CORE POLARIZATION EFFECTS OF SOME ODD SD-SHELL NUCLEI
USING M3Y EFFECTIVE NUCLEON-NUCLEON INTERACTION

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Inelastic electron scattering form factors in some odd-A sd-shell nuclei (17O, 27Al and 39K) have been calculated taking into account higher energy configurations outside sd-shell model space, which are called core-polarization (CP) effects. The two-body wildenthal interaction is used for the sd-shell model space. The two-body Michigan three Yukawas (M3Y) interactions are used for the core-polarization CP matrix elements. This interaction was given in LS-coupling. A transformation between LS and jj must performed to get the relation between the two-body shell model matrix elements and the relative and center of mass coordinates, using the harmonic oscillator radial wave functions with Talmi-Moshinsky transformation. The sd-shell model calculations fail to describe the data in both the transition strength and the form factor; the inclusion of CP effects modifies the form factors markedly and describes the experimental data very well in both the absolute strength and the momentum transfer dependence.

Key words: Inelastic electron scattering, form factor, Shell model.
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1. INTRODUCTION

The central problem in nuclear physics has long been the nucleon-nucleon interaction. As it is well known, this interaction is usually described by a potential based on a meson exchange. There are a number of realistic models for potentials. They all describe the nucleon-nucleon scattering data very well [1]. Several theoretical models have been introduced for this purpose, shell-model is one of them and it leads to a very successful description. The sd-shell is considered in the present work, this model deals with only the distribution and coupling of the valence nucleons within a few model-space orbits. According to this model the 16O is considered as an inert core. The active orbits of the model-space are labeled by the quantum numbers: nlf=1d5/2, 2s1/2, 1d3/2. The nucleons of the core (16O) are 8 protons and 8 neutrons which are inert in the (1s1/2,1p3/2,1p1/2)J=0,T=0 configuration and the remaining A-16 nucleons are distributed over all possible combinations of the 1d5/2,2s1/2 and 1d3/2 orbits according to Pauli Exclusion Principle. To investigate

the effects of the higher configurations outside the sd-shell model space on the 
closed core, the core-polarization effects are introduced. Core-polarization effects 
are taken into account through first order perturbation theory, which allows 
particle-hole excitation from the sd-shell core orbits and also from the valence 
sd-shells to the higher allowed orbits with 6\(\omega\) excitations. Several shell-model 
calculations for sd-shell nuclei are available. The work of Wildenthal [2] is adopted 
here. Coulomb form factors of C4 transitions in even-even nuclei are discussed [3] 
taking into account core-polarization effects. Higher configurations are taken into 
account through a microscopic theory, which allow particle-hole excitations from 
the 1s and 1p shells core orbits and also from the 2s1d-shell orbits to the higher 
allowed orbits with excitations up to 4\(\omega\). The core-polarization is found essential 
in both the transition strengths and momentum transfer dependence of form factors, 
and gives a remarkably good agreement with the measured data with no adjustable 
parameters. The calculations are based on the wildenthal interaction for the sd-shell 
model space and the Modified Surface Delta Interaction (MSDI) for the 
core-polarization effects. A microscopic model has been recently used [4] in order 
to study the first order CP effects on C2 form factor of p-shell nuclei. Those 
calculations depend on the realistic two-body effective interaction (M3Y) as a 
residual interaction to generate the core-polarization matrix elements that be added 
to the model-space matrix elements. The results are quite successful and describe 
the data very well in both the transition strength and momentum transfer 
dependence. The longitudinal form factors for electron scattering have been 
calculated for p-shell nuclei [5] using enlarged model space includes all orbits in 
1p and 2s-1d shells.

The aim of the present work is to consider the particle-hole excitations of the 
core and model space to calculate the longitudinal form factors for electron 
scattering from some sd-shell odd-odd nuclei. A more realistic nucleon-nucleon 
interaction is adopted in the present work for core-polarization calculation which is 
called the Michigan three Yukawas (M3Y) realistic two-body interactions [6] 
where its parameters are adjust from the nucleon-nucleon scattering data. So, we do 
not adjust any parameters in the calculations of the various matrix elements.

