CALCULATION THE MAGNETIC DIPOLE MOMENTS AND QUADRUPOLE MOMENTS FOR SOME EXOTIC CHROMIUM ISOTOPES USING DIFFERENT INTERACTIONS

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Abstract. Calculations of the magnetic dipole moments and the electric quadrupole moments in the shell model are presented for isotopes of ^{24}Cr neutron-rich nuclei using fp-model space and different interactions. These Cr isotopes are ^{49}Cr ($5/2^-$), ^{50}Cr (2^+), ^{51}Cr ($7/2^-$), ^{52}Cr (2^+), ^{53}Cr ($3/2^-$), and ^{54}Cr (2^+). The present calculations are made of *model space* (MS) and with core-polarization effects which are included through effective charge and effective g factors. The radial wave functions of the single-particle matrix elements are calculated by using the *harmonic oscillator* (HO) potential with size parameters b which are calculated for each isotope. The one-body density matrix is calculated through the *kbf*, *fpm*, *fpv* interactions which in turn are carried out in the fp-model space using the NuShellX@MSU code. Finally, these results are compared and discussed with the experimental values.

Key words: Quadrupole moments; magnetic moments; shell model; model space; effective charges; effective g factors.

1. INTRODUCTION

One of the primary sensors is to obtain information about the nuclear structure throughout the entire nuclear diagram *via* electromagnetic moments [1]. Several theoretical models have been developed in order to describe the static and dynamic moments for exotic and stable nuclei. By comparing the experimental data with the predictions of the nuclear models, the effective interactions can be improved [2]. The electric quadrupole and the magnetic dipole moments can help us to understand some features and aspects of nuclear structure. Deviations from the spherical distribution of electrical charges in the nucleus, represented by the electric *quadrupole moment* (Q. M), are sensitive to mix collective components. In particular, for example if the valence nuclei are of a neutron type, then the Q. M calculation gives a useful measurement of how the nucleus is polarized by the presence of added molecules [1]. Magnetic moments (μ) provide a good test of particular configuration and nuclear magnetic moments are very sensitive to any orbits occupied by valence particles (or holes), and very sensitive to mix elements of the spin matrix in wave function [2]. The investigation of the neutron-rich nuclei in the *pf*-shell is one of the interesting topics for the nuclear structure studies. Especially, the structure of the exotic nuclei in the *pf*-shell region displays some interesting phenomena such as the appearance of new magic numbers and disappearance of well-known ones [3]. Nuclear magnetic and quadrupole moments have been an indispensable clue for the investigation of nuclear shell structure and nucleon interactions in the nucleus [4]. Effective interactions are initially derived microscopically from realistic nucleon potential. Therefore, it is found that such an interaction is not necessarily successful in describing experimental data especially for cases with many valence nuclei. They find one possible way to solve this problem is to modify the reaction by being appropriate to experimental data, as the states had the *p*-shell [5], *sd*-shell [6], and also *pf*-shell [7, 8]. The present work studies the microscopic structure of the Cr nucleus through nuclear moments are important to know the nuclear compositions and measuring its deformation by taking the effect of g factor and effective charges.

2. THEORY

The electric quadrupole moment is related to the deformation of nuclei where the nuclear charge distribution in state $|JM\rangle$ is given by [9]:

$$\rho^{e}_{JM}(r) = \langle JM| \sum_{k=1}^{A} e(k)\delta\big(r - r(k)\big)|JM\rangle \quad (1)$$

For axial symmetric systems, the quadrupole moment is can be represented by:

$$Q(JM) = \int \rho^{e}_{JM}(r)(3z^2 - r^2)dr \quad (2)$$

The connection between $Y_{20}(\hat{r})$ and cartesian coordinates is given by [9].

$$r^2Y_{20}(\hat{r}) = \sqrt{\frac{5}{16\pi}}\,(2z^2 - x^2 - y^2) = \sqrt{\frac{5}{16\pi}}(3z^2 - r^2) \quad (3)$$

