt{6}acdn\ddot{\phi} - 9\sqrt{6}acH^2 - 3\sqrt{6}ac\dot{H} \\ + 3\sqrt{6}aH^2 - 3b^2H^2\lambda_1 - 6c\ddot{\phi} + \ddot{\phi}, \end{aligned} \tag{34}$$

$$\begin{aligned} 0 = 3\sqrt{6}e^3H^2(m-1)mpr^2 - 9\sqrt{6}eH^2p - 3\sqrt{6}eH^2 - 3\sqrt{6}ep\dot{H} \\ + 2\sqrt{6}empr\ddot{\sigma} - 3f^2H^2\lambda_2 - 6p\ddot{\sigma} - \ddot{\sigma}, \end{aligned} \tag{35}$$

$$\begin{aligned} 2\dot{H} = -6\sqrt{6}a^3cdH^2n + 18a^2cH^2 - 6a^2H^2 - \sqrt{6}ac\ddot{\phi} - 6\sqrt{6}e^3H^2mpr \\ + 18e^2H^2p + 6e^2H^2 - \sqrt{6}ep\ddot{\sigma} - 3H^2\Omega_m. \end{aligned} \tag{36}$$

Finally, the Friedmann constraint equation is reduced to the following relation:

$$\Omega_m = -\sqrt{6}a^3cdn + 6a^2c - a^2 - b^2 - \sqrt{6}e^3mpr + 6e^2p + e^2 - f^2 + 1. \quad (37)$$

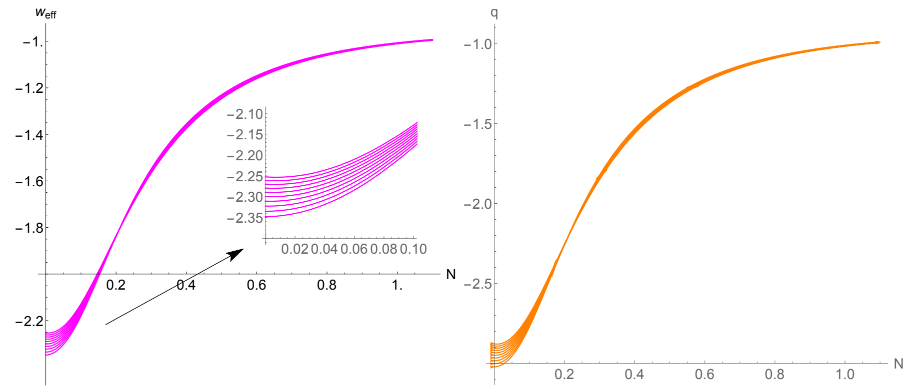

Fig. 1 – The numerical evolution of the effective equation of state $w_{\mathbf{eff}}$ for various initial conditions with respect to N, the logarithm of the cosmic scale factor (left panel); the variation of the corresponding deceleration parameter q (right panel). We have considered the following values for the parameters: $m = 3, n = 2, \lambda_1 = 1, \lambda_2 = 1, w_m = 0$.

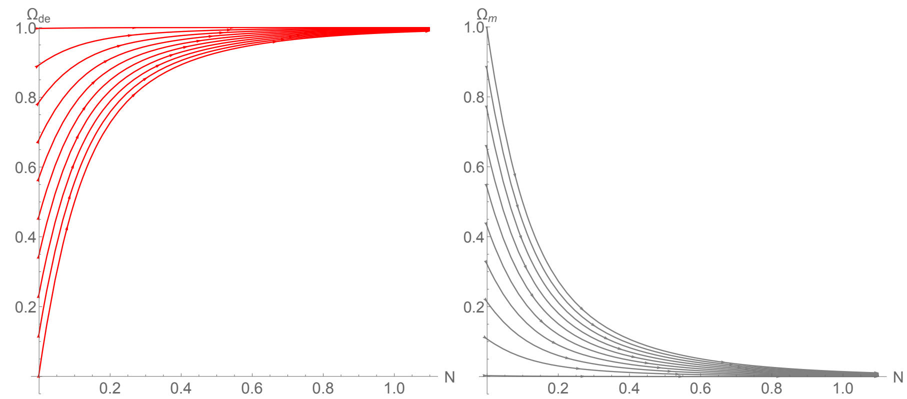

Fig. 2 – The evolution of the density parameters associated to the matter and dark energy fluids for different initial conditions: the variation of the dark energy density parameter Ω_{de} (left panel); b) the behavior of the matter density parameter Ω_m (right panel).