2. THEORY

The reduced matrix elements of the electron scattering operator \(\hat{T}_\Lambda^\mu\) consist of 
two parts, one is for the “Model space” matrix elements, and the other is for the 
“Core-polarization” matrix elements

\[
\left\langle \Gamma_j \left| \hat{T}_\Lambda^\mu \right| \Gamma_i \right\rangle = \left\langle \Gamma_j \left| \hat{T}_\Lambda^\mu \right| \Gamma_i \right\rangle_{MS} + \left\langle \Gamma_j \left| \delta \hat{T}_\Lambda^\mu \right| \Gamma_i \right\rangle_{CP} \quad (1)
\]
Where the state $|\Gamma_i\rangle$ and $|\Gamma_f\rangle$ are described by the model-space wave functions. Greek symbols are used to denote quantum numbers in coordinate space and isospin, i.e. $\Gamma_i \equiv J_i T_i$, $\Gamma_f \equiv J_f T_f$ and $\Lambda \equiv JT$.

The Model Space (MS) matrix elements are expressed as the sum of the product of the one-body density matrix elements (OBDM) times the single-particle matrix elements, which is given by:

$$\langle \Gamma_f \parallel \hat{T}_\Lambda^{\eta \eta} \parallel \Gamma_i \rangle_{MS} = \sum_{\alpha, \beta} OBDM(\alpha, \beta) \langle \alpha \parallel \hat{T}_\Lambda^{\eta \eta} \parallel \beta \rangle_{MS}$$  \hspace{1cm} (2)

where $\alpha$ and $\beta$ denote the final and initial single particle states respectively (isospin is included) for the model space.

Similarly, the core-polarization matrix element in equation (1) can be written as follows:

$$\langle \Gamma_f \parallel \delta \hat{T}_\Lambda^{\eta \eta} \parallel \Gamma_i \rangle_{cp} = \sum_{\alpha, \beta} OBDM(\alpha, \beta) \langle \alpha \parallel \delta \hat{T}_\Lambda^{\eta \eta} \parallel \beta \rangle_{cp}$$  \hspace{1cm} (3)

According to the first order perturbation theory, the single-particle matrix element for the higher-energy configurations is given by [7]:

$$\langle \alpha \parallel \delta \hat{T}_J^{\eta \eta} \parallel \beta \rangle = \langle \alpha \parallel V_{res} \frac{Q}{E_i - H^{(0)}_J} \hat{T}_J^{\eta \eta} \parallel \beta \rangle + \langle \alpha \parallel \hat{T}_J^{\eta \eta} \parallel V_{res} \parallel \beta \rangle$$  \hspace{1cm} (4)

The operator $Q$ is the projection operator onto the space outside the model space. For the residual interaction, $V_{res}$, we adopt the M3Y [6]. $E_i$ and $E_f$ are the energies of the initial and final states, respectively. $H^{(0)}$ is the unperturbed Hamiltonian. Equation (4) is written as [7]

$$\langle \alpha \parallel \delta \hat{T}_J^{\eta \eta} \parallel \beta \rangle = \sum_{\alpha_1, \alpha_2} \sum_{\beta_1, \beta_2} (-1)^{b+\alpha_2 + \Gamma} \frac{(2\Gamma + 1)!}{\alpha_2 \alpha_1 \Gamma} \times \langle \alpha_1 \parallel V_{res} \parallel \beta_1 \parallel \alpha_2 \parallel \hat{T}_J^{\eta \eta} \parallel \alpha_2 \parallel V_{res} \parallel \beta_2 \rangle \times (1+\delta_{\alpha_1,\alpha_2})(1+\delta_{\alpha_2,\beta_1})(1+\delta_{\alpha_2,\beta_2})$$  \hspace{1cm} (5)

where the index $\alpha_1$ runs over particle states and $\alpha_2$ over hole states and $e$ is the single–particle energy. The core-polarization parts are allowing particle-hole excitations from the 1s-, 1p- and 2s1d-shell orbits into higher orbits. These excitations are taken up to $4\hbar\omega$.