Combining eqs. (1), (2) and (3) gives:

$$Q(JM) = \sqrt{\frac{5}{16\pi}}\sum_{k=1}^{A} e(k)\big\langle JM\big|r^2(k)Y_{20}\big(\hat{r}(k)\big)\big|JM\big\rangle, \quad (4)$$

where $e(k)$ denotes the charge of nucleons of numbered k, where $e(k) = 0$ for a neutron and $e(k)$=e for a proton. The quadrupole moment of an axially symmetric body is conventionally defined as $\langle 3z^2 - r^2\rangle$. The expectation value in Eq. (4) still depends on the projection quantum number M. In nuclear physics, when M=J one obtains the largest quadrupole moment for spin J. By definition, it is referred as the spectroscopic or static quadrupole moment [9]:

$$Q(JJ) = \sqrt{\frac{5}{16\pi}}\sum_{k=1}^{A} e(k)\big\langle JJ\big|r^2(k)Y_{20}\big(\hat{r}(k)\big)\,\big|JJ\big\rangle \; = \sqrt{\frac{5}{16\pi}}\,\langle JJ20|JJ\rangle\frac{\big\langle J\big|\big|\hat{O}(E2)\big|\big|J\big\rangle}{\sqrt{2J+1}}$$

where the operator electric transition $\hat{O}(E2)$ is defined as [10]:

$$O(ELM) = \sum_{k=1}^{A} r^L(k) Y_{LM}(\hat{r}(k)) \quad (5)$$

The quadrupole moment is calculated between the same initial and final states. The nuclear matrix element of the electromagnetic $\langle J||\hat{O}(E2)||J\rangle$ operators between the initial (J_i) and final (J_f) nuclear states for a given multipolarity λ is expressed as the sum of the products of the *one-body density matrix* (OBDM) times the single-particle matrix elements [11]:

$$\langle J_f||\hat{O}(\lambda)||J_i\rangle = \sum_{jj'} OBDM(J_i, J_f, j, j', \lambda)\, \langle j'||\hat{O}_J(\lambda)||j\rangle \quad (6)$$

with j and j' label single-particle stands for the shell model space.

The electric matrix element can be represented only in terms of the model space matrix elements by assigning effective charges ($e^{eff}(t_z)$),

$$M(EJ) = \sum_{t_z} e^{eff}(t_z) \langle J_f||\hat{O}_2(\vec{r}, t_z)||J_i\rangle_{MS} \quad (7)$$

In Ref. [12], an expression for the effective charges that depends on the neutron excess is given:

$$\begin{aligned} e^{eff}(t_z) &= e(t_z) + e\delta e(t_z), \delta e(t_z) = \\ &= Z/A - 0.32(N-Z)/A - 2t_z[0.32 - 0.3(N-Z)/A] \end{aligned} \quad (8)$$

The electric quadrupole moment in a state $|J = 2\ M = 0>$ for $J_i = J_f$ is [9]:

$$Q(J=2) = \begin{pmatrix} J_i & J & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{16\pi}{5}} \langle J||\hat{O}(E2)||J\rangle = \begin{pmatrix} J_i & 2 & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{16\pi}{5}} M(EJ) \quad (9)$$

For $J = 1$, the magnetic dipole moment is defined in terms of the $m1$ operator as [13]

$$\mu(J=1) = \begin{pmatrix} J_i & 1 & J_i \\ -J_i & 0 & J_i \end{pmatrix} \sqrt{\frac{4\pi}{3}} \langle J||\hat{O}(m1)||J\rangle \mu_N \quad (10)$$

where $\langle J||\hat{O}(m1)||J\rangle$ the operator of the magnetic transition, and $\mu_N = \frac{e\hbar}{2m_p c} = 0.1051 e.fm.$ is the nuclear magnetrons, with m_p the proton mass.

The orbital and spin free nucleon g factors g(free) are: $g_l^p = 1$, $g_s^p = 5.585$ for proton and $g_l^n = 0$, $g_s^n = -3.826$ for neutron [9].

