DIAMAGNETISM OF IDEAL ELECTRON GAS: SELF-CONSISTENT APPROACH

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The diamagnetic response of the ideal electron gas in a constant magnetic field is considered in a self consistent fashion: the magnetic field which act on the electron system is the resultant of the external constant magnetic field and the magnetic field created by the electron current itself. For this purpose we consider a finite system submitted to an inhomogeneous magnetic field $\mathbf{B}$ and we express the equilibrium induced current as function of $\mathbf{B}$. The self consistent equation for $\mathbf{B}$ is Ampere's law $\mathbf{j}(\mathbf{B}) = \nabla \wedge \mathbf{B}$ with boundary condition that $\mathbf{B}$ coincide at infinity with the applied external magnetic field. We prove, in the thermodynamic limit for semi-infinite system, that the self consistent equation has an unique solution.

Key words: Landau diamagnetism, surface currents, self-consistent methods.

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1. INTRODUCTION

The problem of metal diamagnetism as a quantum effect was theoretical solved by Landau [1] who obtained the free energy density for a gas of free electrons in homogeneous magnetic field. His approach, where the surface states where completely neglected, has been subject to repeated reassessment. The thermodynamic of electron gas in homogeneous magnetic field, as a topic in scientific literature, contains many contradictory results (for a short review see [2]). Some correct results where obtained for the bulk term of free energy [3], and magnetization [4], for the surface correction of susceptibility at zero magnetic field [2], for the free energy at non-zero magnetic field [5] and for the current distribution [6].

In this paper we reconsider the problem of the magnetic field homogeneity in a metal (free electron approximation), which is subject of an external, constant magnetic field. In section 2 we consider the free electron gas in a box with repulsive walls (Dirichlet conditions) in an inhomogeneous magnetic field and we obtain the functional representation of the equilibrium currents. The main technical result is the correlation between the local modification of the magnetic field in a region K and the induced currents in the neighborhood of K. In section 3 we write the self-consistency equation for the internal magnetic field and we prove the existence and the unicity of the solution.

2. CURRENT DISTRIBUTION FOR INHOMOGENEOUS MAGNETIC FIELD

Let $\Lambda = \left\{\mathbf{x} = (x_1, x_2, x_3) \in R^3; 0 \leq x_1 \leq L_1, |x_2| \leq L_2, |x_3| \leq L_3\right\}$ be a parallelepipedic box with edges along the coordinate axis defined by the unit vectors $\mathbf{e}_1$,$\mathbf{e}_2$, $\mathbf{e}_3$. We consider the inhomogeneous magnetic field $\mathbf{B} : \Lambda \to R^3$, to be a $C^1(\Lambda)$ function with $\nabla \mathbf{B} = 0$ and the vector potential $\mathbf{a}(\mathbf{x}) : \mathbf{B} = \nabla \wedge \mathbf{a}$. We define the one-particle Hamiltonian:

$$H_{\mathbf{a}}^{\Lambda} = \frac{1}{2}(-i\nabla_{\mathbf{x}} - \mathbf{a}(\mathbf{x}))^2 \tag{1}$$

imposing 0-Dirichlet condition on $\partial\Lambda$. $T_{\mathbf{a}}^{\Lambda}(t) = \exp\left(-tH_{\mathbf{a}}^{\Lambda}\right)$ is an integral kernel semigroup which admits the Feynman-Kac-Itô representation:

$$T_{\mathbf{a}}^{\Lambda}(\mathbf{x}, \mathbf{x}') = \int_{\Omega_\Lambda} \exp[-i\mathcal{F}(t, \mathbf{a}, \omega)] P_{\mathbf{x},\mathbf{x}'}(d\omega), \tag{2}$$

$$\mathcal{F}(t, \mathbf{a}, \omega) = \int_0^t \mathbf{a}(\omega(\mathbf{s})) \cdot d\omega(s) + \frac{1}{2}\int_0^t \text{div}\mathbf{a}(\omega(s))ds. \tag{3}$$

In Eq.(2) $P_{\mathbf{x},\mathbf{x}'}(d\omega)$ is the conditional Wiener measure for 3-dimensional Brownian path $\omega(s)$ starting from $\mathbf{x}$ at $s = 0$ and ending in $\mathbf{x}'$ at $s = t$, Ω_Λ is the set of Wiener trajectories which remain in Λ and the first integral in Eq.(3) is an Itô integral along the $\omega(\cdot)$ trajectory.

