TRANSPORT COEFFICIENTS OF STRONGLY INTERACTING QUARK-GLUON PLASMA USING DUAL QCD HADRONIC BAG

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Abstract. Utilizing the magnetic symmetry based dual QCD model of non-Abelian gauge theory, the qualitative feature of the strongly interacting QGP have been understood within the framework of dual QCD hadronic bag. The study of the nonequilibrium and dissipative effects observed during the phase transition have been studied by investigating the temperature behavior of transport coefficients along with the relaxation times for quarks and gluons. These transport parameters provide proper understanding about the non-equilibrium dynamics of the system. The behavior of shear and bulk viscosity confirms the non-monotonic and non-conformal behavior of QGP near the critical temperature. The high temperature behavior of QGP appears to be conformal and behaves like an ideal relativistic gas.

Key words: Gauge field theories, General properties of QCD (dynamics, confinement, etc.), Quark-gluon plasma.

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1. INTRODUCTION

One of the crucial areas of high energy physics research is to examine the study of Quantum Chromodynamics (QCD) [1–3] contemplated as the fundamental theory of quarks and gluons. In the ultraviolet (UV) or weak coupling regime of QCD, the perturbative calculations for deep inelastic scattering agree well with experimental data. However, in the infrared (IR) regime, the description of QCD vacuum as well as hadron properties and non-perturbative processes still remains as distinct challenge in the formulation of QCD as a local quantum field theory. One of its most peculiar characteristic conjecture is that at sufficiently high temperatures \( T \) or chemical potentials \( \mu_B \), QCD is believed to be in quark gluon plasma (QGP) phase, whereby color charges are screened rather than confined [4–7]. Exploring the aforesaid new states of matter under extreme conditions is necessary quest and such a one concisely prevail the Universe at about a few microseconds after the Big Bang [8], and has now been regenerated as the hottest matter ever in laboratory by heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) [9–15] as well as at the Large Hadron Collider (LHC) [16–23]. However, the physical properties conjectured...
for such phase are not yet evidently understood and does not enlighten the inquest about the relevant degrees of freedom to formulate the proper statistical description of the Strongly Interacting QGP (sQGP). Lattice QCD Simulation [24–31] also provide a non-perturbative approach to study the QCD properties at finite temperature but exclusively from the equation of state (EoS), lattice QCD simulations cannot distinguish a weakly interacting QGP (wQGP) from a sQGP. Therefore, in order to understand the properties of sQGP at extreme conditions, the transport coefficients for quark matter are the prime physical quantities to be studied [32–34]. Transport properties of sQGP describes the hydrodynamic response of the system to energy and momentum density fluctuations and have attracted remarkable attention in the last years. The experimental findings of the heavy-ion collisions at the RHIC have reported to a small viscosity, leading to the revelation of the discovery of the nearly prefect fluidity of the sQGP. Therefore, several studies on the viscosities have been accomplish in the last few years, for the confined [35–37] as well as for the deconfined phase [38–42]. Indeed, the idea of a strongly coupled QGP, which behaves like an almost perfect liquid with very low viscosity needs to be addressed theoretically from first principle. In view of these facts, the only so far well-established method to perform an analytic study of strongly interacting matter as well as their associated transport properties based on the first principles is an effective field theory of infrared effective dual QCD discussed in the next section.

2. SU(2) DUAL QCD FORMULATION AND MONOPOLE CONDENSATION

In order to demonstrate that monopole condensation is essential to understand color confinement in QCD, let us start from the SU(2) color gauge group and review the dual QCD formulation first [43–50]. Imposing the magnetic isometry condition on the gauge potential \( W_\mu \) leads to the restricted potential in the following form,

\[
D_\mu \hat{m} = 0, \text{ i.e. } (\partial_\mu + g W_\mu \times ) \hat{m} = 0, \quad (1)
\]

\[
W_\mu = A_\mu \hat{m} - g^{-1} (\hat{m} \times \partial_\mu \hat{m}) = A_\mu + B_\mu, \quad (2)
\]

and

\[
A_\mu \equiv \hat{m} \cdot W_\mu, \quad B_\mu = -1/g(\hat{m} \times \partial_\mu \hat{m}).
\]