3. RESULTS AND DISCUSSION

3.1. MAGNETIC DIPOLE MOMENTS

The magnetic dipole moments (μ) are calculated for Cr isotopes in *fp*-model space using three interactions, *kbf* [21], and *fpm* [22], *fpv* [23] with free nucleon g factors g free [9], reducing the g free to g_1= 0.9 g free and with using the effective nucleon g factors g_2, were compared with the experimental values [15]. These Cr isotopes are composed of the core ^{40}Ca plus the valence active nucleons distributed in *fp*-shell. The calculated and experimental values of the magnetic dipole moments are presented in Table 1. The calculated magnetic dipole moments for ^{49}Cr isotope with *g* free are $\mu_{calc} = -0.5249, -0.7690$, and -0.6116 nm by using *kpf*, *fpm* and *fpv* interactions, respectively, where are overestimated with experimental values, while the negative sign indicates to the z-components are opposite. The inclusion of the effective nucleon g_1 factors with *kbf*, *fpm*, *fpv* interactions are decreasing the magnetic moment values are μ_{calc}= –0.4724, –0.6773 and –0.4741 nm, respectively, these values of magnetic moments remain to overestimate with the experimental value 0.476(3) nm [15] and with the wrong sign. The calculated magnetic moments μ_{calc} for ^{49}Cr are μ_{calc}=0.4765, 0.4725 and 0.4741 nm, by using the effective nucleon g_2 factors ($g_l^p = 1.62, g_l^n = 0.32, g_s^p = 5.0, g_s^n = -3.0$) and *kpf* interaction, ($g_l^p = 1.85, g_l^n = 0.38, g_s^p = 5.0, g_s^n = -3.0$) and *fpm* interaction, ($g_i^p = 1.66$, $g_l^n = 0.35, g_s^p = 5.0, g_s^n = -3.0$) and *fpv* interaction, respectively, these results are very close to the experimental value [15]. The calculated magnetic moments for ^{50}Cr of the excited 2^+ state (783 KeV) with three interactions (*kbf, fpm* and *fpv*) and with *g* free are μ_{Calc}= 1.1389, 0.9157 and 1.1621 nm, respectively, where the results are underestimated the experimental values [15]. Effective nucleon with g_1 factors, the results μ_{Calc} = 1.0251, 0.8241 and 1.0459 nm, respectively, predicts still are underestimated to the experimental value [15]. Using the effective nucleon g_2 factors, ($g_l^p = 1.1, g_l^n = 0.0, g_s^p = 5.0, g_s^n = -3.0$), ($g_l^p = 1.25, g_l^n = 0.1, g_s^p = free$, $g_s^n = free$), ($g_l^p = 1.09, g_l^n = 0.0, g_s^p = free, g_s^n = free$) and *kbf, fpm*, *fpv* interactions, respectively, these results of calculated magnetic moment which agree with the experimental value +1.24(6) nm [15]. The calculated of magnetic dipole moments for ^{51}Cr isotope with *g* free and with three interactions (*kbf, fpm* and *fpv*) are μ_{Calc} = –0.7709, –1.0092 and –0.8038 nm, respectively, these results are not close to the experimental value [(–)0.934(5) nm] [15]. The calculations with the effective g_1 factors decreased the magnetic moment values for *kbf* and *fpv* interactions with the experimental value, while being close when *fpm* interaction used. Using the effective nucleon g_2 factors, ($g_l^p = 1.1, g_l^n = 0.0, g_s^p = 5.0$, $g_s^n = -3.0$), ($g_l^p = 0.58, g_l^n = 0.0, g_s^p = 5.9, g_s^n = 3.99$), ($g_l^p = 0.82, g_l^n = 0.0$, $g_s^p = 5.9, g_s^n = -3.66$) and *kbf, fpm*, *fpv* interactions, respectively, are calculated the magnetic moment which is well agreement with the experimental value [15]. The calculated magnetic moments for ^{52}Cr of the excited 2^+ state (1434 keV) with three interactions (*kbf*, *fpm* and *fpv*) and with g free are μ_{calc} = 3.0112, 1.6591 and 2.3099 nm, respectively where are discrepancy between the theoretical prediction and previous experimental value +2.41(13) nm [15] for *kbf* and *fpm* interactions except for the magnetic moment of *fpv* interaction are close with the experimental value. The magnetic moments are calculated using the effective g_1 factors and *kbf, fpm*, *fpv* interactions, respectively. These results of *kbf*, *fpv* interactions are agree with the experimental value while no agreement with *fpm* interaction. Using the effective nucleon g_2 factors ($g_l^p = 0.73, g_l^n = 0.0, g_s^p = 5.0, g_s^n = -3.0$), ($g_l^p = 0.58, g_l^n = 0.0, g_s^p = 5.0, g_s^n = -3.0$), ($g_l^p = 1.08, g_l^n = 0.0, g_s^p = free, g_s^n = free$) and *kbf, fpm, fpv* interactions, respectively, the results of calculated magnetic moment which agree with the experimental value +1.24(6) nm [15]. The calculated magnetic moments for ^{53}Cr isotope adopt are different values for *g* free [9] factors and by using *kpf*, *fpm* and *fpv* interactions give μ_{calc}=–1.1434, –1.2611 and –1.3643 nm, respectively, which are greater than the experimental value –0.47454(3) nm [15].

