TIME EVOLUTION OF THE GAUSSIAN ENTROPIC DISCORD IN A SQUEEZED THERMAL ENVIRONMENT

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Abstract. We describe the Markovian dynamics of the Gaussian quantum discord of a system composed of two uncoupled bosonic modes in two cases: when they are in contact with a common squeezed thermal bath, and when each of them is interacting with its own squeezed thermal bath. This study is done in the framework of the theory of open quantum systems, based on completely positive quantum dynamical semigroups. We take an initial squeezed thermal state and we show that the behaviour of the Gaussian quantum discord depends on the bath parameters (temperature, dissipation coefficient, squeezing parameter and squeezing phase), and on the initial state of the system (squeezing parameter and average photon numbers). We show that Gaussian quantum discord is decaying in time, tending asymptotically to zero, due to the effect of the environment. We also compare the Gaussian quantum discord with the Gaussian geometric discord.

Key words: Gaussian entropic discord, squeezed thermal baths, Gaussian states, open quantum systems.

1. INTRODUCTION

In the past years quantum information theory has obtained recognition though studying quantum correlations, which enable interesting applications in quantum information processing. Recent theoretical and experimental results indicate that some non-entangled mixed states can improve performance in some quantum computing tasks [1–3]. Quantum discord was defined as a measure of quantum correlations for a bipartite system that can exist in a separable state [4, 5]. In the previous papers, the dynamics of quantum correlations, such as entanglement and discord, in continuous variable open systems has been thoroughly studied [6–17].

In this paper we study, in the framework of the theory of open quantum systems based on completely positive quantum dynamical semigroups, the dynamics of Gaussian quantum discord of a subsystem consisting of two bosonic modes (harmonic oscillators), in two cases: they are interacting with a common squeezed thermal bath, and each one is interacting with its own squeezed thermal bath. Starting with an initial squeezed thermal state, we describe the evolution in time of the Gaussian quantum discord. We show that the quantum discord strongly depends on the parameters characterising the initial state (squeezing of the modes and average number of the thermal photons) and on the parameters characterising the reservoir (temperature, squeezing parameter and squeezing phase, and dissipation parameter). We compare, for the considered system, the time evolution of Gaussian quantum discord and geometric quantum discord. The paper is organised as follows. In Sec. 2 we introduce the covariance matrix formalism used for the description of the dynamics of open quantum systems. In Sec. 3 we describe the behaviour of the Gaussian quantum discord for the considered system, interacting with the environment. A summary in given in Sec. 4.

2. COVARIANCE MATRIX OF TWO BOSONIC MODES INTERACTING WITH SQUEEZED THERMAL BATHS

We study the dynamics of a subsystem consisting of two bosonic modes (harmonic oscillators) weakly interacting with squeezed thermal baths. Using the axiomatic formalism of the theory of open quantum systems, based on completely positive dynamical semi-groups, the Markovian irreversible dynamics is determined by the following master equation for the density operator $\rho(t)$ [18–21]:

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H,\rho(t)] + \frac{1}{2\hbar}\sum_k \left(2L_k\rho(t)L_k^\dagger - \left\{\rho(t), L_k^\dagger L_k\right\}_+\right), \tag{1}$$

where H is the Hamiltonian of the system and L_k are operators defined on the Hilbert space of H, representing the interaction of the open system with the environment. The Hamiltonian of two bosonic modes with identical masses $m_1 = m_2 = 1$ and frequencies ω_1 and ω_2 is given by

$$H = \frac{1}{2}\left(p_x^2 + p_y^2 + \omega_1^2 x^2 + \omega_2^2 y^2\right), \tag{2}$$

where x, y are the coordinates and p_x, p_y are the momenta of the two quantum oscillators. The Gaussian form of the states is preserved during the interaction with the environment by taking the operators L_k as polynomials of the first degree in the canonical variables. The covariance matrix of the bi-modal system that entirely specifies a two-mode Gaussian state is given by