The restricted potential retains the full non-Abelian gauge degrees of freedom and is gauge invariant. Further, the decomposition separates outs the non-topological Abelian part \( A_\mu \) from the topological monopole part \( B_\mu \). The topological part is further viewed as an isolated point singularity representing the monopole topology as,

\[
\hat{m} : S_R^2 \to S^2 = SU(2)/U(1), \quad (3)
\]

where, \( S_R^2 \) is the two dimensional sphere of the three dimensional space and \( S^2 \) is the group coset space completely fixed by \( \hat{m} \). The dual structure of the gauge potential,
is further understood in terms of the field strength and may be given in the following form,

\[ G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + g (W_\mu \times W_\nu) = (F_{\mu\nu} + B^{(d)}_{\mu\nu}) \hat{m}, \]  

(4)

where

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \text{ and } B^{(d)}_{\mu\nu} = -g^{-1} \hat{m} \cdot (\partial_\mu \hat{m} \times \partial_\nu \hat{m}) = \partial_\mu B_\nu - \partial_\nu B_\mu. \]  

(5)

Let \((\xi_1, \xi_2, \xi_3)\) be a right-handed orthonormal basis and choosing \(\hat{m} = \xi_3\) to be the Abelian direction, the gauge potential and field strength reduces in the following form,

\[ W_\mu \rightarrow W'_\mu = (A_\mu + B_\mu) \xi_3, \quad G_{\mu\nu} \rightarrow (F_{\mu\nu} + B^{(d)}_{\mu\nu}) \xi_3, \]  

(6)

where \(U\) is the gauge transformation given in the following form, \(U = \exp(-\alpha t_2 - \beta t_3)\).

More importantly, the dual QCD formulation describes the sub-dynamics of QCD in terms of the \(SU(2)\) QCD Lagrangian,

\[
\mathcal{L} = -\frac{1}{4} F^2_{\mu\nu} - \frac{1}{4} B^2_{\mu\nu} - \frac{1}{2} F_{\mu\nu} B^{\mu\nu} + \bar{\psi}_r i \gamma^\mu \left[ \partial_\mu + g \left( \frac{1}{2} \right) (A_\mu + B_\mu) \right] \psi_r \\
+ \bar{\psi}_b i \gamma^\mu \left[ \partial_\mu - g \left( \frac{1}{2} \right) (A_\mu + B_\mu) \right] \psi_b + m_0 (\bar{\psi}_r \psi_r + \bar{\psi}_b \psi_b) \\
+ \left[ \partial_\mu + i \frac{4\pi}{g} (A^{(d)}_\mu + B^{(d)}_\mu) \right] \phi \]  

(7)

where, \(\psi_r\) and \(\psi_b\) are the red and the blue quarks, \(A_\mu\) and \(B^{(d)}_\mu\) are the regular potentials. It describes the full \(SU(2)\) gauge degrees of freedom and retains the non-Abelian topology of QCD containing the monopole degrees of freedom explicitly.

The monopole condensation in QCD may be triggered in a natural way by prevailing the one-loop effective potential known as the Coleman-Weinberg potential. However, in order to bring out a stable monopole condensation, one need to manifest a phase transition in QCD which is signified through the introduction of the phenomenological quartic potential given as,

\[ V(\phi^* \phi) = 3\alpha_s \frac{1}{2}\phi^2 \phi \phi^2, \]  