Table 1

The calculated magnetic dipole moments (μ) of $^{49,50,51,52,53,54}Cr$ isotopes are compared with the experimental values [15]

Nucleus	$J^{\pi}T$	E_X (keV)	**Technique**	$\mu_{Calc.}$**(nm) using *kbf* interaction**	$\mu_{Calc.}$**(nm) using *fpm* interaction**	$\mu_{Calc.}$**(nm) using *fpv* interaction**	$\mu_{exp.}$ **(nm) [15]**
^{49}**Cr**	5/2⁻ 1/2	0	g(free)	–0.5249	–0.7690	–0.6116	0.476(3)
			g(eff.) = 0.9g(free)	–0.4724	–0.6773	–0.5504	
			g(eff.)	0.4765	0. 4725	0.4741	
^{50}**Cr**	2⁺ 1	783	g(free)	1.1389	0.9157	1.1621	+1.24(6)
			g(eff.) = 0.9g(free)	1.0251	0.8241	1.0459	
			g(eff.)	1.2402	1.2436	1.2494	
^{51}**Cr**	7/2⁻ 3/2	0	g(free)	–0.7709	–1.0092	–0.8038	(–)0.934(5)
			g(eff.) = 0.9g(free)	–0.6938	–0.9083	–0.7234	
			g(eff.)	–0.9325	–0.9342	–0.9349	
^{52}**Cr**	2⁺ 2	1434	g(free)	3.0112	1.6591	2.3099	+2.41(13)
			g(eff.) = 0.9g(free)	2.7101	1.4931	2.0789	
			g(eff.)	2.4157	2.4132	2.4113	
^{53}**Cr**	3/2⁻ 5/2	0	g(free)	–1.1434	–1.2611	–1.3643	– 0.47454(3)
			g(eff.) = 0.9g(free)	–1.0291	–1.1349	–1.2279	
			g(eff.)	–0.4746	–0.4742	–0.4745	
^{54}**Cr**	2⁺ 3	835	g(free)	2.2724	2.0224	1.5949	+1.68(11)
			g(eff.) = 0.9g(free)	2.0452	1.8202	1.4358	
			g(eff.)	1.6802	1.6814	1.6809	

While using three interactions (*kpf, fpm* and *fpv*) and effective nucleon g_1 factors, the results of μ_{calc} = –1.0291, –1.1349 and –1.2279 nm, respectively, still overestimate the experimental value. The magnetic moments are calculated with the effective nucleon g_2 factors, ($g_l^p = 1.21, g_l^n = 0.29, g_s^p = 5.99, g_s^n = -3.0$),

$(g_l^p = 1.38, g_l^n = 0.38, g_s^p = 5.98, g_s^n = -3.0)$, $(g_l^p = 1.45, g_l^n = 0.5, g_s^p = 5.98, g_s^n = -3.0)$ and *kbf*, *fpm*, *fpv* interactions, respectively, these results are close to the experimental value [15]. The calculated magnetic moments for ^{54}Cr of the excited 2^+ state (783 keV) with g free and *kbf, fpm, fpv* interactions are μ_{calc}= 2.2724, 2.0224 and 1.5949 nm, respectively, these results by using *kbf* and *fpm* interactions are overestimated with the experimental value 1.68(11) nm [15], while the calculated value with *fpv* interaction is close to the experimental value. Using the effective nucleon g_1 factors and the interactions (*kbf, fpm, fpv*) the results are μ_{calc}= 2.0452, 1.8202 and 1.4358 nm, respectively, where are overestimated with the experimental value 1.24(6) nm [15]. The calculated magnetic moments using the effective nucleon g_2 factors $(g_l^p = 0.76, g_l^n = -0.32, g_s^p = 5.0, g_s^n = -3.99)$,$(g_l^p = 0.86, g_l^n = -0.11, g_s^p = 5.0, g_s^n = -3.9)$, $(g_l^p = 1.1, g_l^n = 0.0, g_s^p = 5.57, g_s^n = -3.85)$ and *kbf, fpm, fpv* interactions, these results are well agreement the experimental value 1.24(6) nm [15].

