STRING COSMOLOGICAL MODEL IN PRESENCE OF MASSLESS SCALAR FIELD IN MODIFIED THEORY OF GENERAL RELATIVITY

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In this paper, the plane symmetric string cosmological model in the presence of massless scalar field is obtained in modified theory of general relativity. The model is constructed for geometric string by assuming the relation between metric potentials; various physical and geometrical properties of the model are discussed.

Key words: Plane Symmetry, Modified Theory of General Relativity, Massless Scalar Field, String Cosmology.

1. INTRODUCTION

The study of modified theory of general relativity has great importance today because several prominent results are in the development of recent study. Barber [1, 2] proposes two self creation theories by modifying Brans and Dicke theory and general theory of relativity. Brans [3, 4] pointed out that, Barber [1] first theory is inconsistent in general, as it violates the principle of equivalence. However, in his second theory the gravitational coupling of Einstein theory field equations is allowed to be variable scalar on the space-time manifold. In this theory the scalar field does not gravitates directly but simply divides the matter tensor acting as reciprocal of gravitational constants in the limits, this theory approaches to Einstein theory in every respect.

The massless scalar field in relativistic mechanics yields significant results regarding both singularities involved and matched the principal. Many authors have examined Barber's second self creation theory on various angles, Pimental [5], Soleng [6] Synge [7], Reddy [8, 9, 10], Reddy and Venkatasweralu [11, 12], Shanti and Rao [13], Shri Ram and Sigh [14], Mohanty *et al.* [15, 16] are some of the authors, who has investigated various aspects of Barber's self creation theory.

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Some cosmologist has studies the various aspects of plane symmetric cosmological model in modified theory of general relativity, Panigrahi and Sahu [17], has studied the micro and macro cosmological model in presence of massless scalar field and perfect field in modified theory of general relativity. Recently Vankateswarlu *et al.* [18] constructed Bianchi-I, II, VIII and IX string cosmological models in Barbers second self creation theory.

The study of string cosmological model was initiated by Vilenkin [19], Latelier [20], Krori *et al.* [21] and studied the problem of cosmic strings in Bianchi type cosmologies with self interacting scalar field. The concept of string theory was developed to describe events at early stages of evaluation of the universe. Kibble [22] and Vilenkin [23] believed that string can be considered as one of the source of density perturbations that are required for formation of the large scale structures in the universe.

In a previous paper of same authors [25], plane symmetric cosmological models of electromagnetic and massless scalar fields in the presence of a charged perfect fluid have been studied. In this paper, we have obtained plane symmetric cosmological model in presence of massless scalar field and cosmic string in modified theory of general relativity. The model is constructed for geometric string $\rho = \lambda'$ by assuming the relation between metric potentials and also discussed the various physical and geometrical properties.

2. SOLUTION OF FIELD EQUATIONS

Consider the plane symmetric space-time as,

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2(\mathrm{d}x^2 + \mathrm{d}y^2) - B^2\mathrm{d}z^2 , \tag{1}$$

where A and B are the function of cosmic time only.
The Einstein-Barber's field equations in the second self creation theory are

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = -8\pi\phi^{-1}(T_{ij}^V + T_{ij}^s) . \tag{2}$$

$$\text{And,} \Box \phi = \frac{8\pi}{3}\lambda(T_{ij}^V + T_{ij}^s) , \tag{3}$$

where ϕ is the Barber scalar, T_{ij}^V is the energy momentum tensor for massless scalar field. T_{ij}^s is the energy momentum tensor for cosmic string. ϕ is the invariant D'Alembertian, T^v is the trace of energy momentum tensor T_{ij}^V , T^S is the stress energy momentum tensor T_{ij}^s , λ is a coupling constant to be determined from experiment where $|\lambda| = \frac{1}{10}$. In the limit $\lambda \to 0$, this theory approaches the Einstein's theory of every respect due to the nature of space-time (1) as a function of 't'.

The energy momentum tensor T_{ij}^{V}, Singh and Deo [24] for a micro quantum matter field representing massless scalar field distribution and energy momentum tensor T_{ij}^{s}, Latelier [21] for cosmic string given by,

$$T_{ij}^{v} = v_i v_j - \frac{1}{2} g_{ij} v_k v^k , \tag{4}$$

with

$$g^{ij} v_{;ij} = \sigma . \tag{5}$$

And

$$T_{ij}^{s} = \rho u_i u_j - \lambda' x_i x_j . \tag{6}$$

Together with,

$$u^i u_j = -x^i x_j = 1 \text{ and } u^i x_i = 0 , \tag{7}$$

the scalar field v and the source density σ are both functions of T only. ρ is the rest energy density of cloud of string, λ' is the tension density of the cloud string, u^i is the four velocity of the cloud particle and x^i is the direction of anisotropy along z-axis.