(8)

where, \(\phi_0 \equiv \phi^* \phi > 0^{1/2}\) is the vacuum expectation value of \(\phi\). The effective potential breaks the magnetic isometry and emphasizes its topological character to establish monopole condensation of QCD vacuum. As a consequence, the existence of mass scales are guaranteed as two massive modes, a scalar \((m_\phi)\) and axial vector \((m_B)\) which determines the confinement scale of the color electric flux tube. The numerical results of the magnetic glueballs masses have been presented in Table I as functions of coupling \((\alpha_s)\) in the infrared sector of QCD. Moreover, the transition from a confined hadronic phase to a QGP phase, is foreseen at high temperatures.
Table 1

<table>
<thead>
<tr>
<th>$\alpha_s$</th>
<th>$\phi_0$ (GeV)</th>
<th>$m_B$ (GeV)</th>
<th>$m_\phi$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0.143</td>
<td>2.11</td>
<td>4.20</td>
</tr>
<tr>
<td>0.22</td>
<td>0.149</td>
<td>1.51</td>
<td>2.22</td>
</tr>
<tr>
<td>0.47</td>
<td>0.167</td>
<td>1.21</td>
<td>2.22</td>
</tr>
<tr>
<td>0.96</td>
<td>0.181</td>
<td>0.929</td>
<td>0.655</td>
</tr>
</tbody>
</table>

and/or baryonic chemical potentials and for some decades, the bag model equation of state (EoS) has been accustomed to characterize QGP. Employing the dual QCD formalism, the bag energy $B$ may be identified and expressed as [46],

$$B^{1/4} = \left( \frac{12}{\pi^2} \right)^{1/4} m_B^{8/12},$$

(9)

where $m_B$ is the non-thermal contribution to the vector glueball mass. To contemplate the QGP phase with two quark flavors $(u,d)$ and gluons, we utilize a Bag model EoS with bag energy. The pressure and energy density for the QGP phase may be expressed in the following form,

$$P_p = \frac{2}{9} \pi^2 T^4 + \frac{2}{3} T^2 \mu_q^2 + \frac{\mu_q^4}{27 \pi^2} - B,$$

(10)

$$\epsilon_p = \frac{2}{3} \pi^2 T^4 + 2 \mu_q^2 T^2 + \frac{\mu_q^4}{27 \pi^2} + B,$$

(11)

In the next section, the numerical results of the transport coefficient are presented that are sensitive to the change from hadronic to quark-gluon degrees of freedom and provide a suitable order to the phase transition in the $T-\mu$ plane.

### 3. TRANSPORT COEFFICIENTS FOR QUARK-GLUON PLASMA

The crucial characteristic feature of QCD at finite temperature is the existence of an approximate phase transition from hadrons to quarks and gluons. The EoS from lattice QCD calculation and the energy density demonstrate a rapid change for the temperature range $T \simeq 170 - 220 MeV$ and is significant for the evenness of the transition from quarks to hadrons. Although, in addition to the EoS, the transport coefficients are ought to be specified in order to govern the transport of energy and momentum and are clearly of high value. The transport coefficients designate the response of a system to various kind of perturbations and determine the dynamics of the system toward the equilibrium state through dissipation. As the QGP produced...
in heavy-ions collisions is a system considerably away from equilibrium, therefore the investigation of transport coefficient is essential. The transport coefficient of the plasma which is promptly associated to its conformal properties is the bulk viscosity \( \zeta \) related to the thermodynamical quantities and to the correlation functions of the trace of energy-momentum tensor \( T^\mu_\nu \) as [51],

\[
\zeta = \lim_{\omega \to 0} \frac{1}{9\omega} \int_0^\infty dt \int d\tau^3 < [T^\nu_\mu(x), T^\nu_\mu(0)] > e^{i\omega t}.
\] (12)

For a confined frequency region, \( \omega \to \omega_0 \equiv \omega_0(T) \sim T \) and accordingly, \( \zeta \) is recognized as [51]

\[
\zeta = \frac{1}{9T} \left[ T^5 \frac{\partial}{\partial T} \left( \frac{\epsilon_p - 3P_p}{T^4} \right) + 16|\epsilon_v| \right] = \frac{1}{9T} \left[ -16\epsilon_p + 9T s_p + TC_V + 16|\epsilon_v| \right].
\] (13)