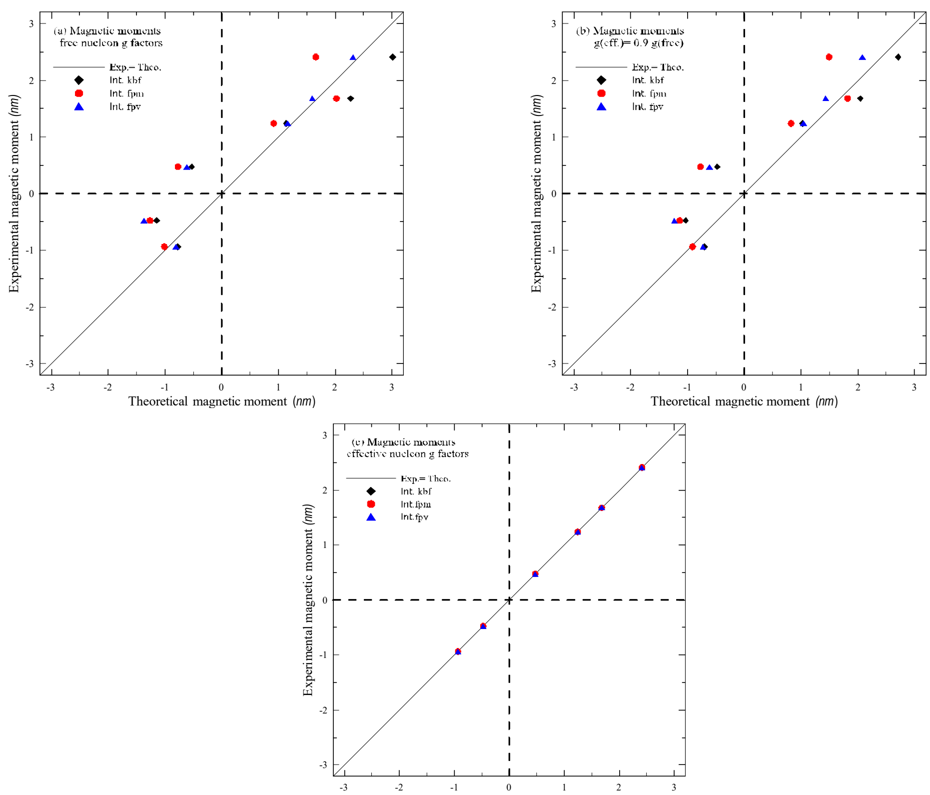

Fig. 1 – Comparison between the experimental and v magnetic moments μ, using (a) free nucleon g factors, b) g(eff) = 0.9 g (free), (c) effective nucleon g factors.
The experimental values are taken from Ref. [15].

Figure 1 represents the comparison between the experimental and calculated magnetic dipole moments for ^{49,50,51,52,53,54}Cr isotopes using free nucleon g factors, reduced effective nucleon g_1 factors, and effective nucleon g_2 factors. It is clear that three interactions are giving values compared with the experimental value. The deviation between the calculated and the experimental values is large when using the free nucleon g factors Fig. 1a and for g_1 factors Fig. 1b, but calculated values with all the interactions are the excellent agreement with the experimental values when using effective nucleon g_2 factors Fig. 1c.

3.2. ELECTRIC QUADRUPOLE MOMENT

The electric quadrupole moments are calculated for Cr isotopes in *fp* model space, using three interactions *kbf* [21], *fpm* [22], *fpv* [23] and the calculated values are compared with the experimental values of the quadrupole moments [15]. The calculations of the shell model are performed by NuShellX@MSU code [14] to obtain the *one body matrix elements* (OBME). The single particle-matrix elements can be calculated by size parameters *b* and the HO (*harmonic oscillator*). The size parameters *b* for some Cr isotopes are calculated which adopted on mass number (*A*) for these isotopes: $b = \sqrt{\frac{\hbar}{M_P\omega}}$, where $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$ (*Mp* = mass of proton) [16]. The effective charges for proton and neutron, e_p and e_n, are taken into account for polarization effects of the valence nucleons on the inert core in the shell model calculations [17]. Four sets of effective charges are adopted, first set of effective charges, the Bohr-Mottelson effective charges (B-M) [18] are calculated according to Eq. 8 for each isotopes in the present work and tabulated in Table 2, the second set of effective charges, *conventional effective charges* (CON) are $e_p = 1.3\ e$ and $e_n = 0.5\ e$ [10], the third standard effective charges (ST) are $e_p = 1.36\ e$ and $e_n = 0.45\ e$ [19], the fourth effective charges are taken from program Nushell are $e_p = 1.5\ e$ and $e_n = 0.5\ e$. Lastly the effective charges empirical (Emp) are $e_p = 1.1e$, $e_n = 0.1e$ [20]. The electric quadrupole moment (*Q. M*$_{calc}$) are calculated for $_{24}$Cr (A = 49, 50, 51, 52, 53 and 54) isotopes with *kbf* interaction [20]. The *Q. M*$_{calc}$ are calculated for (^{49}Cr;5/2$^-$) with B-M effective charges, CON effective charges, ST effective charges, the Nushell program effective charges and the Emp effective charges and using *kbf* interaction [20]. These results of the *Q. M*$_{calc}$ cannot be compared with the quadrupole moments experimental values (*Q. M*$_{exp}$) because we do not have experimental values so we need more search. The *Q. M*$_{calc}$ for (^{50}Cr;2$^+$) with, B-M, CON, ST, the Nushell program and Emp effective charges and using *kbf* interaction are –30.2, –29.9, –28, –30.9 and –18.4 $e\,fm^2$, respectively. These results are close to the *Q. M*$_{exp}$ is –36 ±7 $e\,fm^2$ [15] and within the experimental error which indicates an oblate deformation for ^{50}Cr isotope except results of the *Q. M*$_{calc}$ with ST and Emp effective charges do not agree with the *Q. M*$_{exp}$ as shown in Fig. 3a and Table 2. The calculated of the *Q. M*$_{calc}$ for (^{51}Cr; 7/2$^-$) cannot be compared with the *Q. M*$_{exp}$ because we do not have experimental values that need further research. The *Q. M*$_{calc}$ for (^{52}Cr; 2$^+$) with B-M, CON, ST, the