Itô's lemma, see [7] p.153:

$$\int_0^t \nabla_{\mathbf{x}}\phi(\omega(s)) \cdot d\omega(s) = \phi(\omega(t)) - \phi(\omega(0)) - \frac{1}{2}\int_0^t (\nabla_{\mathbf{x}}\phi)(\omega(s)) \cdot d\omega(s) \tag{4}$$

allows us to use this representation with any other vector potential associated to the same field, $\mathbf{a} \longmapsto \mathbf{a} + \nabla\phi$:

$$T_{\mathbf{a}+\nabla\phi}^{\Lambda}(t)(\mathbf{x}, \mathbf{x}') = \exp(i\phi(\mathbf{x}) - i\phi(\mathbf{x}'))T_{\mathbf{a}}^{\Lambda}(t)(\mathbf{x}, \mathbf{x}'). \tag{5}$$

We consider a spinless electron gas at thermal equilibrium, in the Λ enclosure, which is described in the grand canonical ensemble by the parameters $\beta = (k_B T)^{-1}$ and activity z. We denote by $g^{\Lambda}(\beta, z, \mathbf{a}; \mathbf{x}, \mathbf{x}')$ the integral kernel of the one-particle reduced density matrix, *i.e.* of the operator

$$\rho_{\Lambda,\mathbf{a}}^{(1)} = zT_{\mathbf{a}}^{\Lambda}(\beta)[1 + zT_{\mathbf{a}}^{\Lambda}(\beta)]^{-1}.$$

Obviously, (5) also implies

$$g^{\Lambda}(\beta, z, \mathbf{a} + \nabla\phi; \mathbf{x}, \mathbf{x}') = \exp(i\phi(\mathbf{x}) - i\phi(\mathbf{x}'))g^{\Lambda}(\beta, z, \mathbf{a}+; \mathbf{x}, \mathbf{x}'). \tag{6}$$

The current density induced by the field $\mathbf{B}(x)$ inside the sample is given by:

$$\mathbf{j}^{\Lambda}(\mathbf{B};\mathbf{x}) = e(-i\nabla_{\mathbf{x}} - \mathbf{a}(\mathbf{x}))g^{\Lambda}(\beta, z, \mathbf{a}; \mathbf{x}, \mathbf{x}')|_{\mathbf{x}=\mathbf{x}'}. \tag{7}$$

We notice, from (6) that $\mathbf{j}^{\Lambda}(\mathbf{B};\mathbf{x})$ is gauge invariant, *i.e.* the substitution $\mathbf{a} \longmapsto \mathbf{a} + \nabla\phi$ does not alter the right term of the (7). We have the main technical result of this section:

Proposition 2.1 *Let* $\delta\mathbf{B}$ *be a variation of the magnetic field with the support* supp$(\delta\mathbf{B}) = K \subset \Lambda$. *There exists constants* $M, \mu > 0$, *independent of* $\Lambda, K, \mathbf{B}$, *such that*

$$|\mathbf{j}^{\Lambda}(\mathbf{B}+\delta\mathbf{B};\mathbf{x}) - \mathbf{j}^{\Lambda}(\mathbf{B};\mathbf{x})| \leq M \parallel \delta\mathbf{B} \parallel^{(1)} \parallel \mathbf{B} \parallel^{(1)} exp(-\mu \,\mathrm{dist}(K,\mathbf{x})), \tag{8}$$

where $\|\cdot\|^{(1)}$ *is the usual norm in* $C^1(\Lambda)$.