The reduced single particle matrix element becomes:
\[
\langle \alpha_2 \mid \hat{T}^n_{jT} \mid \beta_1 \rangle = \sqrt{\frac{2T+1}{2}} \sum_{t_z} I_T(t_z) \langle \alpha_2 \mid \hat{T}^n_{jT} \mid \alpha_1 \rangle
\]  

(6)

where:

\[
I_T(t_z) = \begin{cases} 
1 & \text{for } T=0 \\
\left( -1 \right)^{t_z} & \text{for } T=1 
\end{cases}
\]

(7)

where \( t_z = \frac{1}{2} \) for proton and \(-1/2\) for a neutron.

Electron scattering form factor involving angular momentum \( J \) and momentum transfer \( q \), between the initial and final nuclear shell model states of spin \( J_{i,f} \) and isospin \( T_{i,f} \) are [8]

\[
\left| F^n_f(q) \right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \sum_{T=0,1} (-1)^{J_f-T_f} \left\{ \begin{array}{ccc} T_f & T & T_i \\ -T_f & M_T & T_{zl} \end{array} \right\} \left| \langle J_f \mid T^n_{jT} \mid J_i \rangle \right|^2 \left| \Gamma_f \right|^2 \left| \Gamma_i \right|^2
\times \left| F_{cm}(q) \right|^2 \times \left| F_{fs}(q) \right|^2
\]

(8)

where \( T_z \) is the projection along the \( z \)-axis of the initial and final isospin states and is given by \( t_z = (z-n)/2 \). The nucleon finite size (fs) form factor is \( F_{fs}(q) = \exp\left(-0.43q^2/4\right) \) and \( F_{cm}(q) = \exp\left(q^2b^2/4a\right) \) is the correction for the lack of translation invariance in the shell model. \( A \) is the mass number and \( b \) is the harmonic oscillator size parameter.

The single-particle energies are calculated according to [7]:

\[
e_{nlj} = (2n + l - \frac{1}{2})\hbar\omega + \begin{cases} 
-\frac{1}{2}(l+1)\langle f(r) \rangle_{nl} & \text{for } j = l - \frac{1}{2} \\
\frac{1}{2}l\langle f(r) \rangle_{nl} & \text{for } j = l + \frac{1}{2} 
\end{cases}
\]

(9)

with:

\[
\langle f(r) \rangle_{nl} \approx 20A^{-2/3}\text{MeV}
\]

\[
\hbar\omega = 45A^{-1/3} - 25A^{-2/3}
\]

(10)
For the two-body matrix elements of the residual interaction \( \langle \alpha \alpha_s | V_{\text{res}} | \beta \alpha_t \rangle \), which appear in equations (5), the Michigan three Yukawas (M3Y) interaction of Bertch et al. [6] is adopted. The interaction is taken between a nucleon in any core-orbits and nucleon that is excited to higher orbits with the same parity and with the required multipolarity \( \Lambda \), and also between a nucleon in any sd orbits and that is excited to higher orbits with the same parity and with the required multipolarity. The form of the potential is defined in equations (1)-(3) in ref. [6]. The parameters of “Elliot” are used which are given in Table 1 of the mentioned reference. This interaction was given in LS-coupling. A transformation between LS and jj must be performed to get the relation between the two-body shell model matrix elements and the relative and center of mass coordinates, using the harmonic oscillator radial wave functions with Talmi-Moshinsky transformation.

3. RESULT AND DISCUSSION

In the present work, the core-polarization effects are included through microscopic theory to discuss electron scattering for the light doubly odd \( N=Z \) sd-shell nuclei \( ^{17}\text{O}, ^{27}\text{Al} \) and \( ^{39}\text{K} \). Core-polarization effects are taken into account through the first-order perturbation theory, which allows particle-hole and two particles-two holes excitation respectively, from the 1s and 1p shell core orbits to the higher allowed orbits with \( 2\hbar \omega \) excitation. The one-body density matrix elements (OBDM) values for even sd-shell nuclei under consideration in the present work are taken from ref. [9]. The core-polarization single-particle matrix elements are calculated according to equation (5). The many-particle matrix element that includes both the model space and the core-polarization effects are calculated according to equation (1). Finally, the nuclear form factor can be obtained from equations (8). The single-particle wave functions are those of the HO potential with size parameters \( b \), chosen to reproduce the root mean square charge radius.