Nushell program and Emp effective charges and using the *kbf* interaction are 1.7, 1.59, 1.57, 1.75 and 0.94 $e\,fm^2$, respectively. These results of $Q.\ M_{calc}$ are not matched between theoretical and the $Q.\ M_{exp}$ is -8 ± 2 $e\,fm^2$ [15] which indicate a prolate deformation, although the isotope of ^{52}Cr is had magic numbers (neutron = 28) which show direct evidence of the collapse of $N = 28$ shell closure. The $Q.\ M_{calc}$ for (^{53}Cr; $3/2^-$) with B-M, CON, ST, the Nushell program and Emp effective charges and using the *kbf* interaction. These results of the $Q.\ M_{calc}$ are agreement with the $Q.\ M_{exp}$ [15] which indicate an oblate deformation for ^{53}Cr isotope while with Emp. effective charges, do not match between $Q.\ M_{calc}$ and $Q.\ M_{exp}$ [15] as shown in Table 2. The $Q.\ M_{calc}$ are calculated for (^{54}Cr; 2^+) with B-M, CON, ST, the Nushell program and Emp effective charges and using the *kbf* interaction. These results of the $Q.\ M_{calc}$ with Con, ST, Emp effective charges are not an agreement between $Q.\ M_{calc}$ and $Q.\ M_{exp}$ while results obtained from B-M and Nushell program effective charges with the $Q.\ M_{exp}$ [15] as shown in Fig. 3a. The results of the $Q.\ M_{calc}$ calculated for (^{49}Cr; $5/2^-$) isotope with set effective charges and *fpm* interaction [23] cannot be compared with the $Q.\ M_{exp}$ values because we do not have experimental values so we need more search as shown in Fig. 3b and Table 2. The $Q.\ M_{calc}$ for (^{50}Cr; 2^+) with B-M, CON, ST, the Nushell program and Emp effective charges and using *fpm* interaction are –34.9, –32.2, –32, –35.7 and –21.1 $e\,fm^2$, respectively. These results are close to the $Q.\ M_{exp}$ is -36 ± 7 $e\,fm^2$ [15] and within the experimental error. The calculated of the $Q.\ M_{calc}$ for (^{51}Cr; $7/2^-$) cannot be compared with the $Q.\ M_{exp}$ because we do not have experimental values for the ^{51}Cr nucleus that need further research. The calculated $Q.\ M_{calc}$ for (^{52}Cr; 2^+) with B-M, CON, ST, the Nushell program and Emp effective charges and using the *fpm* interaction are –12.5, –10.4, –10.1, –11.24 and –5.2 $e\,fm^2$, respectively. These results of $Q.\ M_{calc}$ are not matched between the $Q.\ M_{calc}$ and the $Q.\ M_{exp}$ is -8 ± 2 $e\,fm^2$ [15] which indicate an oblate deformation, except the $Q.\ M_{calc}$ of calculated with B-M, ST effective charges are agreement with the $Q.\ M_{exp}$. The breakdown of the $Z = 24$ magicity at $N = 28$ with *fpm* interaction seems linked to this behavior of $f_{7/2}$ orbit when it is full by a neutron [24]. The results of the $Q.\ M_{calc}$ for (^{53}Cr; $3/2^-$) are agreement with the experimental values [15] while with Emp. effective charges; do not match between the $Q.\ M_{calc}$ and the $Q.\ M_{exp}$ values [15]. The results of $Q.\ M_{calc}$ for (^{54}Cr; 2^+) calculated with CON, ST, Emp. effective charges which indicate a small oblate deformation and agree very well with the $Q.\ M_{exp}$ values [15] except effective charges are a result obtained from Emp effective charges overestimated with the $Q.\ M_{exp}$ [15] as shown in Fig. 3b. The results of the $Q.\ M_{calc}$ for (^{49}Cr;$5/2^-$) isotope with set effective charges and *fpv* interaction, cannot be compared with the $Q.\ M_{exp}$ values because we do not have $Q.\ M_{exp}$ values so we need more search. The results of the $Q.\ M_{calc}$ for (^{50}Cr;2^+) are close to the $Q.\ M_{exp}$ is -36 ± 7 $e\,fm^2$ [15] and within the experimental error except result of the $Q.\ M_{calc}$ obtained from Emp effective charges where overestimated with experimental values as shown in Table 2. The results of the $Q.\ M_{calc}$ for (^{52}Cr; 2^+) are not matched between the $Q.\ M_{calc}$ and the $Q.\ M_{exp}$ values [15] is -8 ± 2 $e\,fm^2$ [15] which indicate an oblate deformation and the breakdown of the $Z = 24$ magicity at $N = 28$ with *fpv* interaction. The results of $Q.\ M_{calc}$ for (^{53}Cr; $3/2^-$) obtained from B-M, CON, ST, Nushell program are an agreement with the experimental values [15] while the result of the *Q. M*$_{calc}$ with Emp effective charges were overestimated with the experimental value [15]. The results of *Q. M*$_{calc}$ for (^{54}Cr; 2^+) are calculated with CON, ST, Emp. effective charges which indicate a small oblate deformation and agree very well with the *Q. M*$_{exp}$ values [15] except result obtained from Emp effective charges where overestimated with the *Q. M*$_{exp}$ values [15] as shown in Fig. 3c.