Proof. We assign to $\mathbf{B}$ the family of vector potentials

$$\mathbf{a}_{\mathbf{x}'}(\mathbf{x}) = \int_0^1 \mathbf{B}((1-s)\mathbf{x}' + s\mathbf{x}) \wedge (\mathbf{x} - \mathbf{x}')s ds.$$

We then have

$$\mathbf{a}_{\mathbf{x}'}(\mathbf{x}) - \mathbf{a}_{\mathbf{x}''}(\mathbf{x}) = -\nabla_{\mathbf{x}}\phi_{\mathbf{B}}(\mathbf{x}, \mathbf{x}', \mathbf{x}''),$$

where $\phi_{\mathbf{B}}(\mathbf{x}, \mathbf{x}', \mathbf{x}'')$ is the magnetic flux through $(\mathbf{x}, \mathbf{x}', \mathbf{x}'')$ triangle. We denote:

$$\delta\mathbf{a}_{\mathbf{x}'}(\mathbf{x}) = \int_0^1 \delta\mathbf{B}((1-s)\mathbf{x}' + s\mathbf{x}) \wedge (\mathbf{x} - \mathbf{x}')s ds.$$

Obviously, $\delta\mathbf{a}_{\mathbf{x}'}(\mathbf{x}) = 0$ on the sphere of center $\mathbf{x}'$ and radius $dist(K, \mathbf{x}')$ (in fact on the maximal star shaped set of center $\mathbf{x}'$ included in $\Lambda \setminus K$). We factorize

$$\rho^{(1)}_{\Lambda,\mathbf{a}} = T^{\Lambda}_{\mathbf{a}}(\beta/2)\left\{zT^{\Lambda}_{\mathbf{a}}(\beta/2)[1 + zT^{\Lambda}_{\mathbf{a}}(\beta)]^{-1}\right\}, \text{ where } \mathbf{a} = \mathbf{a}_{\mathbf{x}'},$$

and we denote $\widetilde{g}^{\Lambda}(\beta, z, \mathbf{a}_{\mathbf{x}'}; \mathbf{x}, \mathbf{y})$ the integral kernel of the operator in the curly brackets. Then,

$$\mathbf{j}^{\boldsymbol{\Lambda}}(\mathbf{B};\mathbf{x}) = \int_{\Lambda} d^3\mathbf{y} \left[e(-i\nabla_{\mathbf{x}} - \mathbf{a}_{\mathbf{x}'}(\mathbf{x}))T^{\Lambda}_{\mathbf{a}}(\beta/2)(\mathbf{x}, \mathbf{y}))\right] \widetilde{g}^{\Lambda}(\beta, z, \mathbf{a}_{\mathbf{x}'}; \mathbf{y}, \mathbf{x})|_{\mathbf{x}=\mathbf{x}'}, \tag{9}$$

So, to estimate the difference $\mathbf{j}^{\mathbf{\Lambda}}(\mathbf{B}+\delta\mathbf{B};\mathbf{x})-\mathbf{j}^{\mathbf{\Lambda}}(\mathbf{B};\mathbf{x})$ we need to estimate the variations of the two kernels: $T_{\mathbf{a}}^{\Lambda}$ and $\widetilde{g}^{\Lambda}$ in Eq.(9).

$$\begin{aligned}\delta\left[(i\nabla_{\mathbf{x}}+\mathbf{a}_{\mathbf{x}'}(\mathbf{x}))T_{\mathbf{a}}^{\Lambda}(\beta/2)(\mathbf{x},\mathbf{y}))\right] &= (i\nabla_{\mathbf{x}}+\mathbf{a}_{\mathbf{x}'}(\mathbf{x}))\delta T_{\mathbf{a}}^{\Lambda}(\beta/2)(\mathbf{x},\mathbf{y}))= \\ &= (i\nabla_{\mathbf{x}}+\mathbf{a}_{\mathbf{x}'}(\mathbf{x}))\int_{\Omega_\Lambda}\exp[-i\mathcal{F}(\beta/2,\mathbf{a}_{\mathbf{x}'};\omega)]\cdot \\ &\cdot\{\exp[-i\mathcal{F}(\beta/2,\delta\mathbf{a}_{\mathbf{x}'};\omega)]-1\}\,P_{\mathbf{x},\mathbf{y}}^{\beta/2}(d\omega).\end{aligned} \tag{10}$$