Using equation (4) and (6), the set of field equation (2) and (3) for the space time (1) reduces to following explicit form,

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = -4\pi\phi^{-1}\dot{v}^2 , \tag{8}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = 8\pi\phi^{-1}\left(-\frac{1}{2}\dot{v}^2 + \lambda'\right), \tag{9}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi\phi^{-1}\left(-\frac{1}{2}\dot{v}^2 + \rho\right), \tag{10}$$

$$\ddot{\phi} + \left[2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right]\dot{\phi} = \frac{8\pi\lambda}{3}\left(-\dot{v}^2 + \lambda' + \rho\right). \tag{11}$$

Here the dot (.) over the letter denotes ordinary differentiation with respective time. There are four equations in six unknowns $A, B, \phi, \rho, \lambda'$ and v. Hence to determine the solution to assume the condition that the geometric string $\rho = \lambda'$, Latelier [21] *i.e.* the rest energy density of the cloud of the string is equal to tension density of cloud of string.
From (9) and (10), we have

$$\frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{A}}{A} = 4\pi\phi^{-1}\dot{v}^2 . \tag{12}$$

Comparing equation (8) and (12), on simplifying we get

$$\frac{2\dot{A}}{A} + \frac{\ddot{B}}{\dot{B}} = 0 . \tag{13}$$

Now applying the condition $A = B^n$ where *n* is real number.
From equation (13) gives,

$$A = (2n+1)^{\frac{n}{2n+1}} (k_1 t + k_2)^{\frac{n}{2n+1}} , \tag{14}$$

$$B = (2n+1)^{\frac{1}{2n+1}} (k_1 t + k_2)^{\frac{1}{2n+1}} , \tag{15}$$

where k_1 and k_2 are constant of integration.
Now transforming the time co-ordinate '*t*' by putting $k_1 t + k_2 = T$ and $2n + 1 = m$, equation (15) and (16) can be reduced to the form

$$A = m^{\frac{n}{m}} \quad T^{\frac{n}{m}} , \tag{17}$$

$$B = m^{\frac{1}{m}} \quad T^{\frac{1}{m}} . \tag{18}$$

With the help of equation (17) and (18), equation (11) can be transformed to the form

$$\ddot{\phi} + \frac{1}{T}\ddot{\phi} - \frac{8\pi\lambda}{3}\left(-\dot{v}^2 + \lambda' + \rho\right) = 0 . \tag{19}$$

From equation (12), we get

$$\dot{v}^2 = v_T^2 = \frac{(-n^2 + nm + n)\phi}{4\pi \quad m^2 \quad T^2} . \tag{20}$$

From equation (9) and (10) we obtained,

$$\rho = \lambda' = \frac{(2n^2 - nm + n)\phi}{8\pi m^2 T^2} \tag{21}$$

Using the equation (20) and (21), equation (19) can be transformed to the form

$$T^2\ddot{\phi} + T\ddot{\phi} + p^2\phi = 0\,, \tag{22}$$

where $p^2 = -\frac{2\lambda\left(3n^2 - 2nm\right)}{3m^2}$, $\frac{\left(3n^2 - 2nm\right)}{m^2} > 0$ and $0 < \lambda < 10^{-1}$.

On integration equation (22) yields two basic solutions for ϕ as,

$$\phi_1 = \cos(p\log T)\,, \tag{23}$$

$$\phi_2 = \sin(p\log T)\,. \tag{24}$$

The value of ϕ_2 given in equation (24) is not accepted as it leads to unphysical situation.

From equation (20), we obtain

$$\dot{v} = \frac{\alpha\sqrt{\phi}}{T}, \text{ where } \alpha = \sqrt{\frac{\left(-n^2 + nm + n\right)}{4\pi\ \ m^2}}\,. \tag{25}$$

On integration equation (25) becomes

$$v = \alpha\int\frac{\sqrt{\phi}}{T}\mathrm{d}T + c_1, \qquad \text{where } c_1 \text{ is constant of integration.} \tag{26}$$

Using equation (23) and (24) we obtained the expression for scalar field v as,

$$v = \alpha\int\frac{\sqrt{\cos(p\log T)}}{T}\mathrm{d}T + c_1\,. \tag{27}$$

From equation (5) the source density of the scalar field v is

$$\sigma = \ddot{v} + \frac{1}{T}\dot{v}\,. \tag{28}$$

Using equation (26) and (28) we get,

$$\sigma = \frac{\alpha\ \ \dot{\phi}}{2T\sqrt{\phi}}\,. \tag{29}$$

Using equation (23) in the equation (29) we find,

$$\sigma = \frac{\alpha \quad p\sin(p\log T)}{2T^2\sqrt{\cos(p\log T)}}. \tag{30}$$

From equation (23), (21) yields,

$$\rho = \lambda' = \beta \frac{\cos(p\log T)}{T^2} \quad \text{where} \quad \beta = \frac{2n^2 - nm + n}{8\pi m^2}. \tag{31}$$

Thus the plane symmetric string cosmological model in modified theory of general relativity for the space-time (1) is given by

$$\mathrm{d}s^2 = \mathrm{d}T^2 - m^{\frac{2n}{m}} \quad T^{\frac{2n}{m}}(\mathrm{d}X^2 + \mathrm{d}Y^2) - m^{\frac{2}{m}} T^{\frac{2}{m}} \mathrm{d}Z^2. \tag{32}$$

The physical and kinematical parameters for the model (32) are

Spatial volume $$V = \sqrt{-g} = T. \tag{33}$$

Expansion scalar $$\theta = \frac{1}{T}. \tag{34}$$

Shear scalar $$\sigma^2 = \frac{1}{6T^2}. \tag{35}$$

3. DISCUSSION

The Barber's scalar ϕ, the scalar field v, the source of density σ, energy density ρ and tension density λ' are the functions of time and given by equation (23), (27), (30) and (31).