Table 2

Quadrupole moments (units efm2) are calculated for some isotopes of Chromium (Cr, Z = 24) with *kbf* [20], *fpm* [22], *fpv* [23] effective interaction and by using, the B-M [18], the CON [10], the ST [19], from NuShellX code [14] and the empirical effective charges [20]. The experimental quadrupole moments are taken from Ref. [15]

Nucleus	J^π	***b***	**Eff. charge** e_p, e_n	***Q. M*$_{calc}$ (efm^2) kbf Inter.**	***Q. M*$_{calc}$ (efm^2) fpm Inter.**	***Q. M*$_{calc}$ (efm^2) fpv Inter.**	***Q. M*$_{exp}$ (efm^2) [15]**
^{49}Cr	$5/2^-$	1.99	1.17, 0.80	39.0	36.9	41.3	
			1.30, 0.50	36.0	34.0	37.7	
			1.36, 0.45	36.0	34.2	37.9	
			1.50, 0.50	40.5	37.82	41.9	
			1.10, 0.10	24.0	22.8	25.1	
^{50}Cr	2^+	2.0	1.16, 0.78	–30.2	–34.9	–31.7	–36 ± 7
			1.30, 0.50	–29.9	–32.2	–29.5	
			1.36, 0.45	–28.0	–32.0	–29.6	
			1.50, 0.50	–30.9	–35.7	–32.7	
			1.10, 0.10	–18.4	–21.1	–19.6	
^{51}Cr	$7/2^-$		1.15, 0.75	27.3	24.0	27.5	
			1.30, 0.50	25.0	21.2	24.6	
			1.36, 0.45	24.9	21.0	24.3	
			1.50, 0.50	27.5	23.3	27.0	
			1.10, 0.10	15.6	12.6	14.8	
^{52}Cr	2^+	2.01	1.14, 0.73	1.77	–12.5	–2.60	–8 ± 2
			1.30, 0.50	1.59	–10.4	–2.10	
			1.36, 0.45	1.57	–10.1	–2.00	
			1.50, 0.50	1.75	–11.2	–2.27	
			1.10, 0.10	0.94	–5.20	–0.98	
^{53}Cr	$3/2^-$	2.016	1.13, 0.71	–11.2	–10.9	–10.6	–15 ± 5
			1.30, 0.50	–11.0	–10.7	–10.1	
			1.36, 0.45	–11.1	–10.8	–10.2	
			1.50, 0.50	–12.3	–11.9	–11.3	
			1.10, 0.10	–7.50	–7.20	–6.60	
^{54}Cr	2^+	2.02	1.12, 0.70	–13.5	–14.1	–15.3	–21. ± 8
			1.30, 0.50	–12.6	–13.2	–14.1	
			1.36, 0.45	–12.5	–13.0	–13.9	
			1.50, 0.50	–13.9	–14.5	–15.5	
			1.10, 0.10	–7.50	–7.80	–8.20	