As can be noted, the dynamics of the system for this specific case is reduced to an 8–dimensional autonomous system of differential equations. Due to the high complexity of the phase space, we shall rely the investigation of the dynamical features only on non–exhaustive numerical evaluations of the autonomous system for various initial conditions, analyzing the behavior of the effective equation of state, the deceleration parameter, the matter and dark energy density parameters, respectively. We have displayed in Figure 1 the behavior of the effective equation of state $w_{\mathbf{eff}}$ for our model with respect to the logarithm of the cosmic scale factor N and the corresponding deceleration parameter q. From these figures it is observed that initially the effective equation of state corresponds to a super–accelerating phase with a phantom pathological behavior and the dynamical system evolves asymptotically from bellow near a de–Sitter epoch. For higher values of the cosmic scale factor, the present quintom model behaves ultimately closely as a cosmological constant. Furthermore, in Figure 2 the evolution of the density parameters associated to the matter and dark energy fluids for different initial conditions are displayed. It is observed that for various initial values of the density parameter the system evolves toward a de–Sitter epoch where the effective equation of state closely mimics the behavior of a cosmological constant, an era characterized by the domination of the dark energy density parameter Ω_{de} over the matter density parameter Ω_m.

4. CONCLUSIONS

Within this paper a new cosmological scenario is proposed in the scalar tensor theories of gravitation by extending our previous work which consisted in a quintom model with nonminimal Galileon corrections, to generalised coupling functions in the corresponding action. In this case we have deduced the resulting Klein–Gordon and modified Friedmann relations, obtaining the basic equations which describe the evolution of the corresponding cosmological model. Assuming power–law behavior for the nonminimal coupling functions we have introduced the auxiliary variables in which the dynamics can be described by an 8 dimensional autonomous system of differential equations. Due to the high complexity of the phase space we have relied our analysis only to a numerical approach, analyzing the dynamical features of the phase space structure. We have solved numerically the autonomous dynamical system by considering different initial conditions, obtaining the evolution for the effective equation of state, the matter and dark energy density parameters, respectively. For various initial conditions the system evolves from a phantom regime with a super–negative equation of state towards a stage where the dark energy dominates completely the cosmic picture in terms of density parameters, mimicking a cosmological constant behavior, a de–Sitter regime which is attained asymptotically.

REFERENCES

1. A. G. Riess *et al.* [Supernova Search Team], *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, Astron. J. **116** (1998) 1009; arXiv:astro-ph/9805201.