Oxygen is one of the important elements in our life. The shell model calculation for \( ^{17}\text{O} \) assumed \( ^{16}\text{O} \) as an inert closed core and the remainder nucleon can occupy \( 2s_{1/2} \) or \( 1d_{5/2} \) or \( 1d_{3/2} \) of the model space. In this transition the nucleus is excited from the ground state \( (J^T_s=5/2^+1/2) \) to the excited state \( (J^T_s=1/2^+1/2) \) by the electron with an excitation energy of 0.87 MeV. The single-particle wave function for this transition is that of HO potential with size parameter \( b_{r,\text{rms}} =1.763 \text{ fm} \) [10]. The result of sd-shell model calculations with
inclusion of CP effect predicts the B(C2) value to be 1.927 e⁻² fm⁴ in comparison with the measured value 2.18±0.16 e⁻² fm⁴ [10], which is more close to the measured value.

The longitudinal C2 electron scattering form factor with core-polarization effect for this transition is shown in Figure 1 (as a solid curve) and compare with the experimental data of ref. [11]. The results estimates the experimental data in the first and the second maximum region; these results with core-polarization effect are very good especially at the second maximum region with the large bar errors and it is generally acceptable. This indicates that the M3Y suitable to describe the data at the high region of momentum transfer upon disagreement of MSDI which is fails to describe the experimental data at the high momentum transfer [3]. The one-body density matrix elements (OBDM) values for this transition are tabulated in Table 1.

![Fig. 1 – The inelastic longitudinal (C2) form factors for the 1/2⁺ 1/2 state at 0.78 MeV in ¹⁷O, the solid curve is the single-particle model space calculations with core-polarization effects. The experimental data are taken from Ref. [13].](image-url)
Fig. 2 – The inelastic longitudinal (C2) form factors for the 1/2+ 1/2 state at 2.53 MeV in \(^{39}\text{K}\), the solid and dashed curves are the single-particle model space calculations with and without core-polarization effects, respectively. The experimental data are taken from Ref. [14].

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>(j_a)</th>
<th>(j_b)</th>
<th>OBDM ((\Delta T=0))</th>
<th>OBDM ((\Delta T=1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{17}\text{O})</td>
<td>5/2</td>
<td>1/2</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>(^{39}\text{K})</td>
<td>3/2</td>
<td>1/2</td>
<td>1.0</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

The one-body density matrix elements for 0.87 MeV (5/2\(^+\)\(\rightarrow\)1/2\(^+\)) and 2.53 MeV (3/2\(^+\)\(\rightarrow\)1/2\(^+\)) transition in \(^{17}\text{O}\) and \(^{39}\text{K}\) respectively.

The results for the nucleus of \(^{39}\text{K}\) are presented for the transition C2 (3/2\(^+\)\(\rightarrow\)1/2\(^+\)) 2.53 MeV. The size parameter for the single-particle wave function is taken to be: \(b_{r_{max}}=1.950\) fm [10]. In this transition, the nucleus is excited from the ground state (\(J_f^T=3/2^+\) 1/2) to the state (\(J_f^T=1/2^+\) 1/2) with an
excitation energy of 2.53 MeV due to electron scattering. The values of the OBDM elements for this transition are given in Table 1. Figure 2 shows the calculations of the transverse C2 electron scattering form factor with and without CP effect as a solid curve and dashed curve, respectively. The sd-shell model fails to describe the data in both the transition strength B(C2) and the form factor. The calculated B(C2↑) value without including core-polarization effects is 11.5 e² fm⁴ which is low in comparison with the measured value 18.90±1.8 e² fm⁴ [10]. When the CP effect is included, the B(C2↑) value becomes 20.5 e² fm⁴ which is closer to the measured value than that calculated with (0+2+4) hω extended model space [12], where their calculated value is underestimated by a factor of 2. The inclusion of core-polarization effect gives an excellent agreement in the first maximum and the second maximum region up to 1.8 fm⁻¹, than overestimates the data beyond that.