$$\sigma(t) = \begin{pmatrix} \sigma_{xx}(t) & \sigma_{xp_x}(t) & \sigma_{xy}(t) & \sigma_{xp_y}(t) \\ \sigma_{xp_x}(t) & \sigma_{p_xp_x}(t) & \sigma_{yp_x}(t) & \sigma_{p_xp_y}(t) \\ \sigma_{xy}(t) & \sigma_{yp_x}(t) & \sigma_{yy}(t) & \sigma_{yp_y}(t) \\ \sigma_{xp_y}(t) & \sigma_{p_xp_y}(t) & \sigma_{yp_y}(t) & \sigma_{p_yp_y}(t) \end{pmatrix}, \tag{3}$$

with entries defined by $\sigma_{ij} = \mathrm{Tr}[(R_iR_j + R_jR_i)\rho]$, where $i,j = 1,2,3,4$, and $\mathbf{R} = \{x, p_x, y, p_y\}$.

From the master equation (1) we obtain the following system of equations for the quantum correlations of the canonical observables, which can be written in the matrix form as follows [20]:

$$\frac{d\sigma(t)}{dt} = Y\sigma(t) + \sigma(t)Y^{\mathrm{T}} + 2\mathcal{D}, \tag{4}$$

with

$$Y = \begin{pmatrix} -\lambda & 1 & 0 & 0 \\ -\omega_1^2 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 1 \\ 0 & 0 & -\omega_2^2 & -\lambda \end{pmatrix} \tag{5}$$

and

$$\mathcal{D} = \begin{pmatrix} D_{xx} & D_{xp_x} & D_{xy} & D_{xp_y} \\ D_{xp_x} & D_{p_xp_x} & D_{yp_x} & D_{p_xp_y} \\ D_{xy} & D_{yp_x} & D_{yy} & D_{yp_y} \\ D_{xp_y} & D_{p_xp_y} & D_{yp_y} & D_{p_yp_y} \end{pmatrix}, \tag{6}$$

where the dissipation coefficient λ and the diffusion coefficients D_{ij} are real quantities.

The solution to the equation (4) is given by [20]

$$\sigma(t) = M(t)(\sigma(0) - \sigma(\infty))M^{\mathrm{T}}(t) + \sigma(\infty), \tag{7}$$

where $M(t) \equiv \exp(Yt) \to 0$ for $t \to \infty$. The asymptotic covariance matrix defined only by the bath parameters is:

$$\sigma(\infty) = \begin{pmatrix} 1+2(N_1+M_{1R}) & 2M_{1I} & 0 & 0 \\ 2M_{1I} & 1+2(N_1-M_{1R}) & 0 & 0 \\ 0 & 0 & 1+2(N_2+M_{2R}) & 2M_{2I} \\ 0 & 0 & 2M_{2I} & 1+2(N_2-M_{2R}) \end{pmatrix}, \tag{8}$$

where $(k = 1,2)$

$$N_k = n_{th,k}\left(\cosh^2 R + \sinh^2 R\right) + \sinh^2 R, \tag{9}$$

M_{1R} and M_{1I} are the real and, respectively, the imaginary part of

$$M_k = -(2n_{th,k} + 1)\cosh R \sinh R \exp(i\varphi), \tag{10}$$

with R and φ being the squeezing parameter and squeezing phase of the baths, respectively. The average photon numbers of the baths are given by (we set the Boltzmann constant $k_B = 1$)

$$n_{th,k} = \frac{1}{2}\left(\coth\frac{1}{2T_k} - 1\right) \tag{11}$$

and T_k are their temperatures.

The real, positive and symmetric covariance matrix (3) can be written in the block structure as follows:

$$\sigma(t) = \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix}, \tag{12}$$

where A, B are the covariance matrices of the individual modes and C contains the cross-correlations between the modes.