2. S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], *Measurements of* Ω *and* Λ *from 42 high redshift supernovae*, Astrophys. J. **517** (1999) 565; arXiv:astro-ph/9812133.
3. E. J. Copeland, M. Sami and S. Tsujikawa, *Dynamics of dark energy*, Int. J. Mod. Phys. D **15** (2006) 1753; arXiv:hep-th/0603057.
4. V. Sahni, *The Cosmological constant problem and quintessence*, Class. Quant. Grav. **19** (2002) 3435; arXiv:astro-ph/0202076.
5. J. Hogan, *Welcome to the Dark Side*, Nature **448** (2007) 240-245.
6. A. Del Popolo and M. Le Delliou, *Small scale problems of the* Λ*CDM model: a short review*, Galaxies **5** (2017) no.1, 17; arXiv:1606.07790.
7. S. Weinberg, *The Cosmological Constant Problem*, Rev. Mod. Phys. **61** (1989) 1.
8. A. V. Astashenok and A. del Popolo, *Cosmological measure with volume averaging and the vacuum energy problem*, Class. Quant. Grav. **29** (2012) 085014; arXiv:1203.2290.
9. G. B. Zhao *et al.*, *Dynamical dark energy in light of the latest observations*, Nat. Astron. **1** (2017) no.9, 627; arXiv:1701.08165.
10. B. Feng, X. L. Wang and X. M. Zhang, *Dark energy constraints from the cosmic age and supernova*, Phys. Lett. B **607** (2005) 35; arXiv:astro-ph/0404224.
11. H. Wei and R. G. Cai, *A note on crossing the phantom divide in hybrid dark energy model*, Phys. Lett. B **634** (2006) 9; arXiv:astro-ph/0512018.
12. A. Vikman, *Can dark energy evolve to the phantom?*, Phys. Rev. D **71** (2005) 023515; arXiv:astro-ph/0407107.
13. M. Kunz and D. Sapone, *Crossing the Phantom Divide*, Phys. Rev. D **74** (2006) 123503; arXiv:astro-ph/0609040.
14. Y. F. Cai, E. N. Saridakis, M. R. Setare and J. Q. Xia, *Quintom Cosmology: Theoretical implications and observations*, Phys. Rept. **493** (2010) 1; arXiv:0909.2776.
15. A. Sangwan, A. Mukherjee and H. K. Jassal, *Reconstructing the dark energy potential*, JCAP **1801** (2018) 018; arXiv:1712.05143.
16. Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, *Cosmological evolution of a quintom model of dark energy*, Phys. Lett. B **608** (2005) 177; arXiv:astro-ph/0410654.
17. Z. K. Guo, Y. S. Piao, X. Zhang and Y. Z. Zhang, *Two-Field Quintom Models in the w-w' Plane*, Phys. Rev. D **74** (2006) 127304; arXiv:astro-ph/0608165.
18. S. Tsujikawa, *Quintessence: A Review*, Class. Quant. Grav. **30** (2013) 214003; arXiv:1304.1961.
19. R. R. Caldwell, *A Phantom menace?*, Phys. Lett. B **545** (2002) 23; arXiv:astro-ph/9908168.
20. R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, , *Phantom Energy: Dark Energy with* $w < -1$ *causes a Cosmic Doomsday*, Phys. Rev. Lett. **91** (2003) 071301.
21. M. Visser and C. Barcelo, *Energy conditions and their cosmological implications*, arXiv:gr-qc/0001099.
22. B. Ratra and P. J. E. Peebles, *Cosmological Consequences of a Rolling Homogeneous Scalar Field*, Phys. Rev. D **37** (1988) 3406.
23. R. R. Caldwell, R. Dave and P. J. Steinhardt, *Cosmological imprint of an energy component with general equation of state*, Phys. Rev. Lett. **80** (1998) 1582; arXiv:astro-ph/9708069.
24. K. J. Ludwick, *The viability of phantom dark energy: A review*, Mod. Phys. Lett. A **32** (2017) no.28, 1730025; arXiv:1708.06981.
25. W. Hu, *Crossing the phantom divide: Dark energy internal degrees of freedom*, Phys. Rev. D **71** (2005) 047301; arXiv:astro-ph/0410680.
26. S. Panpanich, P. Burikham, S. Ponglertsakul and L. Tannukij, *Resolving Hubble Tension with Quintom Dark Energy Model*, arXiv:1908.03324.