The factor in the curly brackets is vanishing for trajectories which do not touch K, therefore the integral in (10) may be restrained to $\Omega_\Lambda(K)$ =the set of trajectories which reach K and consequently $\delta[T_{\mathbf{a}_{\mathbf{x}'}}^{\Lambda}(\beta/2)(\mathbf{x},\mathbf{y})]$ is bounded in modulus by

$$P_{\mathbf{x},\mathbf{y}}^{\beta/2}(\Omega_\Lambda(k))\leq C\beta^{-3/2}\exp[\frac{|x-y|^2+dist(K,x)^2}{\beta}].$$

To estimate the gradient, is convenient to change the integration variable from $\omega(s)$ to the 'Brownian bridge' $\alpha(s)=\omega(s)-(1-\frac{2s}{\beta})x-\frac{2s}{\beta}x'$. This transformation moves the x dependency of the measure into the function at the exponent, which allows the representation of derivatives as functional integral. We can now apply the estimations from [8]. On the other hand, the estimation of $\widetilde{g}^{\Lambda}(\beta,z,\mathbf{a}_{\mathbf{x}'};\mathbf{y},\mathbf{x}')$ is analogous with that from [4] as a function of $\delta T_{\mathbf{a}_{\mathbf{x}'}}^{\Lambda}$. Considering both estimations we obtain the estimation (8). In the end of this section we make a remark which is the immediate result of the previous result.

Corollary 2.2 Let $\Lambda^{(n)}$*be a sequence of parallelepipeds like those described at the beginning of the section, such that* $L_1^{(n)},L_2^{(n)},L_3^{(n)}\to\infty$, *i.e.* $\Lambda^{(n)}\to\mathbf{R}_+^3=\{\mathbf{x}\mid x_1\geq 0\}$. *We suppose that the sequence of magnetic fields* $\mathbf{B}^{(n)}:\Lambda^{(n)}\to\mathbf{R}^3$ *converges on any compact* $K\subset\mathbf{R}_+^3$, *in* $\mathcal{C}^1(K)$ *norm, towards a bounded magnetic field* $\mathbf{B}:\mathbf{R}_+^3\to\mathbf{R}^3$. *Let* $\mathbf{a}$ *be a vector potential associated to* $\mathbf{B}$. *We denote* $H_{\mathbf{a}}=\frac{1}{2}(-i\nabla_{\mathbf{x}}-\mathbf{a}(\mathbf{x}))^2$ *the differential operator in* $\mathbf{R}_+^3$*with 0-Dirichlet conditions on* $x_1=0$. *In this conditions,* $\forall\mathbf{x}\in\mathbf{R}_+^3$

$$\exists\lim_{n\to\infty}\mathbf{j}^{\Lambda^{(n)}}(\mathbf{B}^{(n)};\mathbf{x})=\mathbf{j}(\mathbf{B};\mathbf{x})=-e(i\nabla_{\mathbf{x}}+\mathbf{a}(\mathbf{x}))g(\beta,z,\mathbf{a},\mathbf{x},\mathbf{x}')|_{\mathbf{x}=\mathbf{x}'}$$

where $g(\beta,z,\mathbf{a},\mathbf{x},\mathbf{x}')$ *is the integral kernel of the reduced density operator*

$$\rho_{\mathbf{a}}^{(1)}=zT_{\mathbf{a}}(\beta)[1+T_{\mathbf{a}}]^{-1};\ T_{\mathbf{a}}(\beta)=\exp(-\beta H_{\mathbf{a}}).$$

Proof. We apply the theorem of dominated convergence in Eq.(9), the necessary majorations being those shown at the end of the previous demonstration.