![Graph](image)

**Fig. 1** – (Color online) The plot of bulk viscosity over entropy density with temperature for QGP phase using dual QCD hadronic bag at different coupling in the infrared sector of QCD.

where \( \epsilon_p, P_p, s_p, C_V \), are the energy density, pressure, entropy density, specific heat of the QGP phase [46] respectively and \( \epsilon_v \) is the vacuum energy density. Keeping in view the significance of dimensionless quantity, the bulk viscosity over entropy density ratio (\( \zeta/s \)) as a function of the temperature has been depicted in figure 1 for QGP phase using dual QCD hadronic bag [46]. It has been found that \( \zeta/s \) takes a reasonably high value in the region close to critical temperature (\( T_c \)) and may be attributed as the breaking of scale invariance around \( T_c \), eventually acquiring a value nearing zero at high temperature. Also near the phase transition region, square of speed of sound (\( c_s^2 \))[46] shows a minimal behavior indicating the deviation from it's conformal value enforcing \( \zeta/s \) to attain a maximum around the transition region. The large \( \zeta/s \) near the phase transition is always associated to the non-conformal behavior of EoS and illustrate a strongly interacting behavior of the system around the transition temperature. As the temperature increases, the \( \zeta/s \) approaching nearly
The numerical value of bulk viscosity over entropy density ratio and bulk relaxation time for QGP phase using dual QCD hadronic bag at different coupling in infrared sector of QCD.

<table>
<thead>
<tr>
<th>$\alpha_s$</th>
<th>$T_c$ (GeV)</th>
<th>$\zeta/s$</th>
<th>$\tau_\Pi$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0.239</td>
<td>2.03</td>
<td>1.40</td>
</tr>
<tr>
<td>0.22</td>
<td>0.171</td>
<td>2</td>
<td>1.93</td>
</tr>
<tr>
<td>0.47</td>
<td>0.136</td>
<td>2.16</td>
<td>2.57</td>
</tr>
<tr>
<td>0.96</td>
<td>0.105</td>
<td>2.17</td>
<td>3.34</td>
</tr>
</tbody>
</table>

zero and moves towards a conformal equation of state close to a weakly interacting system of quarks and gluons. For the typical value of critical temperatures, the $\zeta/s$ ratio have been studied and represented in table 2 for dual QCD hadronic bag. Although, recent lattice QCD result reveal that $\zeta/s$ rises definitely upto the order of 1.0 close to $T_c$ and has been identified as a strongly interacting system associated with large non-conformal behavior. The behavior of $\zeta/s$ once the quark chemical potential is modulated to non-zero values has been shown in figure 2 for dual QCD hadronic bag at different couplings in the infrared sector of QCD. It has been viewed that with the increase of the chemical potential, the upward cusp becomes sharper and the height of the cusp increases as the system moves from far-ultraviolet to near infrared sector of QCD. Beyond the critical end point ($\mu_E$), $\zeta/s$ diverses owing its
origin to the occurrence of different $T_c$ at different values of chemical potential. This illustrate the shift in peak values of the $\zeta$ as chemical potential is changed. The sharp rise of the $\zeta$ near phase transition creates an instability in the hydrodynamic flow of the plasma and such mode acquire the system to move toward droplets formation.

Further we have also examine the performance of the associated transport coefficients related with the bulk viscous pressure in terms of bulk relaxation times ($\tau_{\Pi}$) and given in the following form [52],

$$\tau_{\Pi} = \frac{\zeta}{14.55(\frac{1}{3} - c_s^2)^2(\epsilon_p + P_p)},$$

(14)

the numerical value has been depicted in table 2 for dual QCD hadronic bag at different coupling in infrared sectors of QCD.

