Here we study some possible consequences of recently observed Tully-Fisher relation for gas rich galaxies, as well as perturbed stellar motions around the black hole in Galactic center on dark matter paradigm. We investigate the ability of some alternative approaches which require modifications of fundamental gravitational and/or dynamical laws to explain these observational results. We found that predictions of modified Newtonian dynamics for orbital precession of S2 star strongly depend on the choice of interpolating function, which in turn could also affect its predictions regarding Tully-Fisher relation. On the other hand, \( R^n \) gravity causes retrograde precession of pericenter of S2 star (which is opposite to the predictions of General Relativity), and hence could reproduce the effects of extended dark matter distributed inside its orbit.

Key words: Dark matter, Galactic Center, Gravitation: Astrophysics, Gravity: Experimental test of gravitational theories.

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1. INTRODUCTION

The standard \( \Lambda \)CDM cosmological model predicts that dark matter constitutes about 5/6 of the total mass of the Universe, while baryonic matter contributes to it with only 1/6 [1]. Dark matter is most likely composed from non-baryonic particles with very small primordial velocity dispersion which are so weakly interacting that they move purely under the influence of gravity (see e.g. [2]). Flat rotation curves of spiral galaxies are usually assumed as the most significant observational evidences for dark matter hypothesis [3]. Namely, rotation curves of most of spiral galaxies cannot be explained by their detected baryonic mass and Newtonian gravitational force law, and therefore the dark matter hypothesis tries to explain this problem by assuming the existence of a spherical dark matter halo. According to this hypothesis, during a merger of two galactic clusters weakly interacting dark matter should be mostly concentrated in outer regions and there should be offset between the dark and visible matter distribution peaks. It was indeed inferred from a weak-lensing mass reconstruction (for more details about different types of gravitational lensing and its cosmological applications see e.g. [4] and references therein) in the case of famous "Bullet Cluster" - 1E 0657-558, which was considered as the first empirical evidence for the existence of dark matter [5].
However, the results of some very recent observational studies of gas rich galaxies contradict to the dark matter hypothesis. Namely, an empirical relation between the observed mass (or luminosity) of a spiral galaxy and its rotation velocity (in its flat part), so called Tully-Fisher relation [3], is one of the most convincing probes for dark matter content of a such galaxy. Recently, baryonic Tully-Fisher relation has been tested for gas rich galaxies [6]. This is the most rigorous observational test that has been taken until now because the mass of gas rich galaxies can be measured much more precisely than the mass of most other galaxies where stars dominate the baryonic mass budget. The obtained results are in the great discrepancy with predictions of $\Lambda$CDM cosmological model (see Fig. 2 in [6]). At the same time, the observations behave precisely as predicted by the modified Newtonian dynamics (MOND) [7], which excludes the existence of dark matter in these galaxies. Until now, this is one of the most significant observational challenges to the standard cosmological model in which dark matter is one of the corner stones.

Basically, there are two possible approaches to explain these observations: (i) the main stream approach which requires revision of the nature of dark matter and its interaction with baryonic matter (see e.g. [8]) in order to explain the observed Tully-Fisher relation and to stay, at the same time, in accordance with the standard $\Lambda$CDM cosmological model and Cosmic Microwave Background (CMB) observations, and (ii) the alternative approach which requires modification of the fundamental gravitational and/or dynamical laws on the galactic scales, in order to exclude the need for dark matter.

The aim of this paper is to investigate whether this alternative approach is also able to explain recently observed perturbed stellar motions around the black hole in the center of our Galaxy, as well as to analyze some possible consequences of the obtained results on dark matter paradigm and our knowledge about fundamental gravitational and/or dynamical laws.

2. RESULTS AND DISCUSSION

Different alternative theories of gravity, such as: MOND, scalar-tensor, conformal, Yukawa-like and $R^n$ modified gravity have been proposed in order to solve the dark matter problem (see e.g. [9] and references therein). However, in addition, any such theory must also satisfy very rigorous constraints from Solar system data (such as e.g. precession of the perihelion of Mercury), as well as those from recently observed perturbed stellar motions in Sgr A* cluster (so called S-stars) around massive black hole (BH) in the center of our Galaxy [10–12].