3. THE SELFCONSISTENCY EQUATION

The main idea of the article is to consider that the Lorentz force that acts on the electrons in the metal is generated by a resultant magnetic field: the external magnetic field composed with the magnetic field created by the electron current itself. We suppose that the arrangement of external currents (placed at infinity) generates a constant magnetic field along the $\mathbf{e_3}$ axis.

$$\mathbf{B}_{\text{ext}} = b_{\text{ext}}\mathbf{e}_3, \;\; (b_{\text{ext}} > 0). \tag{1}$$

The resultant magnetic field $\mathbf{B} : \mathbf{R}^3 \to \mathbf{R}^3$, restricted to Λ such it appears in the one-particle Hamiltonian 1, must be then computed from the relation

$$\nabla \wedge \mathbf{B}(\mathbf{x}) = \begin{cases} \mathbf{j}(\mathbf{B},\mathbf{x}) & \text{if} \quad \mathbf{x} \in \Lambda \\ 0 & \text{if} \quad \mathbf{x} \notin \Lambda \end{cases}, \quad \lim_{x\to\infty} \mathbf{B}(\mathbf{x}) = \mathbf{B}_{\text{ext}} \tag{2}$$

where $\mathbf{j}(\mathbf{B},\mathbf{x})$ is computed as a function of $\mathbf{B}\mid_\Lambda$ according to Eq. (7).

Eq. (2) is a self-consistency equation which determines uniquely the magnetic field $\mathbf{B}$. Since its solution is not the constant function, we have to consider, unlike all previous approaches of diamagnetic electron gas, inhomogeneous magnetic fields. The proof of existence and solution uniqueness for equations (2), (7) for a finite volume Λ implies, on one hand, technical difficulties (we need to consider magnetic fields $\mathbf{B}(\mathbf{x})$ on the hole space $\mathbf{R}^3$ and with variable direction) and, on the other hand, it doesn't bring relevant information about the described phenomena. Therefore, for clarity we will limit ourselves to the case when Λ is a slab $(L_2, L_3 = \infty)$ of thickness $L_1 \to \infty$. The slab geometry has the advantage that all physical quantities are translation invariant in $(\mathbf{e_2},\mathbf{e_3})$ plane and reflexion invariant about $x_1 = L_1/2$. This significantly simplifies the equations and we have the following reformulation of the problem in the case $\Lambda = \mathbf{R}^3_+$.

Let $\mathbf{B} : \mathbf{R}^3_+ \to \mathbf{R}^3$ be a magnetic field of the form $\mathbf{B}(\mathbf{x}) = b(x_1)\mathbf{e}_3$ where $b : \mathbf{R}_+ \to \mathbf{R}$ is a derivable function for which there exists $\lim_{x\to\infty} b(x) = b_\infty$. We associate to $\mathbf{B}$ the vector potential:

$$\mathbf{a}(\mathbf{x}) = \mathbf{e}_2 a(x_1) = \mathbf{e}_2[-\int_{x_1}^{\infty}(b(x) - b_\infty)dx + b_\infty x_1] = \mathbf{e}_2[\delta a(x_1) + b_\infty] \tag{3}$$

Let H be the self-adjoint operator on $L_2(\mathbf{R}^3_+)$ given by the differential operator (1) with 0-Dirichlet condition on the $x_1 = 0$ plane. Denoting $T(\beta) = \exp(-\beta H)$ and $g(\beta, z, a; \mathbf{x}, \mathbf{x}')$ the integral kernel of $\rho^{(1)} = zT(\beta)(1 + zT(\beta))^{-1}$ we obtain the expresion for the current $\mathbf{j}(\mathbf{B};\mathbf{x}) = \mathbf{e_2} j(b;x_1)$, where

$$j(b;x_1) = (-i\partial_2 - a(x_1))g(\beta, z, a; \mathbf{x}, \mathbf{x}')|_{\mathbf{x}=\mathbf{x}'} \tag{4}$$