3. DYNAMICS OF GAUSSIAN QUANTUM DISCORD

Quantum discord is defined as the difference between two quantum analogues of classically equivalent expressions of the mutual information, measuring total correlations in a quantum state [3]. If we have pure entangled states, then quantum discord coincides with the entropy of entanglement. For a bipartite state ρ_{12} quantum discord is defined as the measure of all quantum correlations in this quantum state [4, 5] :

$$D(\rho_{12}) = I(\rho_{12}) - C(\rho_{12}), \tag{13}$$

where $I(\rho_{12})$ is the quantum mutual information [22] expressing total correlations and $C(\rho_{12})$ are the classical correlations in the considered bipartite quantum state [23]. Extending the notion of discord to bipartite continuous variable systems, closed formulas of the Gaussian quantum discord have been obtained for bipartite squeezed thermal states [24] and for general two-mode Gaussian states [25].

The Gaussian quantum discord of a general two-mode Gaussian state can be defined as the quantum discord where the conditional entropy is restricted to generalized Gaussian positive operator valued measurements (POVM) on the mode 2. In terms of four symplectic invariants [25], defined as: $\alpha = \det A$, $\beta = \det B$, $\gamma = \det C$ and $\delta = \det \sigma$ one gets, for the Gaussian quantum discord, the following expression:

$$D = f(\sqrt{\beta}) - f(\nu_-) - f(\nu_+) + f(\sqrt{\varepsilon}), \tag{14}$$

where

$$f(y) = \frac{y+1}{2}\log\frac{y+1}{2} - \frac{y-1}{2}\log\frac{y-1}{2}, \tag{15}$$

$\nu_\pm$ are the symplectic eigenvalues of the state, given by

$$2\nu_{\pm}^2 = \Delta \pm \sqrt{\Delta^2 - 4\delta}, \;\; \Delta \equiv \alpha + \beta + 2\gamma \tag{16}$$

and

$$\varepsilon = \begin{cases} \frac{2\gamma^2+(\beta-1)(\delta-\alpha)+2|\gamma|\sqrt{\gamma^2+(\beta-1)(\delta-\alpha)}}{(\beta-1)^2} \\ \text{if } (\delta-\alpha\beta)^2 \leq (\beta+1)\gamma^2(\alpha+\delta), \\ \\ \frac{\alpha\beta-\gamma^2+\delta-\sqrt{\gamma^4+(\delta-\alpha\beta)^2-2\gamma^2(\delta+\alpha\beta)}}{2\beta} \\ \text{otherwise.} \end{cases} \tag{17}$$

In order to study the dynamics of Gaussian quantum discord we take as an initial state a squeezed thermal one, having the following covariance matrix [26]:

$$\sigma(0) = \begin{pmatrix} a & 0 & c & 0 \\ 0 & a & 0 & -c \\ c & 0 & b & 0 \\ 0 & -c & 0 & b \end{pmatrix}, \tag{18}$$

where

$$a = 2n_1 \cosh^2 r + 2n_2 \sinh^2 r + \cosh 2r, \tag{19}$$

$$b = 2n_2 \cosh^2 r + 2n_1 \sinh^2 r + \cosh 2r, \tag{20}$$

$$c = (n_1 + n_2 + 1)\sinh 2r. \tag{21}$$

Here r is the squeezing parameter of the initial state and n_1, n_2 are the average photon numbers of the initial state.

3.1. COMMON SQUEEZED THERMAL BATH

In the following we describe the time evolution of Gaussian quantum discord when the two bosonic modes interact with a common squeezed thermal bath. In Fig. 1 it is illustrated the time evolution of the discord as a function of the squeezing parameter and temperature of the bath. We observe that quantum discord is increasing with the squeezing parameter, but it is decreasing by increasing the temperature. In time, quantum discord asymptotically tends to a zero value. From Fig. 2 we notice that the Gaussian quantum discord is decreasing as we increase the squeezing parameter of the bath and has an oscillatory behaviour as a function of the squeezing phase of the bath.