The S-stars represent a valuable tool for testing the General Relativity (GR) in regimes far stronger than those tested so far (see e.g. [13]), as well as for constraining different types of modified gravity theories. The motion of one of them, called S2,
was observed during its complete orbital period of about 15 years and deviations from its Keplerian orbit were detected. GR predicts that pericenter of S2 star should advance by $0.18^\circ$ per orbital revolution [11]. Besides, an extended dark mass which probably exists in the Galactic center [14–16] (more precisely its part inside the orbit of S2 star) could cause Newtonian retrograde precession (NRP) of pericenter of S2 star by $\approx 0.08^\circ$ [17, 18].

Therefore, here we will study whether the alternative theories which are very successful in explaining the flat rotational curves of spiral galaxies could also satisfactorily explain these deviations from the Keplerian orbit of S2 star. We assumed the following parameters of S2 star [14]: orbital eccentricity $e = 0.87$, major semi-axis $r = 919$ AU and mass of central black hole $M_{BH} = 3.4 \times 10^6 M_\odot$, where $M_\odot$ is solar mass.

### 2.1. MOND AND S2 STAR

MOND is probably the most famous alternative to dark matter hypothesis which successfully explains the flat rotational curves of spiral galaxies. It assumes that the Newton’s second law of motion is modified in the following way [7]:

$$\vec{F} = m \cdot \vec{a} \cdot \mu \left( \frac{a}{a_0} \right),$$

(1)

where $a_0 \approx 1.2 \times 10^{-10}$ m s$^{-2}$ is a new physical constant with units of acceleration and $\mu(x)$ is an interpolating function which must have the asymptotic form: $\mu(x) = x$ when $x \ll 1$ (MONDian regime) and $\mu(x) = 1$ when $x \gg 1$ (Newtonian regime). This leads to the following relation between the asymptotically flat rotation velocity $v_f$ and the total mass $M$ [6]:

$$a_0 GM = v_f^4.$$

A quite general class of interpolating functions can be expanded in powers of $1/x$ for $x \to \infty$ to give the approximate expression [7, 19]: $\mu(x) \approx 1 - A x^{-n}$, leading to the following rough estimate for the pericenter precession per revolution [7]:

$$\delta \phi \approx 2 \pi A \frac{4n + 1}{2n - 1} \left( \frac{a_0 r^2}{G M_{BH}} \right)^n.$$

(2)

The pericenter precession in the case of standard interpolating function $\mu(x) = \frac{x}{\sqrt{1+x^2}}$ can be obtained by assuming [19]: $A = 1/2$, $n = 2$, while the case $A = 1$, $n = 1$ corresponds to the following interpolating function: $\mu(x) = \frac{x^2}{1+x^2}$, which triggers a slower transition from the MONDian to the Newtonian regime [19]. Substituting the values for major semi-axis of S2 star ($r = 919$ AU) and mass of central black hole ($M_{BH} = 3.4 \times 10^6 M_\odot$) in above expression results with the following values for the pericenter precession of S2 star: $\delta \phi \approx 5'' \times 10^{-11}$ in the first case, and $\delta \phi \approx 0''.03$ in the second case. Comparison with the previously mentioned predictions in the case of GR and NRP shows that MOND gives much smaller values. Besides, although
the both interpolating functions provide reasonable fits to rotation curves of a wide range of galaxies [19], at the same time, they give drastically different values for pericenter precession of S2 star. Hence, the precession of S2 star orbit, as predicted by MOND, strongly depends on the selection of interpolating function. Therefore, the future measurements of this precession could be used to restrict the choice of MOND interpolating functions, which on the other hand, may have significant influence on its predictions related to Tully-Fisher relation and dark matter distribution.