Translation invariance in $\mathbf{e_2}, \mathbf{e_3}$ plane allows us to write the above equation using the Fourier transform of the kernel g as a function of x_2, x_3 variables, in the form:

$$j(b;x) = \int dk_2 dk_3 (k_2 - a(x))\widetilde{g}(\beta, z, a; k_2, k_3; x, x), \qquad x \geq 0.$$

After an integration along a rectangle in $\mathbf{e_1}, \mathbf{e_3}$ plane limited by $x_1 = 0$ and $x_1 = x$, the self consistency equation (2) becomes the fixed point equation:

$$b(x) = b_{\text{ext}} - \int_0^x j(b, x')dx' \tag{5}$$

Proposition 3.1 *There is a constant* $c > 0$ *such that equation* (5) *has a unique solution in the space* $\mathcal{C}(\mathbf{R}_+, b_\infty)$ *of continuous functions with the norm*

$$\|f\|_c = \sup_{x>0} e^{cx} |f(x) - b_\infty|,$$

where b_∞ *is the solution of the equation*:

$$b_{\text{ext}} - b_\infty = m(\beta, z, b_\infty) \tag{6}$$

and $m(\beta, z, b_\infty)$ *denotes the magnetization density of the free electron gas in the homogeneous filed* b_∞ (Landau formula).

Proof. We first obtain a more convenient form of the functional $b \longmapsto j(b, x)$. For this, we notice that, using the Fourier transform, H can be written as a direct integral of the differential operators on $\mathbf{R}_+$.

$$H_{k_2,k_3} = \frac{1}{2}[-\partial_1^2 + (k_2 - a(x_1))^2 + k_3^2],$$

with 0-Dirichlet condition in 0. We first consider the case of small z and then the representation of current as Wiener integral becomes:

$$\begin{aligned} j(b;x) = \sum_{n=1}^{\infty} \int e^{-n\beta k_3^2/2} dk_3 \int (k_2 - a(x)) dk_2 \cdot \\ \cdot \int_{\omega \geq 0} P_{xx}^{n\beta}(d\omega) \exp[-\frac{1}{2}\int_0^{n\beta} (k_2 - a(\omega(t))^2) dt] \end{aligned} \tag{7}$$

The integrals over k are gaussian and, after computing them, the equation (7) becomes:

$$j(b,x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^n}{2\pi n\beta} \cdot \\ \cdot \int\limits_{\omega \geq 0} P_{xx}^{n\beta}(d\omega)[\overline{a \circ \omega_n} - a(x)] \exp[-\frac{n\beta}{2} \overline{(a \circ \omega - \overline{a \circ \omega_n})^2}_1] \tag{8}$$

where, for $f : [0, n\beta] \to \mathbf{R}$ we used the notation

$$\bar{f}_n = \frac{1}{n\beta} \int\limits_0^{n\beta} f(t)dt.$$

To extend the integral representation to an arbitrary $z > 0$, we will use the fact that $j(b,x)$ is an analytical function of z and we will introduce the 'Brownian bridge' variable in each term of (8):

$$\alpha(s) = \frac{1}{\sqrt{n\beta}}(\omega(n\beta s) - x),$$

which allows us to use an unique functional measure $D\alpha$ and to interchange the summation with the functional integral:

$$j(b,x) = \int D\alpha \cdot \Phi(z,x,\alpha)$$

where

$$\Phi(z,x,\alpha) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^n}{2\pi n\beta} \chi_{\{\alpha \geq -x/\sqrt{n\beta}\}} \cdot \overline{[a \circ (x + \sqrt{n\beta}\alpha)_1} - a(x)] \cdot \\ \cdot \exp\left[-\frac{n\beta}{2} \overline{\left(a \circ (x + \sqrt{n\beta}\alpha) - \overline{a \circ (x + \sqrt{n\beta}\alpha)_1} \right)^2}_1 \right]$$