The energy condition $\rho = \lambda' \geq 0$ is satisfied for $\beta \geq 0$ and $T \neq 0 \text{ or } \infty$. When $T \to 0$, the metric potentials *A* and *B* tend to zero, *i.e.* the space-time collapses. As $T \to \infty$, the metric potentials *A* and *B* tends to infinite *i.e.* it admits a singularities. The quantities ϕ, v, σ, ρ and λ' are undetermined for *T* tends to zero, infinity. From equation (27), we observed that the massless scalar field v is a logarithmic function of time and its presence avoided big-bang of the universe. When $T \to 1$, the metric potential *A* and *B* becomes constant *i.e.* space-time reduces to flat, $\phi \to 1$, v is $\mathrm{cons}\tan t$, and $\rho = \lambda' = \mathrm{cons}\tan t$. Here the spatial volume is given by $V = T$ and as $T \to \pm\infty$ $V \to \pm\infty$ these results shows that the universe starts expanding with zero volume. The equation (34) shows that $\theta \to 0$ as $T \to \infty$ and $\theta \to \infty$ as $T \to 0$ thus the universe is expanding with increase of time but the rate of expansion becomes slow as time increases. The equation (35) shows that for $T \to \infty$, $\sigma \to 0$ and, $T \to 0$, $\sigma \to \infty$ it means that the shape of universe changes uniformly. When the coupling parameter $\lambda \to 0$ ($T \neq 0$ or ∞) then $\phi \to 1$, it shows that second self creation theory leads to Einstein's theory as $\lambda \to 0$.

It is observed that the value $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$, the model does not approach the isotropy for large values of *T*.

REFERENCES

1. Barber G.A., *Gen. Rel. Grav*.4, 117, 1982.
2. Barber G.A., *Astrophysics and space science* 282, 4, 683–731, 2002.
3. Brans C.S., Dicke, R. H.: *Phys.Rev.*124, 925, 1961.
4. Brans C.S., *Gen. Rel. Grav*.19, 949, 1987.
5. Pimentel L.O., *Astrophysics and space science* 116, 395, 1985.
6. Soleng H.H., *Astrophysics and space science* 139, 373, 1987.
7. Synge., *J. Math. Phys.* 27, 1074, 1986.
8. Reddy D.R.K., *Astrophysics and space science* 132, 401–403, 1987a.
9. Reddy D.R.K., *Astrophysics and space science* 133, 189–191, 1987b.
10. Reddy D.R.K., *Astrophysics and space science* 134, 201–204, 1987c.
11. Reddy D.R.K., Venkateshwarlu, R., *Astrophysics and space science* 168, 193, 1989.
12. Reddy D.R.K., Venkateshwarlu, R., *Astrophysics and space science* 179, 147, 1990.
13. Shanti K., Rao V.U.M., *Astrophysics and space science* 79, 147, 1991.
14. Ram S., Singh C.P., *Astrophysics and space science* 257, 123, 1998.
15. Mohanty G., Panigrahi U.K., Sahu R.C., *Astrophysics and space science* 281, 633–640, 2002.
16. Mohanty, Panigrahi U.K., Sahu R.C., *Astrophysics and space science* 284, 1055–1062, 2003.
17. Panigrahi U.K. and Sahu R.C., *Science letter,* vol. 25, No. 11 and 12, 2002.
18. Venkateswarlu R., Rao U.U.M., Pavan Kumar K., *Int. J. Theo. Phys.,* DoI 10.1007/s10773-007-9488-X, 2007, online.
19. Vilenkin A., *Phys. Rev.* 121, 263, 1985.
20. Letelier P.S., *Phys. Rev. D* 20, 1294, 1979.
21. Krori K.D., Chaudhari T., Mahanta C.R. and Mazumdar A.: *Gen. Relat. Gravit*. 22, 123, 1990.
22. Kibble T.W.B., *J. Phys. A* 9, 1387, 1976.
23. Vilenkin A.: *Phys. Rev. D*23, 852, 1981.
24. Singh R.K.T., Deo, Shridhar., *Acta Physica Hungarica,* 59, 321, 1986.
25. Bayaskar S.N., Pawar D.D., Deshmukh A.G., Rom. J. Phys. Vol. 54, Nos. 7-8, 763–740, 2009.