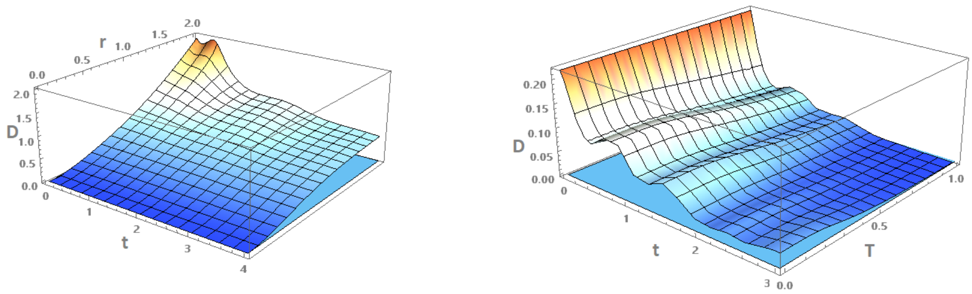

Fig. 1 – Gaussian quantum discord D as a function of time t and squeezing parameter of the modes r (left) for resonant modes ($\omega_1 = \omega_2 = 1$) and $n_1 = 2$, $n_2 = 1$, $T = 0.1$, $\lambda = 0.1$, $R = 0.1$, $\varphi = 5$. Gaussian quantum discord D as a function of time t and temperature T (right), for non-resonant modes ($\omega_1 = 1$ and $\omega_2 = 4$), for $n_1 = n_2 = 1, r = 0.5, R = 0.5, \lambda = 0.5$ and $\varphi = 0$.

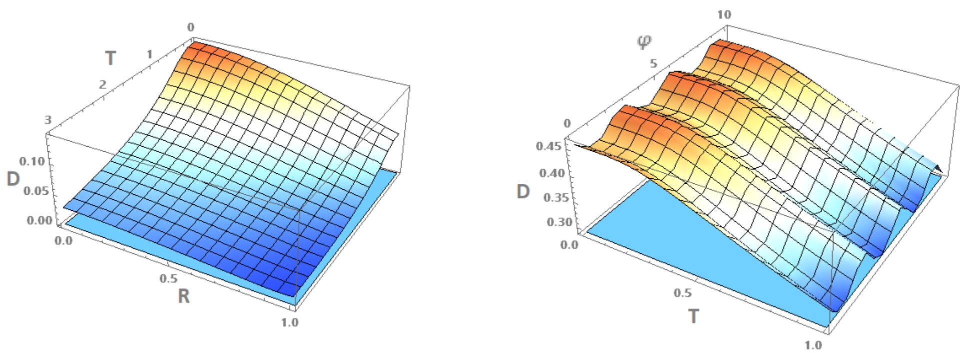

Fig. 2 – Gaussian quantum discord D as a function of temperature T and squeezing parameter of the bath R (left), for $t = 3, n_1 = 1, n_2 = 2, r = 0.5, \lambda = 0.1$ and $\varphi = 1$. Gaussian quantum discord D as a function of temperature T and the squeezing phase φ (right), for $t = 2, r = 1, R = 0.1, n_1 = 1, n_2 = 2$ and $\lambda = 0.1$. In both cases we consider resonant modes ($\omega_1 = \omega_2 = 1$).

3.2. TWO SQUEEZED THERMAL BATHS

In Fig. 3 it is shown the time evolution of Gaussian quantum discord in the case when each bosonic mode interacts with its own squeezed thermal bath. Like in the case of a common thermal bath, we observe that quantum discord is increasing with the squeezing parameter of the modes, it has an oscillatory behaviour as a function of the squeezing phase of the baths, and in time it asymptotically tends to a zero value. In addition, from Fig. 4 we can see that quantum discord is increasing with the average thermal photon numbers of the two modes.

The general behaviour of the Gaussian quantum discord is that it is increasing with the parameters characterising the initial state of the system – the squeezing parameter of the bosonic modes and the average thermal photon numbers of the two modes, and is decreasing by increasing the parameters describing the environment – temperatures, dissipation constant and squeezing of the baths.