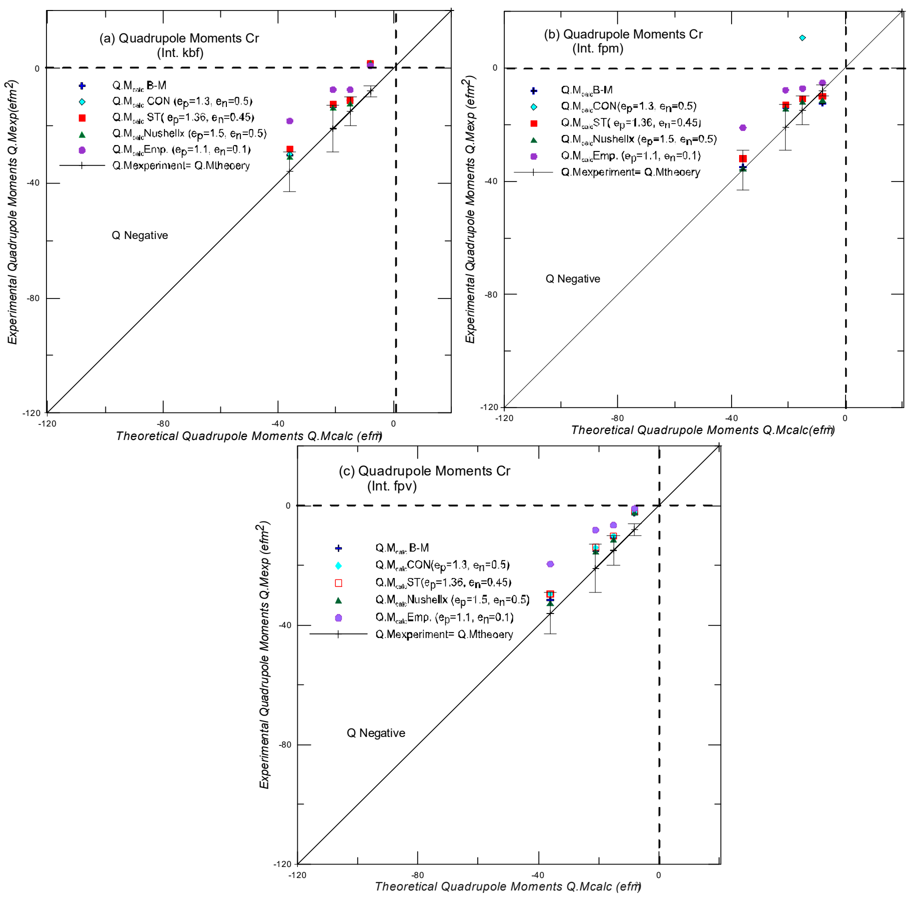

Fig. 2 – Comparison between the experimental values ($Q.\ M_{exp}$) [15] and the calculated of quadrupole moments ($Q.\ M_{calc}$) for Cr (*A*= 49, 50, 51, 52, 53 and 54) isotopes using B-M [18], the CON [10], the ST [19], from NuShellX code [14] and the empirical effective charges [20], and using (a) the *kbf* interaction [21], (b) the *fpm* interaction [22], (c) the *fpv* interaction [23].

4. CONCLUSIONS

We have efficiently calculated the magnetic dipole moment based on *fpv* interaction for free nucleon g factors obtaining a good agreement with the experimental values, ranging within the experimental errors for all Cr isotopes, except ^{53}Cr isotope. The magnetic dipole moments by the inclusion of the core polarization with reduced effective free nucleon g factors to 0.9 free nucleon g factors, which increases the discrepancy with the experimental value, but when using effective nucleon g factors, the results are adequate to obtain a good agreement between the predicted and experimental values. All theoretical results of the quadrupole moments (*Q. M_{calc}*) using, B-M, CON (e_p = 1.3 *e* and e_n = 0.5 *e*), ST (e_p = 1.36 *e* and $e_{\rm n}$ = 0.45 *e*), Nushell program ($e_{\rm p}$ = 1.5 *e* and e_n = 0.5 *e*), Emp. (e_p = 1.1 *e* and $e_{\rm n}$ = 0.1 *e*) effective charges and the *kbf*, *fpm* and *fpv* interactions are well agreement with the experimental values of the quadrupole moments except ^{52}Cr. The theoretical calculations of the quadrupole moment (*Q.* $M_{\rm calc}$) using empirical effective charges and interactions *kbf, fpm* and *fpv* with the experimental values of the quadrupole moment are not agreement. The better results of the *Q. M_{calc}* obtained from B-M, Con, ST, Nushell program and using *fpm* interaction. The investigation of the observed breakdown of the N = 28 magicity in this region ^{52}Cr.

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