is analytic in the neighbourhood of $[0, \infty)$ and $D\alpha$ - integrable. We notice that, in the case of constant magnetic field, $a^{(0)}(x) = b_\infty x$, Φ is dependent on x only through the characteristic functions $\chi_{\{\alpha \geq -x/\sqrt{n\beta}\}}$. Choosing b_∞ as solution of equation (6) allows us to write equation (5) in a more convenient form for estimations. For this, we consider the constant field $b^{(0)}(x) = b_\infty$ and we use the following results from [6]:

$$\exists\, c > 0, \quad \|j(b^{(0)}, x)\|_c \leq \infty\,; \tag{9}$$

$$\int_0^\infty j(b^{(0)},x)dx = m(\beta,z,b_\infty). \tag{10}$$

Particularly, we notice that this choice shows $\lim_{x\to\infty} b^{(1)}(x) = b_\infty$, where $b^{(1)}(x) = b_{ext} - \int_0^x j(b^{(0)},x')dx'$. Substracting equation (6) form equation (5) and using the notation

$$\delta\beta = b - b^0, \quad \delta j = j(b,\cdot) - j(b^{(0)},\cdot) \text{ and } \quad F(\delta b) \stackrel{\text{def}}{=} \int_0^x \delta j(x')dx',$$

we obtain equation (5) in the form:

$$\delta b = F(\delta b) + \int_x^\infty j(b^{(0)},x')dx'. \tag{11}$$

In the end of the demonstration we apply contraction mapping theorem for equation (11). Indeed, the application from the right term of the equation leaves invariant $\mathcal{C}(\mathbf{R}_+,0)$ with the norm $\|f\|_c$ because

$$\left\| \int_x^\infty j(b^{(0)},x')dx' \right\|_c \leq \frac{1}{c} \left\| j(b^{(0)},\cdot) \right\|_c,$$

and $\|F(\delta b)\|_c \leq C\|\delta b\|_c$ according to Proposition 2.1, where $C < 1$. In the end, we mention that the requirement of field differentiability is automatically satisfied by the solution of equation (11), since $j \in \mathcal{C}(\mathbf{R}_+,0)$.

4. CONCLUSIONS

In this paper we approached the problem of metals in thermodynamic equilibrium, in magnetic field, through quantum statistics methods, considering that the assumption of homogeneity for the magnetic field is reasonably inside the metallic sample but not near the surface. Unlike the other previous approaches, it has the advantage of fulfilling the thermodynamic laws. The main result of the paper, Proposition 3.1, is the proof of existence and uniqueness of self-consistency equation which shows the possibility of a non-contradictory treatment of gas electron in metals; even more, this treatment doesn't affect the state equation in the thermodynamic limit. From a technical point of view, this result is the consequence of Proposition 2.1 which states that local perturbations of magnetic field induce well localized currents in the neighborhood of the perturbation. On the other hand, the field near the metal surface is higher than the field inside the metal and hence the surface corrections of thermodynamic quantities, such as magnetization, are modified. This implies a growth of the current density near the surface, without modifying the overall current, so that the surface correction of magnetization diminishes. At low temperatures, we can expect to see a dependence between the current density and the distance from the surface.

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REFERENCES

1. L.D. Landau, *Z.Physick* **64**, 629 (1930).
2. N. Angelescu, M. Bundaru and G. Nenciu, *Commun.Math.Phys.* **42**, 9 (1975).
3. N. Angelescu, A. Corciovei, *Rev.Roum.Phys.* **20**, 661 (1975).
4. H.D. Cornean, *Commun.Math.Phys.* **212**, 1 (2000).
5. H. Kunz, *J.Statistical Phys.* **76**, 183 (1994).
6. N. Macris, Ph. Martin and J. Pulé, *Commun.Math.Phys.* **117**, 215 (1988)
7. B. Simon, *Functional Integration and Quantum Physics.* (Academic Press N.Y., 1979).
8. N. Macris, Ph. Martin and J. Pulé, *Ann.Inst.H.Poincaré, Phys.Théor.* **66**, 147 (1997).