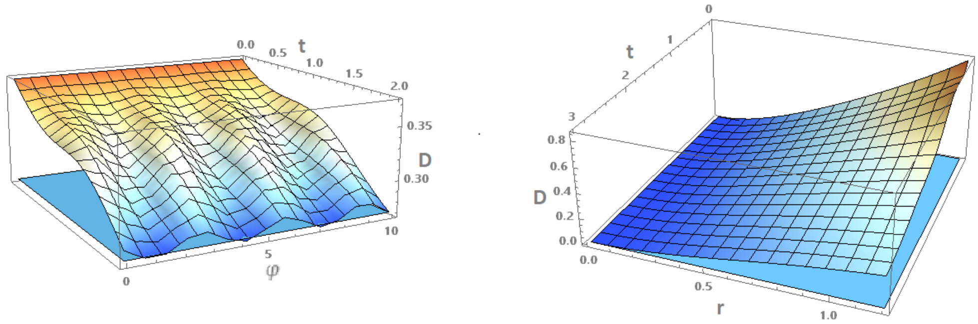

Fig. 3 – Gaussian quantum discord D *versus* time t and squeezing phase φ (left), for $n_1 = 1, n_2 = 2, r = 0.7, T_1 = 1, T_2 = 0.5, R = 0.1$ and $\lambda = 0.1$. Gaussian quantum discord D as a function of time t and squeezing parameter of the modes r (right), for $n_1 = 2, n_2 = 1, T_1 = 0.1, T_2 = 2, R = 0.1, \varphi = 5$ and $\lambda = 0.1$. In both cases we consider resonant modes ($\omega_1 = \omega_2 = 1$).

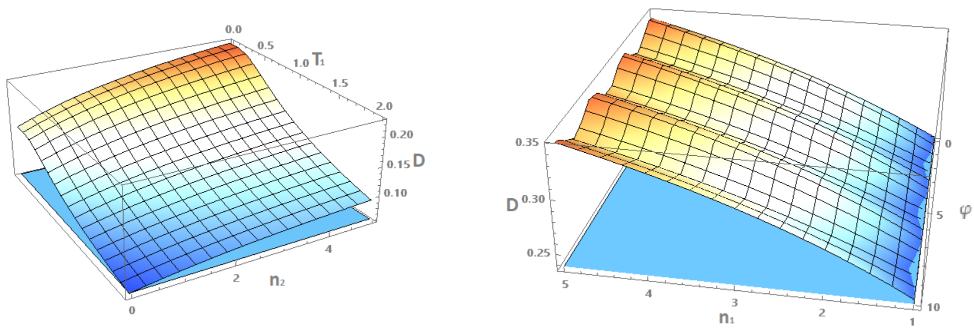

Fig. 4 – Gaussian quantum discord D as a function of temperature T_1 and average photon number n_2 (left), for $t = 2, n_1 = 1, r = 1, \lambda = 0.1$, $T_2 = 0.1, R = 0.1, \varphi = 0$. Gaussian quantum discord D as a function of the average photon number n_1 and the squeezing phase φ (right), for $t = 1, r = 0.5, R = 0.1, n_2 = 0.5$, $T_1 = 1.2$, $T_2 = 0.5$ and $\lambda = 0.1$. In both cases we consider resonant modes ($\omega_1 = \omega_2 = 1$).

In Fig. 5 it is shown an example of comparison between the time evolutions of the Gaussian quantum discord when the two bosonic modes interact with a common squeezed thermal bath, and when each mode interacts with its own squeezed thermal bath, the baths having different temperatures. We can see that the behaviour of the quantum discord strongly depends on the temperatures of the baths.