Moreover, as a matter of fact, one more transport coefficient called shear viscosity ($\eta$) also turn out to be a crucial parameter around the phase boundary and reveals the phase transition from the sQGP to a wQGP in the temperature plane. The shear viscosity to entropy density ratio ($\frac{\eta}{s}$) for the QGP phase is shown to be inversely proportional to the trace anomaly ($\Delta(T)$) and may be given in the following form [53, 54],

$$\frac{\eta}{s} = \frac{1}{\Delta(T)},$$

(15)

The $\frac{\eta}{s}$ for the QGP phase has been plotted in figure 3 using dual QCD hadronic bag at different values of coupling. It has also been observed that a minimum of

![Fig. 3 – (Color online) The plot of shear viscosity over entropy density with temperature for QGP phase using dual QCD hadronic bag at different coupling in the infrared sector of QCD.](image-url)
Table 3

The numerical value of shear viscosity over entropy density ratio and shear relaxation time for QGP phase using dual QCD hadronic bag at different coupling in infrared sector of QCD.

<table>
<thead>
<tr>
<th>$\alpha_s$</th>
<th>$T_c(GeV)$</th>
<th>$\eta/s$</th>
<th>$\tau_\pi(fm)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0.239</td>
<td>0.138</td>
<td>0.568</td>
</tr>
<tr>
<td>0.22</td>
<td>0.171</td>
<td>0.139</td>
<td>0.801</td>
</tr>
<tr>
<td>0.47</td>
<td>0.136</td>
<td>0.137</td>
<td>0.992</td>
</tr>
<tr>
<td>0.96</td>
<td>0.105</td>
<td>0.137</td>
<td>1.28</td>
</tr>
</tbody>
</table>

measured on lattice[32] produces $\eta/s = 0.134, 0.102$ for $T = 1.65T_c, 1.24T_c$ respectively which is in agreement with our dual QCD bag model findings. The RHIC measurement also indicate a maximum value of $\eta/s \sim 0.16$ and $\eta/s \sim 0.24$ for different initial conditions [55–59]. For different coupling in the infrared sector of QCD, $\eta/s$ have been calculated and depicted in table 3 for dual QCD hadronic bag.

The associated shear relaxation time ($\tau_\pi$) for QGP is given in the following form, [60, 61],

$$\tau_\pi = \frac{5\eta}{\epsilon + P},$$

and their respective numerical value at different coupling for dual QCD hadronic bag have been depicted in table 3.

4. SUMMARY AND CONCLUSIONS

Using the gauge field topology in terms of the magnetic symmetry structure of non-Abelian gauge theories, the mathematical foundations of dual QCD have been reviewed and analyzed to study the main qualitative feature observed for a strongly interacting QGP within the framework of dual QCD hadronic bag. The study of the non-equilibrium and dissipative effects during the QGP formation have been studied in terms of transport coefficients and their associated relaxation time which characterizes the interaction and collective motion of QGP around the transition region. These coefficients are of particular interest to quantify the properties of strongly interacting relativistic fluid and the fluctuations causing the system to depart from equilibrium to non-equilibrium phase. The response of such system to fluctuations is essentially described in terms of shear and bulk viscosity and their corresponding relaxation time. In general, the stronger the interparticle interaction, the smaller the shear viscosity over entropy density ratio, on the other hand, the large bulk viscosity near the phase transition is related to the non-conformal equation of state and represent a strongly interacting system. The bulk viscosity over entropy density ratio, also exhibits a sharp rising behavior around the critical temperature and the rising behav-
ior shows the equation of state is highly non-conformal around the phase transition. There are strong non-conformal, non-perturbative dynamics going near the critical region, such dynamics leads to non-monotonic behavior in QGP thermodynamics as shown by the strong peak in bulk viscosity over entropy density ratio. In general, among the various experimental evidence for sQGP around the critical temperature, the enhancement of these parameters provide a significant contribution towards the theoretical analysis of the phase structure of QCD and to identify the strongly interacting nature of hadronic matter.

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