Fig. 3 – The inelastic longitudinal (C2) form factors for the 1/2⁺ 1/2 state at 0.844 MeV in ²⁷Al, the solid and dashed curves are the single-particle model space calculations with and without core-polarization effects, respectively. The experimental data are taken from Ref. [15].
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Fig. 4 – The inelastic longitudinal (C2+C4) form factors for the 7/2+ 1/2 state at 2.211 MeV in 27Al, the solid and dashed curves are the single-particle model space calculations with and without core-polarization effects, respectively. The experimental data are taken from Ref. [15]

Table 2

The values of the OBDM elements for the longitudinal C2 transition to the 1/2+ 1/2 at $E_x = 0.844$ MeV for 27Al

<table>
<thead>
<tr>
<th>$j_a$</th>
<th>$j_b$</th>
<th>OBDM ($\Delta T=0$)</th>
<th>OBDM ($\Delta T=1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/2</td>
<td>5/2</td>
<td>0.4197</td>
<td>-0.0584</td>
</tr>
<tr>
<td>5/2</td>
<td>1/2</td>
<td>-0.0237</td>
<td>0.4181</td>
</tr>
<tr>
<td>5/2</td>
<td>3/2</td>
<td>0.3346</td>
<td>-0.0337</td>
</tr>
<tr>
<td>1/2</td>
<td>5/2</td>
<td>0.6674</td>
<td>-0.0174</td>
</tr>
<tr>
<td>1/2</td>
<td>3/2</td>
<td>0.0158</td>
<td>0.0189</td>
</tr>
<tr>
<td>3/2</td>
<td>5/2</td>
<td>-0.3702</td>
<td>-0.0053</td>
</tr>
<tr>
<td>3/2</td>
<td>1/2</td>
<td>0.0016</td>
<td>-0.0034</td>
</tr>
<tr>
<td>3/2</td>
<td>3/2</td>
<td>0.1503</td>
<td>0.0217</td>
</tr>
</tbody>
</table>
The nucleus of the $^{27}$Al has been studied both theoretically and experimentally. It is considered as $^{16}$O and eleven nucleons are occupying the 2s1d-shell space according to Pauli Exclusion Principle. Calculation are presented for the transitions from the ground state ($\frac{5}{2}^+ 1/2$) to the $1/2^+ 1/2$ state and $7/2^+ 1/2$ at 0.844 MeV and 2.211 MeV, respectively. The size parameter for the single-particle radial wave function is taken to be $b_{r.m.s}=1.804$ fm [10]. The experimental reduced transition strength $B(C2\uparrow)$ for $1/2^+ 1/2$ state at 0.844 MeV is equal to $12.79\pm0.5$ e$^2$ fm$^4$ [14] and the calculated one is 17 e$^2$ fm$^4$, which is higher than the experimental value. The OBDM values for this transition are listed in table (2,3,4) and are taken from ref. [3]. The sd-shell model calculation of C2 form factor including CP effects in $1/2^+ 1/2$ state at 0.844 MeV gives a good result in the first maximum up to $q=1.8$ fm$^{-1}$ and the result is gradually shifted from the experimental data in which the curve underestimates this data, the deviation can be interpreted within the large bar errors as show in Figure 3. The experimental form factor for 2.211 MeV ($7/2^+ 1/2$) in $^{27}$Al doublet of the C2 and C4. The theoretical results of (C2+C4) form factor for this state, which shown in Figure 4 give a good results in the first maximum region up to $q=1.6$ fm$^{-1}$, but at the second maximum region it overestimates the data after 1.8 fm$^{-1}$.
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REFERENCE