27. S. Mishra and S. Chakraborty, *Dynamical system analysis of quintom dark energy model*, Eur. Phys. J. C **78** (2018) no.11, 917; arXiv:1811.08279.
28. G. Leon, A. Paliathanasis and J. L. Morales-Martnez, *The past and future dynamics of quintom dark energy models*, Eur. Phys. J. C **78** (2018) no.9, 753; arXiv:1808.05634.
29. S. Dutta, M. Lakshmanan and S. Chakraborty, *Quintom cosmological model and some possible solutions using Lie and Noether symmetries*, Int. J. Mod. Phys. D **25** (2016) no.14, 1650110; arXiv:1607.03396.
30. K. Nozari, K. Asadi and F. Rajabi, *Cosmological dynamics of a quintom field on the warped DGP brane*, Astrophys. Space Sci. **349** (2014) 549.
31. A. Nikjou, R. Baghbani, M. T. Darvishi and F. Khani, *The phase space of quintom cosmology*, Astrophys. Space Sci. **345** (2013) 421.
32. G. Leon, Y. Leyva and J. Socorro, *Quintom phase-space: beyond the exponential potential*, Phys. Lett. B **732** (2014) 285; arXiv:1208.0061.
33. M. R. Setare and E. N. Saridakis, *Quintom dark energy models with nearly flat potentials*, Phys. Rev. D **79** (2009) 043005; arXiv:0810.4775.
34. E. N. Saridakis, *Quintom evolution in power-law potentials*, Nucl. Phys. B **830** (2010) 374; arXiv:0903.3840.
35. M. Marciu, *Prospects of the cosmic scenery in a quintom dark energy model with generalized nonminimal Gauss-Bonnet couplings*, Phys. Rev. D **99** (2019) no.4, 043508.
36. M. Marciu, D. M. Ioan and F. V. Iancu, *Dynamical features of a quintom dark energy model with Galileon corrections*, Int. J. Mod. Phys. D **28** (2018) no.01, 1950018.
37. S. Bahamonde, M. Marciu and P. Rudra, *Generalised teleparallel quintom dark energy non-minimally coupled with the scalar torsion and a boundary term*, JCAP **1804** (2018) 056; arXiv:1802.09155.
38. M. Marciu, *Quintom dark energy with nonminimal coupling*, Phys. Rev. D **93** (2016) no.12, 123006.
39. N. Behrouz, K. Nozari and N. Rashidi, *Interacting quintom dark energy with Nonminimal Derivative Coupling*, Phys. Dark Univ. **15** (2017) 72.
40. M. R. Setare and E. N. Saridakis, *Non-minimally coupled canonical, phantom and quintom models of holographic dark energy*, Phys. Lett. B **671** (2009) 331; arXiv:0810.0645.
41. K. Nozari, M. R. Setare, T. Azizi and N. Behrouz, *A Non-minimally Coupled Quintom Dark Energy Model on the Warped DGP Brane*, Phys. Scripta **80** (2009) 025901; arXiv:0810.1427.
42. M. Marciu, D. M. Ioan and F. V. Iancu, *Dynamical features of a quintom dark energy model with Galileon corrections*, Int. J. Mod. Phys. D **28** (2018) no.01, 1950018.
43. T. Qiu, J. Evslin, Y. F. Cai, M. Li and X. Zhang, *Bouncing Galileon Cosmologies*, JCAP **1110** (2011) 036; arXiv:1108.0593.
44. A. Ali, R. Gannouji, M. W. Hossain and M. Sami, *Light mass galileons: Cosmological dynamics, mass screening and observational constraints*, Phys. Lett. B **718** (2012) 5; arXiv:1207.3959.
45. S. Zaabat and K. Nouicer, *Cosmology of the interacting Cubic Galileon*, arXiv:1807.00700.
46. M. Shahalam, S. K. J. Pacif and R. Myrzakulov, *Galileons, phantom fields, and the fate of the Universe*, Eur. Phys. J. C **76** (2016) no.7, 410; arXiv:1602.03176.
47. G. Leon and E. N. Saridakis, *Dynamical analysis of generalized Galileon cosmology*, JCAP **1303** (2013) 025; arXiv:1211.3088.
48. A. Liddle, *An Introduction to Modern Cosmology* (2nd edn., John Wiley and Sons, 2003).