2.2. $R^n$ GRAVITY AND S2 STAR

Besides MOND, $R^n$ gravity has been also recently considered as a good candidate to solve the dark matter problem on galactic scales, since it can satisfactorily fit the flat galactic rotational curves [20]. It belongs to power-law fourth-order theories of gravity obtained by replacing the scalar curvature $R$ with $f(R) = f_0 R^n$ in the gravity Lagrangian, which leads to the gravitational potential in the following form [20, 21]:

$$\Phi(r) = -\frac{Gm}{2r} \left[ 1 + \left( \frac{r}{r_c} \right)^{\beta} \right],$$

where $\beta$ is a universal parameter and $r_c$ is an arbitrary parameter, depending on the typical scale of the considered system. A good agreement between the observed and theoretical rotation curves of spiral galaxies is found when the slope $n$ of the gravity Lagrangian is set to the value $n = 3.5$ (giving $\beta = 0.817$), obtained by fitting the Type Ia supernova Hubble diagram with the assumed power-law $f(R)$ model and no dark matter [20]. However, Zakharov et al. [9] used the Solar system data to put strong constraints on the parameters of $R^n$ gravity and found that these parameters must be very close to those corresponding to the Newtonian limit of the theory. Also, the same authors [14] discussed the possibility that the S-stars move inside the dark matter concentration, which probably exists in Galactic center. They found that apoastron shift of S2 star could be used for improving limits on the amount and distribution of dark matter [14].

In order to test whether $R^n$ gravity is able to provide reasonable explanation for recently observed orbital precession of S2 star, we performed two-body calculations of its orbit in gravitational potential (3). Two comparisons between the simulated orbits in Newtonian and $R^n$ gravity are presented in Fig. 1, from which one can see that $R^n$ gravity causes pericenter shifts, resulting in rosette shaped orbits of S2 star [22].

The exact expression for precession angle during one revolution in the case of $R^n$ gravitational potential is given in terms of hypergeometric function by [22], who found that this type of modified gravity has a similar effect as extended mass distribution in Newtonian gravity, since it also produces a retrograde precession of
Fig. 1 – The simulated orbits of S2 star in Newtonian (red dashed line) and $R^n$ gravity (blue solid line) for $r_c = 102$ AU and $\beta = 0.0415$ during 10 (left) and 73 revolutions (right).

S2 star orbit for any $\beta$ and $r_c$ such that $0 < \beta < 1$ and $r_c > 0$. This effect can be also seen from left panel of Fig. 1, where the direction of precession is opposite in respect to the motion of S2 star along its orbit. According to these results any potentially observed retrograde precession of pericenter of S2 star due to extended dark matter inside its orbit could be also explained by $R^n$ gravity, while possible pericenter advance could not be reproduced by this type of modified gravity.

3. CONCLUSIONS

The presented results of our investigation can be summarized as follows:

1. some alternative theories, like MOND and $R^n$ gravity, which successfully explain flat rotation curves of the spiral galaxies without need for dark matter could be rigorously constrained by the observed deviations from the Keplerian orbits of the S-stars;
2. in the case of MOND, the predicted precession of pericenter of S2 star is much smaller than the corresponding values predicted by GR and NRP, and it strongly depends on the choice of interpolating function $\mu(x)$;
3. $R^n$ gravity results with retrograde precession, which is in opposite direction in respect to GR, and hence it could only explain the potentially observed precession due to extended dark matter distributed inside the S2 star orbit;
4. the observed constraints from S-stars could significantly affect the predictions of MOND and $R^n$ gravity regarding the Tully-Fisher relation and absence of dark matter in the spiral galaxies.
However, one should take into account that the astrometric limit for bright stellar sources near the Galactic Center with current telescopes is on the order of 10 mas, and thus it is not still possible to measure the exact deviation of the S2 star orbit from its Newtonian case. In the future the astrometric errors will be several times smaller which will significantly improve the S2 star constrains on MOND and $R^n$ gravity theories.

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