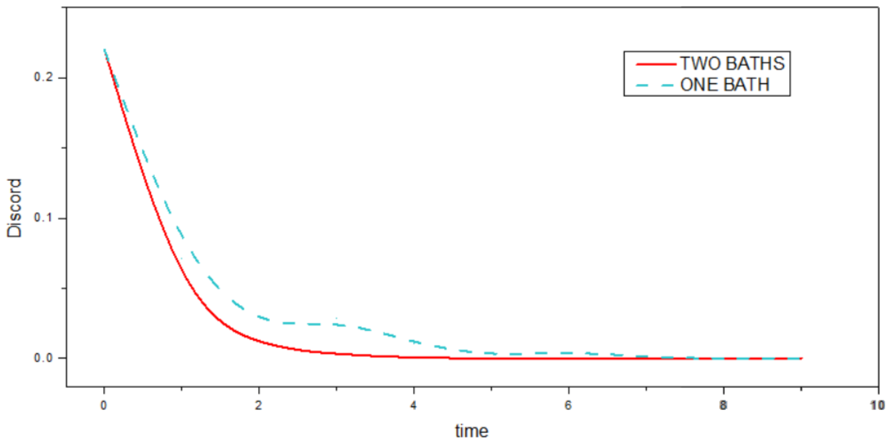

Fig. 5 – Gaussian quantum discord D as a function of time, for $r = 1, n_1 = 2, n_2 = 1, R = 0.1, \varphi = 5$, $\lambda = 0.1$, in the case of a common squeezed thermal bath with temperature $T = 0.1$ (dashed line), and in the case of two squeezed thermal baths with temperatures $T_1 = 0.1$ and $T_2 = 2$ (solid line).

3.3. COMPARING GAUSSIAN QUANTUM DISCORD AND GAUSSIAN GEOMETRIC DISCORD

The geometric discord is another type of quantum discord, defined as the minimum squared Hilbert-Schmidt distance between the given state and the closest classical-quantum state obtained after a local generalized Gaussian positive operator valued measurement performed on one subsystem [27].

In Fig. 6 we compare the time evolutions of Gaussian quantum discord [28] and geometric discord [29] for two resonant bosonic modes interacting with a squeezed thermal bath, for an initial squeezed thermal state. We can see that both Gaussian quantum discord and Gaussian geometric discord decay in time asymptotically to the value zero, corresponding to an asymptotic product state. This is the result of the influence of the environment, which destroys the quantum correlations existing in the quantum system.

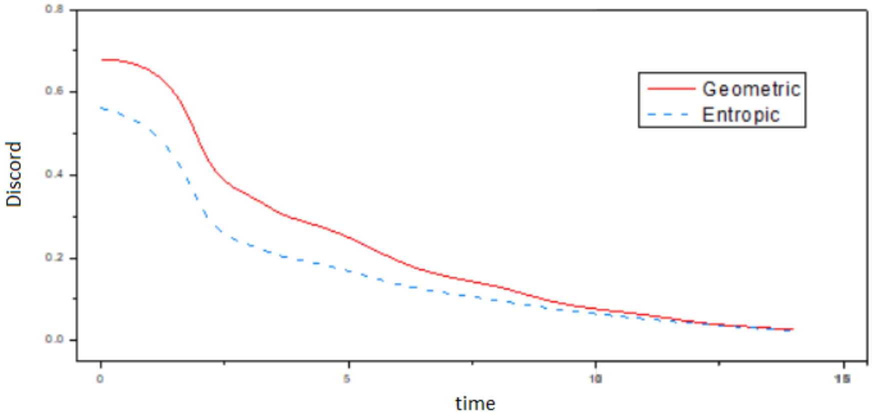

Fig. 6 – Gaussian entropic discord (dashed line) and Gaussian geometric discord (solid line) as a function of time for resonant modes ($\omega_1 = \omega_2 = 1$) and for $r = 1, n_1 = 1, n_2 = 2, R = 0.1, \varphi = 0$, $T = 1, \lambda = 0.1$.

4. SUMMARY

We studied the Markovian dynamics of the Gaussian quantum discord of two bosonic modes, in two cases: both modes interact with a common squeezed thermal bath, and each mode interacts with its own squeezed thermal bath. We worked in the framework of the theory of open quantum system, based on completely positive quantum dynamical semigroups. We have shown that the behaviour of the Gaussian quantum discord strongly depends on the parameters characterising the initial state of the system and the squeezed thermal baths. Gaussian quantum discord is increasing with the parameters characterising the initial state (squeezing parameter and average numbers of thermal photons) and asymptotically decays towards zero value, due to the interaction with the environment.

For the considered system we compared the Gaussian quantum discord and the geometric discord and have shown that their behaviour is in general similar, both of them decaying in time due to the influence of the environment.

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